Homework 1: DOC 2023, Biomedical Imaging

Due: May 17th, 2023. Version 2.0

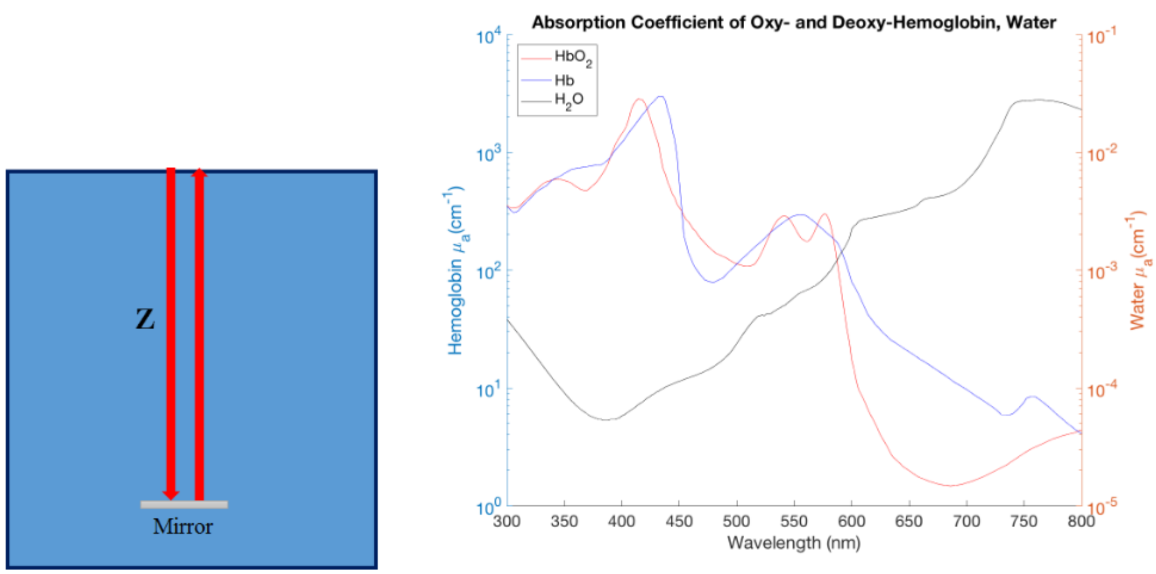
Instructions

Please show work for all problems, and email solutions to ***hall.er@northeastern.edu*** as ***"Homework_#_Group_#.pdf"***.

1 Beer's Law

1.1 Oxygen Sensor

You are working at a fancy startup, which is working on adding oxygen concentration sensing to their device. However, your company is unsure what wavelength of light to use. To find out, you design the following experiment.



(a) (b) Data from www.omlc.org

Figure 1: Experiment Design (1a), and Absorption Coefficients (1b).

As seen in Figure 1, a mirror is placed inside a tank of fluid at a certain distance Z from the top surface, where both the laser and detector are placed (you may assume they are at the same location). The laser emits 10mW of power.

1.1a.) What is the maximum depth the mirror can be placed ***in de-oxygenated blood*** to where 1% of power can still be recorded at the detector? Calculate for the following wavelengths:

i.) $\lambda = 650nm$

ii.) $\lambda = 550nm$

iii.) $\lambda = 450nm$

1.1b.) Repeat for ***oxygenated*** blood.

1.1c.) What is the difference between the two?

1.1d.) Which wavelength would you choose for this device, and why?

2 Lenses and X-Rays (Diffraction)

You have just finished creating a lens with NA = .8, and are now curious what spot sizes it will create with different lasers. You design an experiment according to figure 2 below.

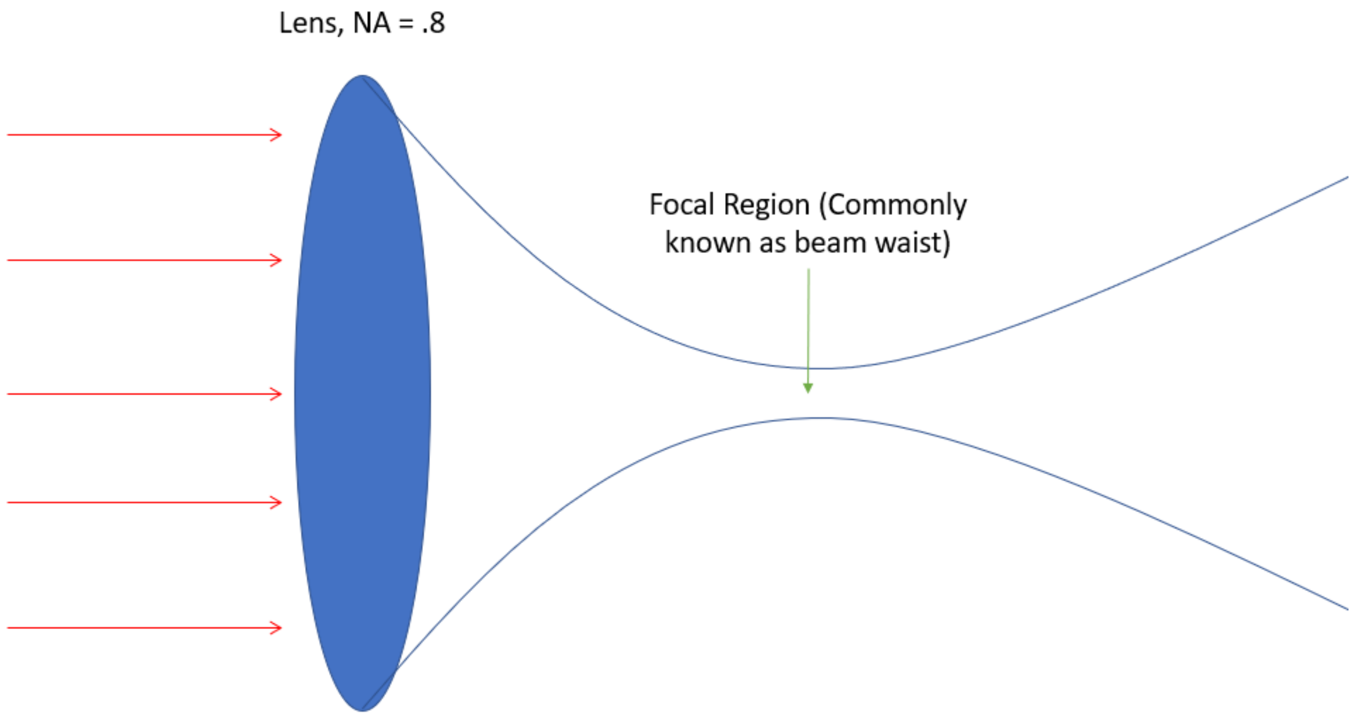


Figure 2: Simple Lens Experiment. Light rays enter parallel to each other, and then are focused to a region commonly known as the beam waist

2.0a.) For you lens, what is the estimated spot size for the following wavelengths?

i.) $\lambda = 400nm$

ii.) $\lambda = 580nm$

iii.) $\lambda = 1220nm$

2.0b.) If you decrease the NA of your lens, how will this affect your spot size?

2.0c.) Eric's cat Winston is in for his routine vet visit, and needs a chest X-Ray. Winston has been working very hard on his diet, and it can be assumed that his chest is 10cm wide. If the X-Ray machine has a wavelength of $1nm$, how small should the spot size be to make sure the depth of focus is at least 1 Winston wide? You can assume $n = 1.5$

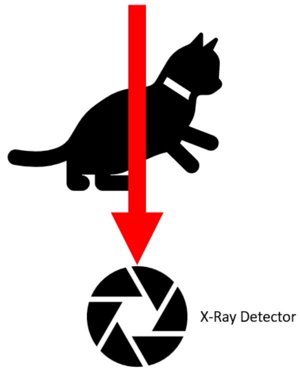


Figure 3: X-Ray for Winston. He is 10cm wide.

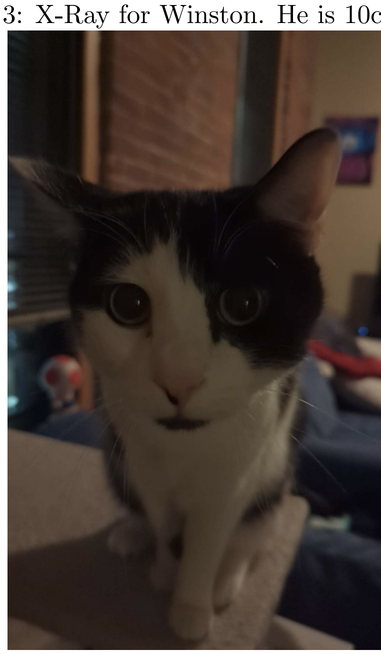


Figure 4: "Feed me silly human"

3 Interference

Light's ability to interfere with itself creates interesting ways to measure distances and acquire images. One of the most famous examples is the Michaelson Interferometer, which is shown in a great example by Dr. James Wyant at the University of Arizona. Here is the link to this example: Michelson Interferometer.

The intensity at the detector depends on how different the two beampaths, B1 and B2, are from each other. The equation given by Prof. Wyant gives the intensity at the detector as a function of d, the difference between the optical path distance of B1 and B2.

3.0a.) Using the provided MATLAB program "interference.m", show the intensity at the following values of d.

i.) $d = 0$

ii.) $d = \lambda$

iii.) $d = 2\lambda$

iv.) $d = 10\lambda$

3.0b.) Why is the intensity zero when the beam paths are the same?

4 Fiber Optics (Reflection)

Fiber optics are quickly becoming a vital part of the medical imaging sector, as they can provide near loss-less power transfer of light over several meters (and beyond!) in a compact form-factor. But how do they work? Consider the following figure:

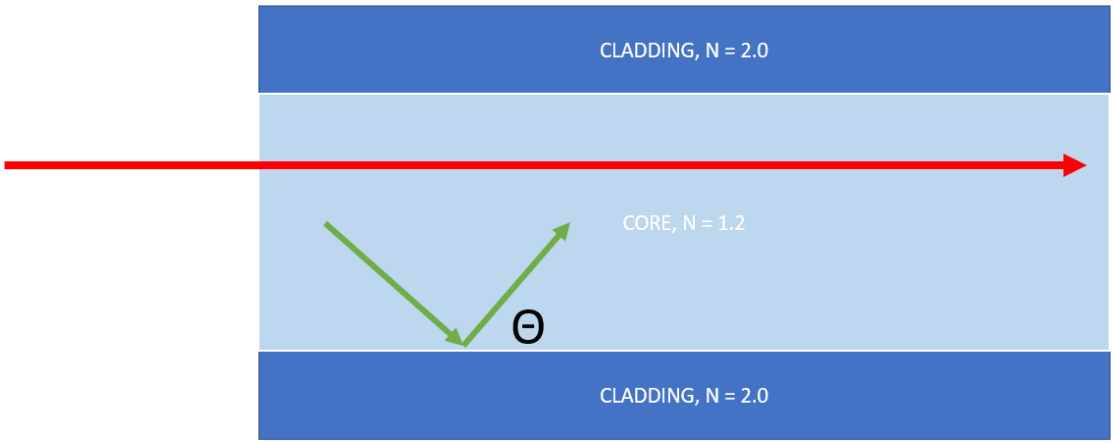


Figure 5: Demo of an optical fiber. The light parallel to the fiber propagates down without attenuation (red). Light with a certain angle θ will continue down the fiber as well, until a critical angle is hit.

Light enters the middle of the fiber, commonly known as the optical core. If the light is perfectly straight (red line), it will travel down the fiber until the end. However, what if the light is not traveling straight, like the green line? It approaches the second layer, commonly known as the cladding of the optical fiber. By varying the indices of refraction and restricting the angle of incoming light, the light will bounce off of the cladding and continue down the fiber. Science!

4.0a.) If $n_1 = 1.2$ and $n_2 = 2.0$, what is the maximum angle θ_{max} where refraction does not occur (all light is reflected back down the fiber)? Critical Angle is given by:

$$\sin \theta_c = n_1/n_2$$

4.0b.) For angles 0° to 90°, calculate and plot the reflection as a function of the incident angle θ (When you submit your homework, you can just attach a picture of your plot. Make sure to label everything!). To do this, you can use the fresnel.m MATLAB code provided by Chuck on the course website. If we assume the light is unpolarized, we can say:

$$R = (\rho_s^2 + \rho_p^2)/2$$

In addition, the code inputs are θ in *degrees*, and the ratio of indices of refraction given by:

$$n_r = n_1/n_2$$