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Biomedical Imaging

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EECE–4649
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Biomedical Imaging

- Principles of Biomedical Imaging (EE/Physics)
- Image Processing (EE/Math and CE/CS)
- Collecting the Image (eg. X–Ray technician)
- Reading the Image (eg. Radiologist)
- Making Decisions

Course Components

- Lectures
- Slides
- Workshops
- Homework (Breadth)
- Projects and Oral Report (Depth)
- Participation
- Teamwork

Course Topics

- Background Material
  - Wave Theory. Tissue Properties.
  - Absorption, Scattering, and Reflection
  - Contrast, Resolution, and penetration
- X–Ray, X–Ray CT
- MRI
- Inverse Problems
- Ultrasound
- Optics
  - Microscopy in the laboratory
  - *In–vivo* Microscopy
  - Optical Coherence Tomography
- Endoscopy
- Experimental Techniques

A Note About the Slides

These slides are not meant to be complete. They are intended to support the lectures, and not to replace them.

They provide reminders to me and to you.

I encourage you to save the slides to paper or pdf, and take notes on them.

Try This Learning Approach

- There are a lot of equations here and it's easy to become confused.
- As engineering students, you can learn the equations when you need them.
- In this course, you have help when you need it.
- In the lectures, concentrate on the concepts and don't get stuck on the equations.
- When you solve a problem, learn the equations you need.

Communication

- I am hearing impaired.
- I can understand you better if I can see your face.
- I hear better on my right side.
- I hear better in a quiet environment.
- I hear better if one person speaks at a time.
- I have a portable microphone that can help.
- Do not let this stop you from asking questions.

Common Themes

- Contrast
- Resolution (x, y, z, t)
- Penetration
- Other Issues
  - Invasiveness
  - Equipment Cost
  - Reimbursement Issues
  - Size
  - Safety
  - Complexity
  - Speed
  - Many More . . .

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Common Math: Waves

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Waves $\frac{\partial^2 ?}{\partial t^2} = c^2 \frac{\partial^2 ?}{\partial z^2}$ *etc.*

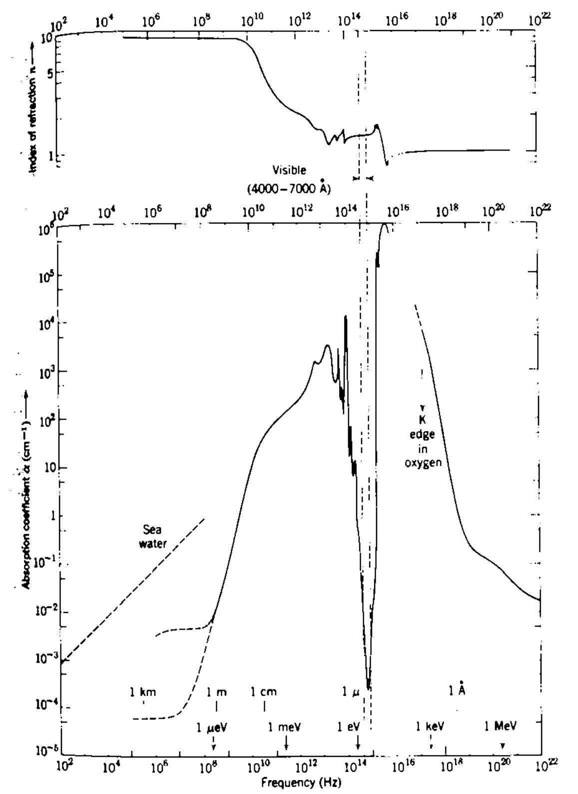
Ocean Waves in Space

λ is Wavelength

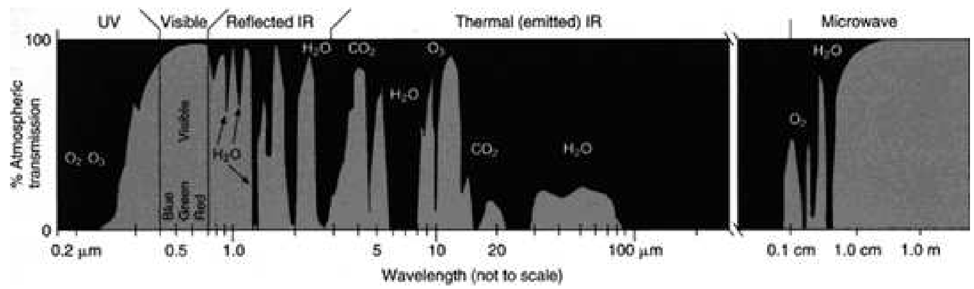
Ocean Waves in Time

T is Period. $f = 1/T$ is frequency.

EM Spectrum



A. Liquid Water



B. Water Vapor

ELECTROMAGNETIC TRANSMISSION. Water strongly absorbs most electromagnetic waves, with the exception of wavelengths near the visible spectrum (A, From Jackson *Classical Electrodynamics*, ©1975). The atmosphere also absorbs most wavelengths, except for very long wavelengths and a few transmission bands (B, NASA's Earth Observatory).

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Earthlight

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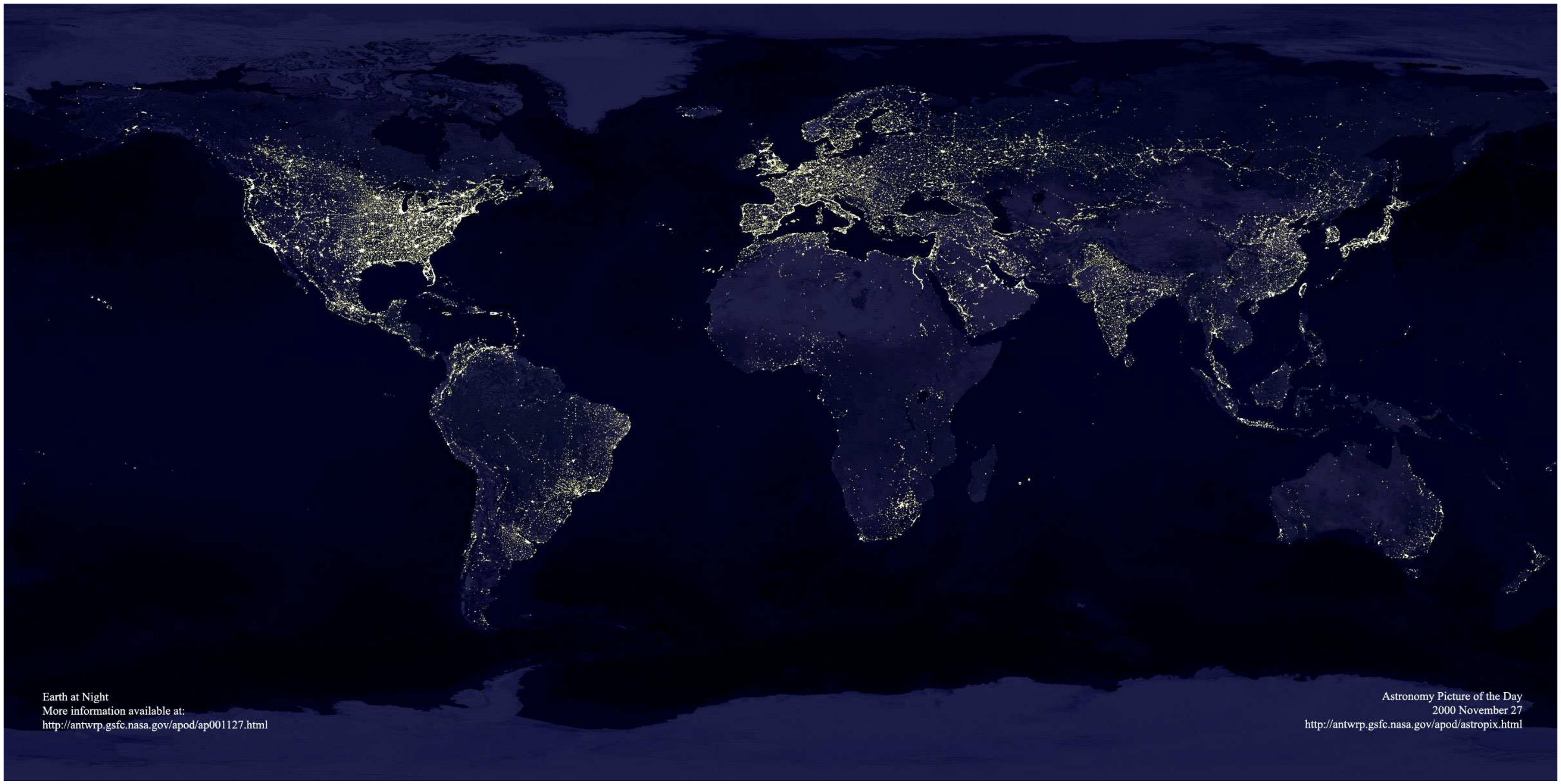

C. Mayhew & R. Simmon (NASA/GSFC), NOAA/ NGDC, DMSP Digital Archive).

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Electromagnetic Waves & Electrons

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A Wave is a Wave is a Wave (EM, US, Other)

$$E = E_0 e^{j\omega t - kz} \qquad p = p_0 e^{j\omega t - kz} \qquad h = h_0 e^{j\omega t - kz}$$

Electromagnetic Waves Interact with Electrons

X–Ray	Ultraviolet	Visible/Near–Infrared
picometers	nanometers	nano– to micrometers
Ionization	Orbit to Orbit	Molecules

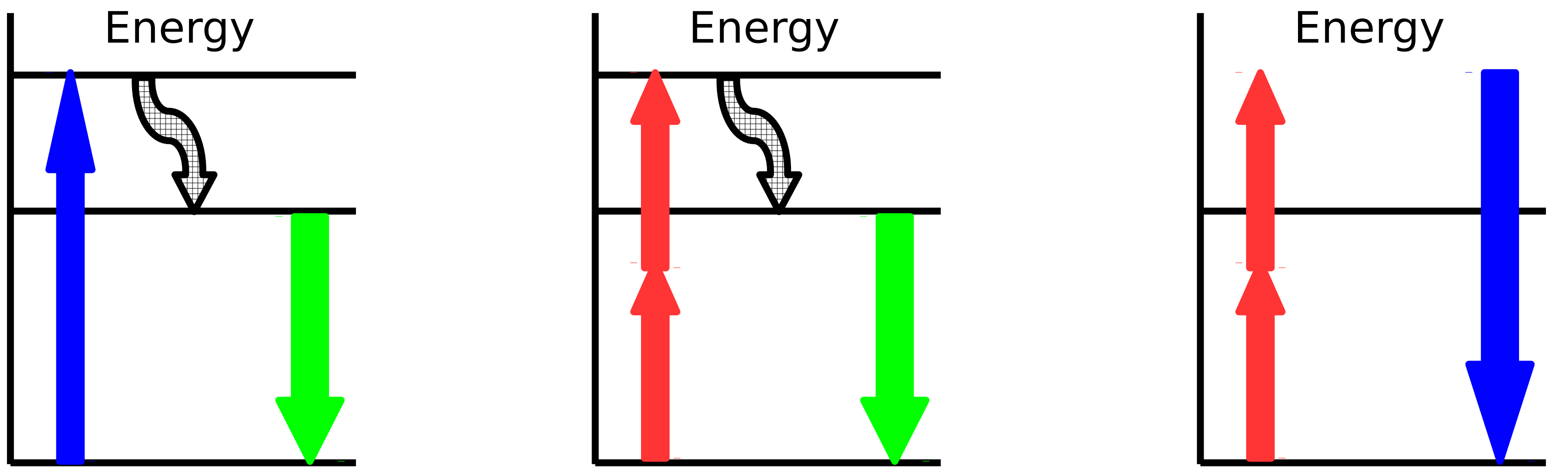
Infrared	Radio Frequency
micrometers	meters (kHz, MHz)
Vibration/Rotation	Nuclear Spins

Energy Levels and Transitions

Absorption, ↑ Stimulated Emission, ↓ Spontaneous Emission ↓ nanoseconds?

Fluorescence, Harmonic Generation, Mixing, and more

Important: Discrete Levels Depend on Material



Fluorescence 2–Photon Fluorescence Second Harmonic

State Populations

Equilibrium:

Boltzman Distribution

$$N\left(E_n\right) \propto e^{-E_n/kT} \qquad kT = 25\text{meV @ } T = 300K$$

$\lambda = 500$nm	2.5eV	$e^{-E_n/kT} = e^{-100} \approx 10^{-43}$
5μm	250meV	10^{-5}
50μm	25meV	$e^{-1} = 0.37$
5mm	250μeV	0.99

$$E = h\nu = \frac{hc}{\lambda}\text{ Joules} \qquad \frac{hc}{e\lambda}\text{ electronVolts(eV)}$$

Absorption and Emission

- Large Population Difference (eg. High Frequency)
  - Many Electrons in Lower State
  - Many Absorption Events (Transition Up)
- Small Population Difference (eg. Low Frequency)
  - Near Equal Absorptions and Emissions
- Population Inversion (Non–Equilibrium)
  - More Emission than Absorption
  - Gain (eg. Laser)

Irradiance

Circuits	EM Waves	
$V = IR$	$\left\|\vec{E}\right\| = \left\|\vec{H}\right\| Z$	$Z = \sqrt{\frac{\mu}{\epsilon}}$
V in Volts	$\vec{E}$ in $\frac{\text{Volts}}{\text{meter}}$	
I in Amperes	$\vec{H}$ in $\frac{\text{Amperes}}{\text{meter}}$	
R in Ohms	Z or η in Ohms	
$P = IV$	$\vec{S} = \vec{E} \times \vec{H}$	
	$\left\|\vec{S}\right\| = \left\|\vec{E}\right\| \left\|\vec{H}\right\|$	For Plane Wave ($\vec{E} \perp \vec{H}$)
$P = \frac{V^2}{R}$	$I = \left\|\vec{S}\right\| = \frac{\left\|\vec{E}\right\|^2}{Z}$	Irradiance
P in Watts	I in $\frac{\text{Watts}}{\text{meter}^2}$	

Wave Interference and Coherence

- Sum of Two Waves

$$E = E_1 + E_2 \qquad I = |E_1 + E_2|^2$$

- Expand

$$I = E_1^2 + E_2^2 + E_1 E_2^* + E_1^* E_2$$

- Coherent Sum depends on phase

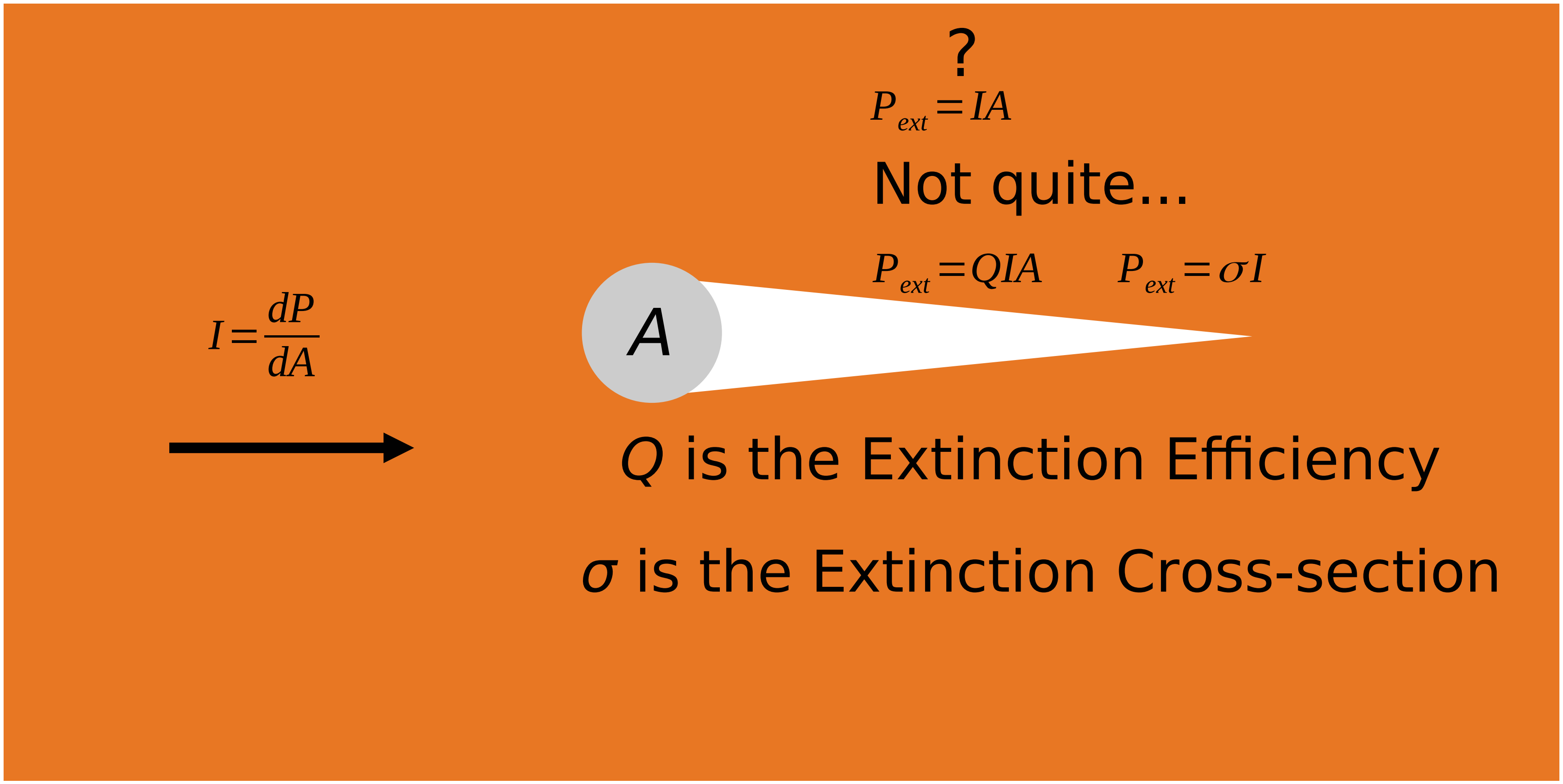
$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta\phi$$

$$\max I = I_1 + I_2 + 2\sqrt{I_1 I_2} \qquad \min I = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

- Incoherent Sum (Average, but over what time?)

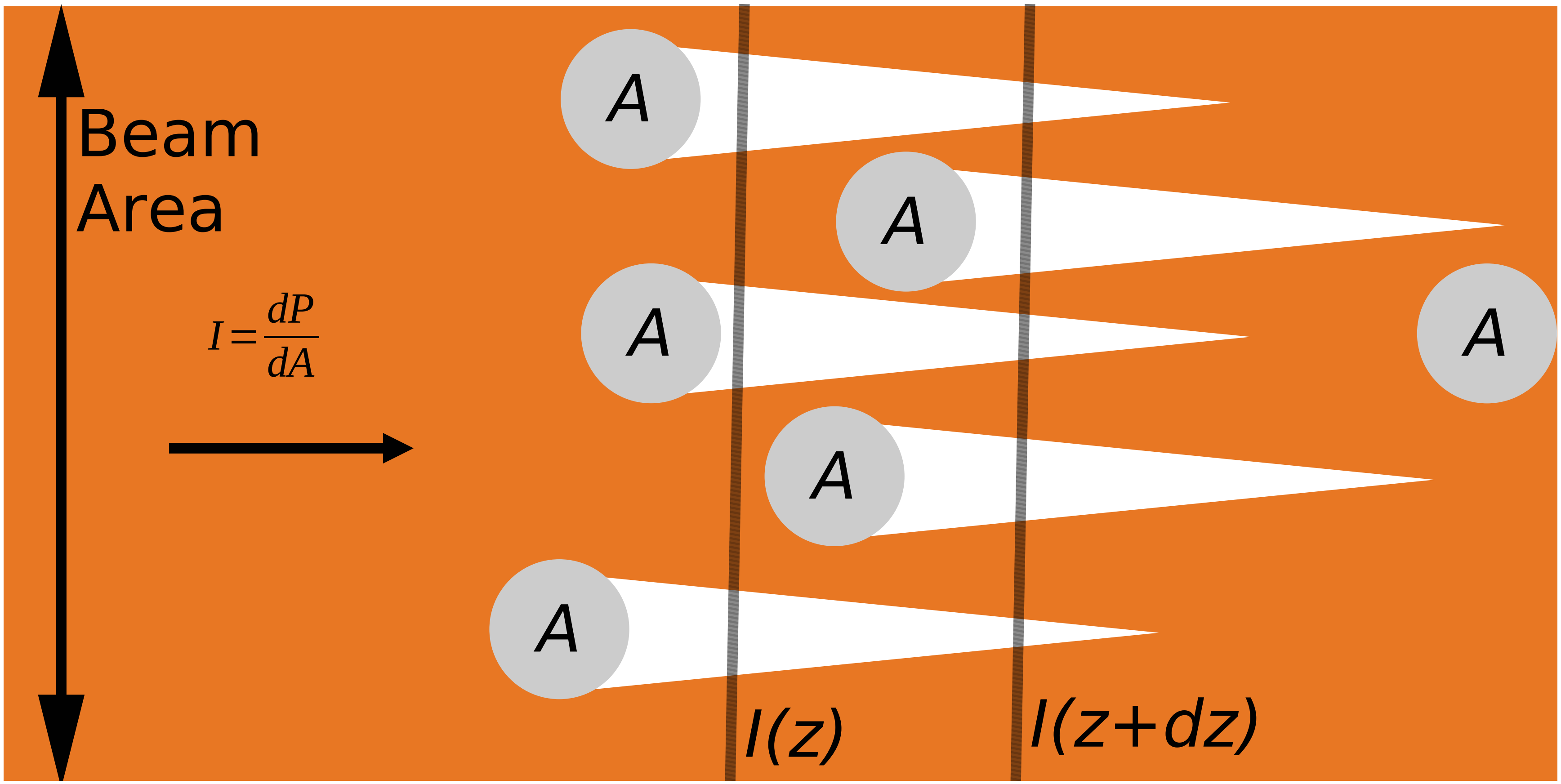
$$\bar{I} = I_1 + I_2 \quad \text{because} \quad \overline{E_1 E_2^*} = 0$$

Interaction with Scatterer



Extinguished light is either scattered or absorbed

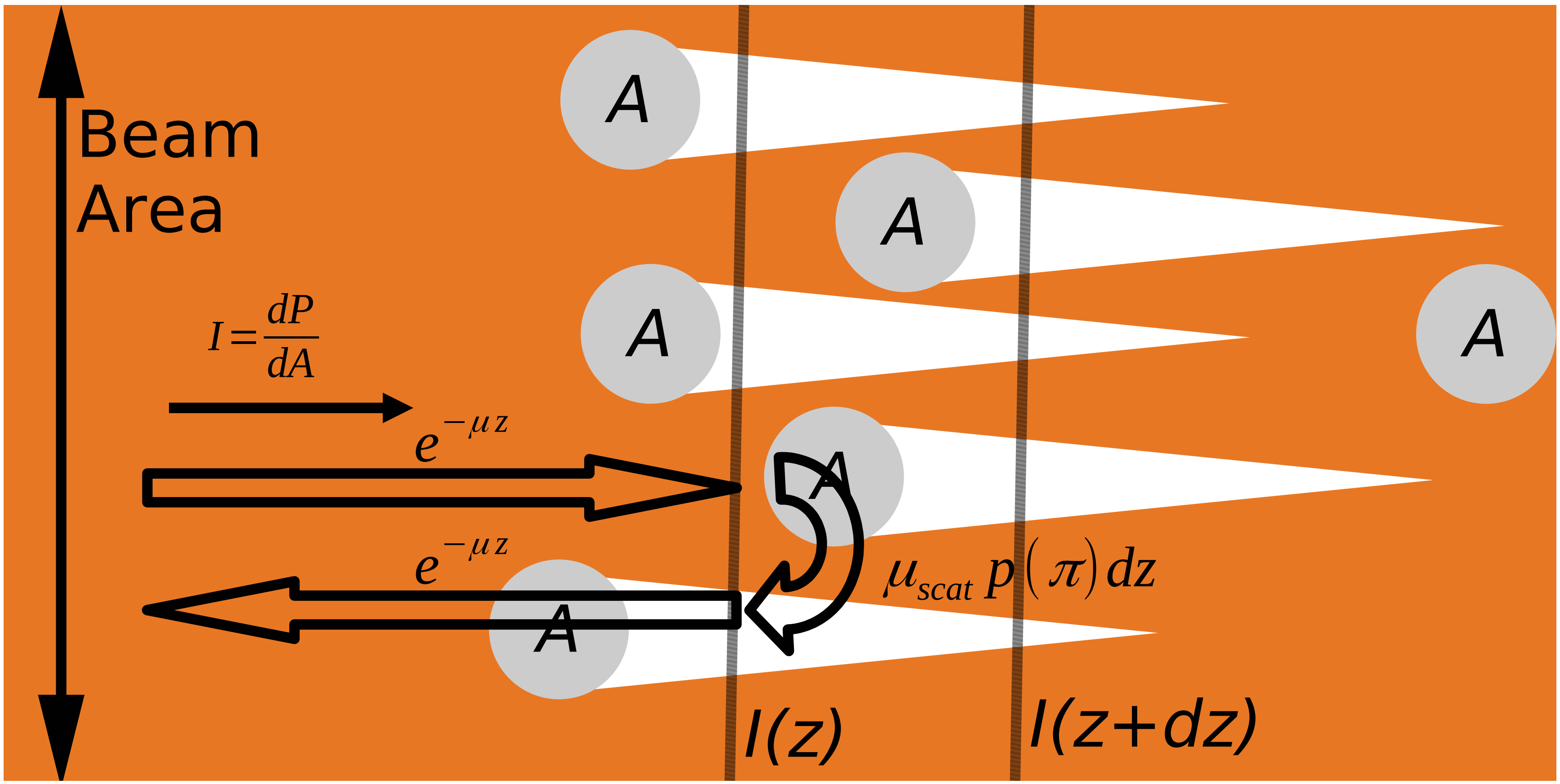
Multiple Scatter (Weak)



Assume light interacts with zero or one particle.

$$P_{ext} = I\sigma N \qquad dP_{ext} = I\sigma N_v A_{beam} dz$$

Backscatter from Slab



Absorption

- Absorption Cross–Section, σ_a, and Coefficient, μ_a
  - Absorption Efficiency, Q_a

$$P_a = \sigma_a I = Q_a \pi r^2 I \qquad \mu_a = N_V \sigma_a$$

- Beer's Law for Absorption
  - Power In: $P(z) = I(z) A$
  - Power Absorbed: $dP = I(z) N \sigma_a \quad N$ absorbers
  - Number: $N = N_V \times A dz$
  - Power Out: $P(z + dz) = P(z) - dP$

$$\frac{dP}{dz} = -P(z) \mu_a$$

$$P(z) = P(0) e^{-\mu_a z} \qquad I(z) = I(0) e^{-\mu_a z}$$

Absorption and Scattering

- Absorption Cross–Section, σ_a, and Coefficient, μ_a

$$P_a = \sigma_a I = Q_a \pi r^2 I \qquad \mu_a = N_V \sigma_a$$

- Scattering Cross–Section, σ_s, and Coefficient, μ_s

$$P_s = \sigma_s I = Q_s \pi r^2 I \qquad \mu_s = N_V \sigma_s$$

- Extinction Cross–Section, σ, and Coefficient, μ

$$P_{extinguished} = \sigma I = Q \pi r^2 I \qquad \mu = N_V \sigma$$

$$\sigma = \sigma_a + \sigma_s \qquad \mu = \mu_a + \mu_s$$

- Beer's Law Again

$$\frac{dP}{dz} = -P(z)\,\mu$$

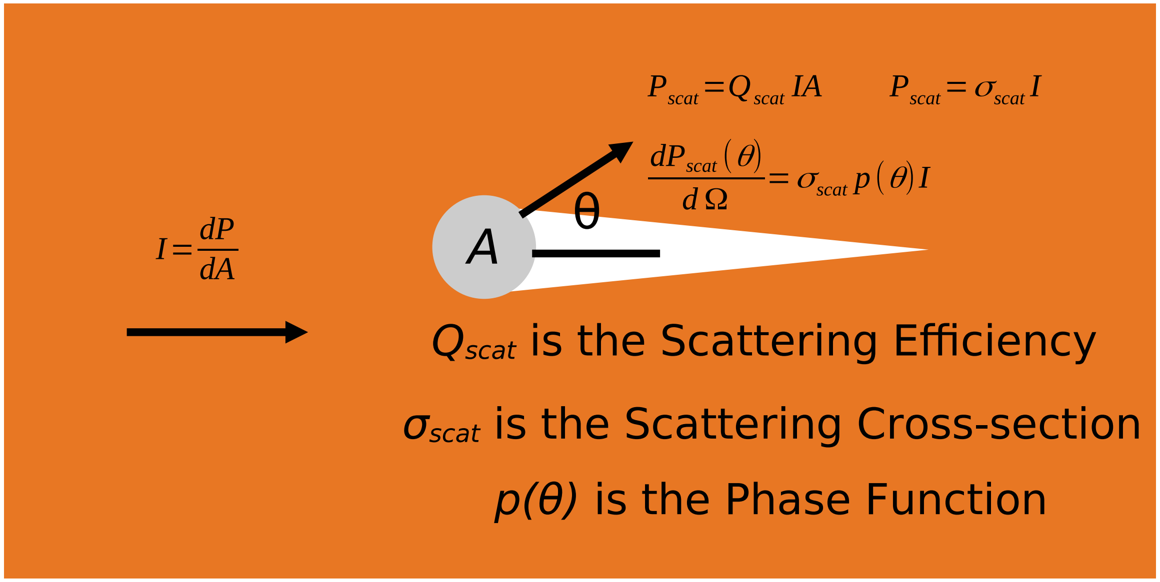
$$P(z) = P(0)\,e^{-\mu z} \qquad I(z) = I(0)\,e^{-\mu z}$$

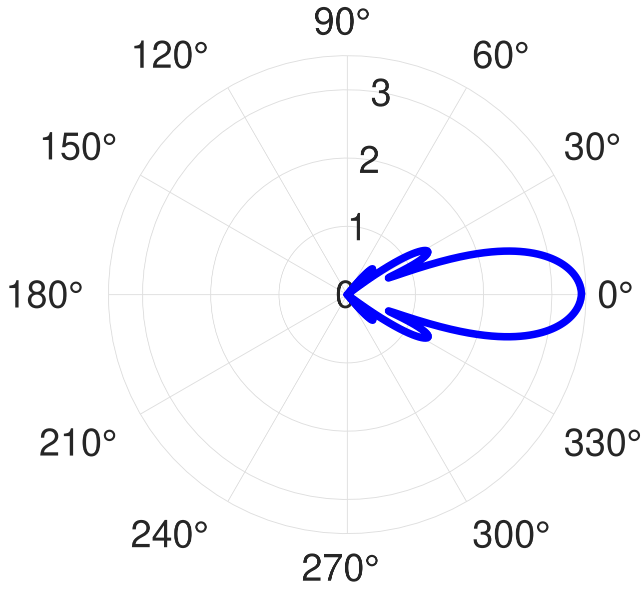
Phase Function

- Where does the Scattered Power go? Scattered Intensity

$$\frac{dP_{scat}}{d\Omega}$$

- Intensity is Power per Unit Solid Angle (*e,g.* Watts/steradian)
- Often Plotted in Polar Coordinates





Scattered Wave

- Where does it go? It's Complicated

$$\mu_s p(\theta, \zeta) \qquad \text{Phase Function}$$

  - Small Particles: Rayleigh Scatter (uniform, $1/\lambda^4$)
  - Large Particles: Forward, independent of wavelength
  - Smooth Surfaces: Fresnel Reflection and Transmission (Impedance Contrast)
- Anisotropy and Transport Scattering Coefficient

$$\mu_s' = (1-g)\mu_s \qquad g = \langle \cos\theta \rangle = \int\int p(\theta, \zeta)\cos\theta d\zeta d\theta$$

- Detailed Calculations
  - Spheres, Cylinders, *etc.*:
    Mie Scattering; https://omlc.org/calc/mie_calc.html
  - Other Shapes: FDTD, Other

How Much Is Scattered Forward?

- Forward Contribution

$$I_{forward} = I(\theta)\cos\theta$$

- p as a Probability Density

$$\int p(\theta)\, d\Omega = 1$$

- Mean Cosine of Scattering Angle

$$\overline{\cos\theta} = \int \cos\theta \times p(\theta)\, d\Omega$$

- Anisotropy

$$g = \overline{\cos\theta}$$

- $g \approx 1$ for forward, $g \approx 0$ for uniform, $g \approx -1$ for backward

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Scattering and Absorption

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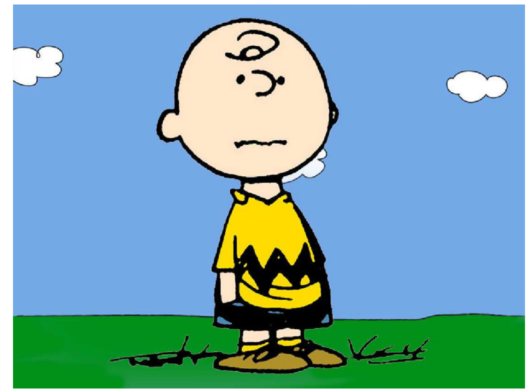
- Multiple Scattering
  - More Chances for Exit (T or R)
  - More Chances for Absorption
- Light Diffusion
- Albedo

$$W = \frac{\text{Scattering}}{\text{Extinction}} = \frac{\mu_s}{\mu_s + \mu_a}$$

Albedo and Charlie Brown

"Doing a good job is like wetting your pants in a dark suit. . . You get a warm feeling, but nobody notices."
Can we explain it with our knowledge of scattering and absorption?



Light Suit

μ_s Large (Fibers in Air)

μ_a Small

$$W = \frac{\mu_s}{\mu_s + \mu_a} \text{ Large}$$

Fibers in Water:

$$\mu_s \downarrow \qquad W = \frac{\mu_s}{\mu_s + \mu_a} \downarrow$$

Dark Suit

μ_s Large (Fibers in Air)

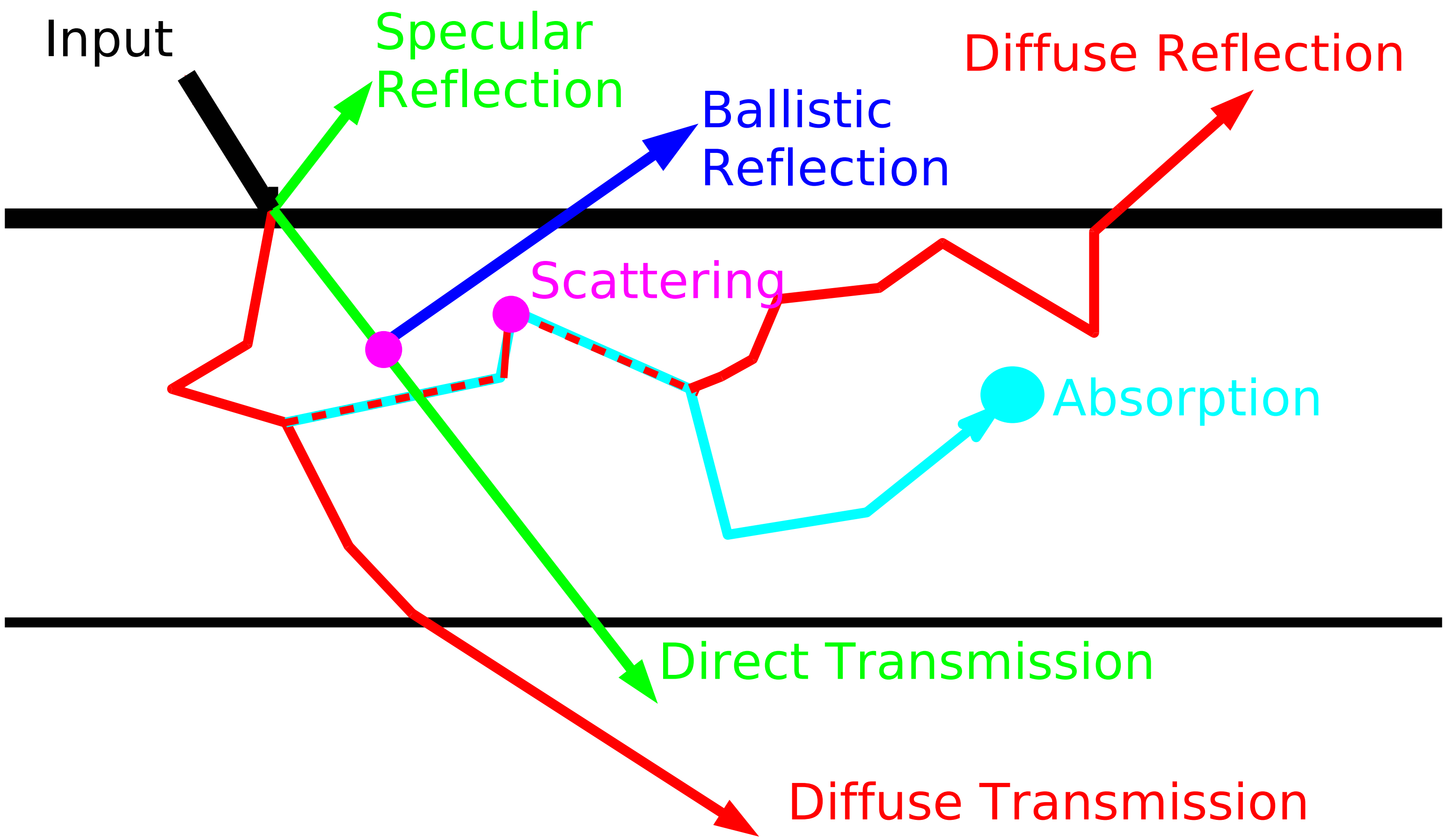
μ_a Very Large

$$W = \frac{\mu_s}{\mu_s + \mu_a} \approx 0$$

Fibers in Water:

$$\mu_s \downarrow \qquad W = \frac{\mu_s}{\mu_s + \mu_a} \approx 0$$

Waves Interactions



(and of course, emission)

Transmission

- Fresnel Reflection
- Absorption: Beer's Law

$$P(z) = P(0)\, e^{-\mu z}$$

- Phase
  - Speed $= c/n$ where n is the index of refraction
  - Time = distance/speed
  - Phase $= 2\pi\times$ Time/period

$$\phi = 2\pi \frac{\int n d\ell}{\lambda}$$

- Refraction: Snell's Law

$$n \sin\theta = n'\theta'$$

Reflection

S Polarization Electric Field

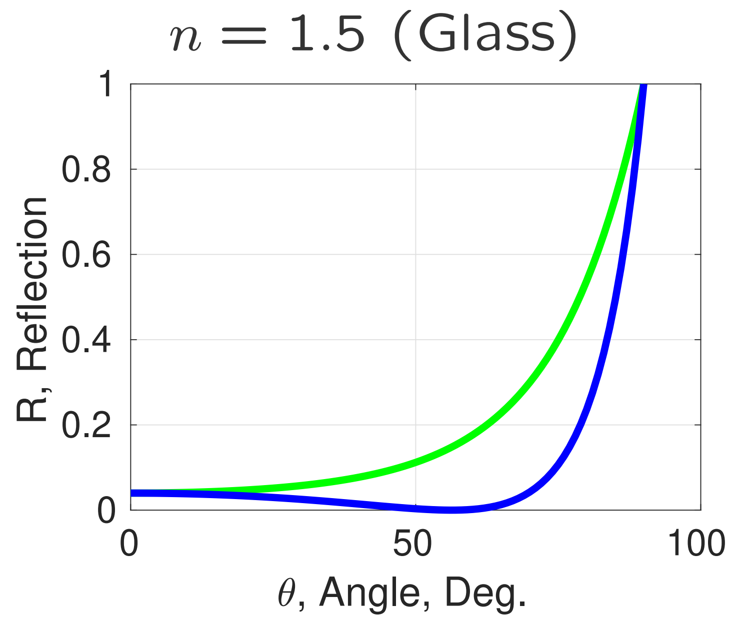
$$\rho_s = \frac{E_r}{E_i} = \frac{\cos\theta_i - \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}}$$

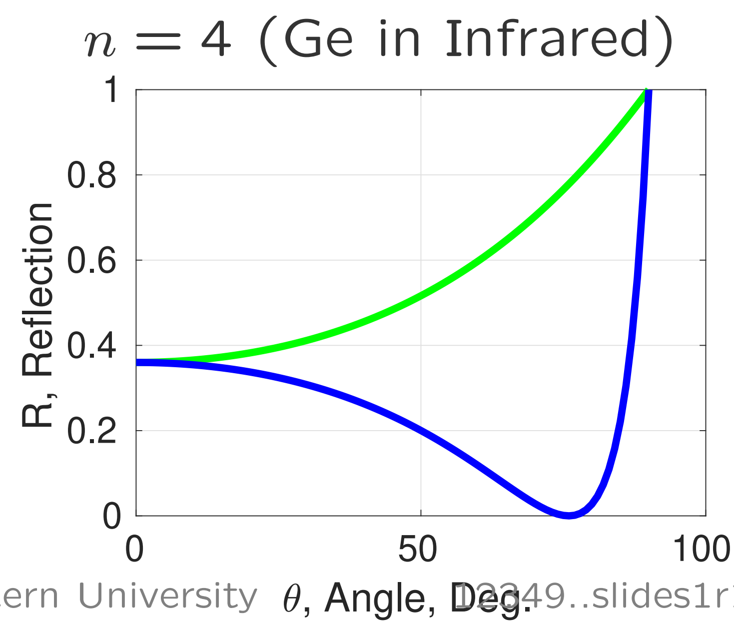
P Polarization Electric Field

$$\rho_p = \frac{\sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i} - \left(\frac{n_2}{n_1}\right)^2 \cos\theta_i}{\sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i} - \left(\frac{n_2}{n_1}\right)^2 \cos\theta_i}$$

Irradiance

$$R = |\rho|^2$$





Diffraction

- Angular Divergence

$$\alpha \approx \frac{\lambda}{D}$$

- Spot Size

$$d \approx \frac{\lambda}{D} z$$

- Numerical Aperture

$$d \approx \frac{\lambda}{NA} \text{ (Resolution)} \qquad NA = n \sin\theta \approx n \frac{(D/2)}{z}$$

$$\Delta z \approx \frac{n\lambda}{NA^2} \text{ (Depth of Focus)}$$

Example: $\lambda = 10^{-6}\text{m}$ $\quad D = 10^{-4}\text{m}$

$\alpha = 10^{-2}\text{radians}$ $\quad d = 10^{-2}\text{m @ 1m}$ $\quad (NA \approx 5 \times 10^{-5})$

What is Imaging?

- Wave Source(s)
- Path to Target (Diffraction, Absorption, Scattering)
- Target Contrast (Absorption, Scattering, Fluorescence, Life-time, Phase, *etc.*)
- Path from Target (Diffraction, Absorption, Scattering)
- Detector(s)
- Signal Processing (Inverse Problem)
- Decision

Decision, Decisions

Suppose we Can Find Probability Density of Detection and False Alarm. How do we (1) Make Good Decisions with a Measurement and (2) Compare Different Instruments?

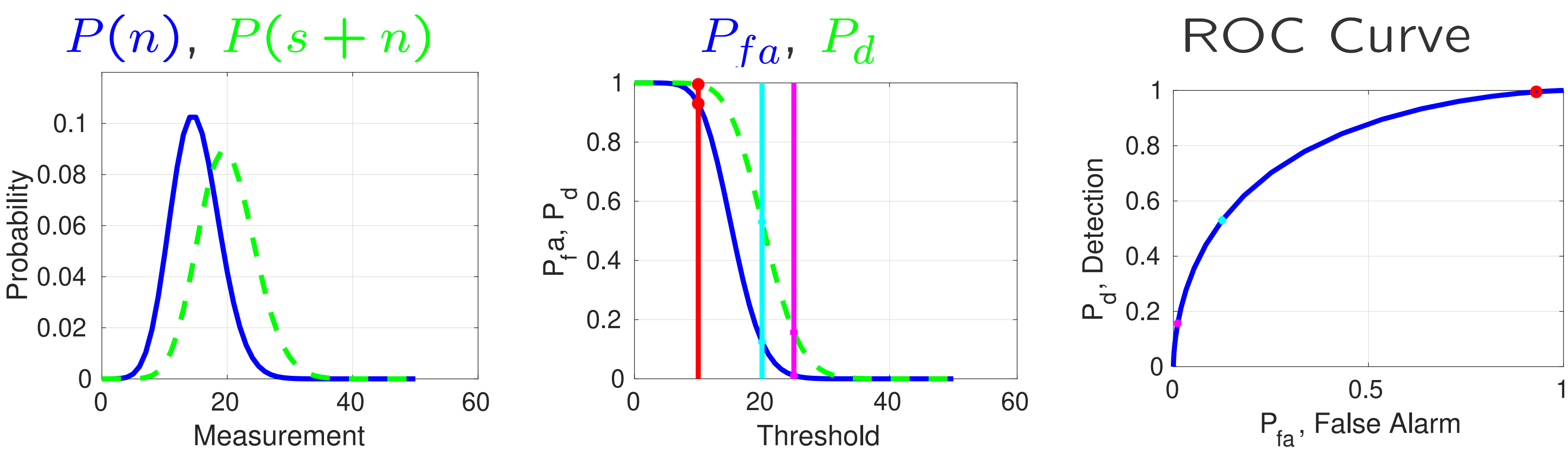
- Cost Function
  - Set a Threshold and Compute "Cost"
  - Vary the Threshold to Minimize Cost

$$C = C_{miss} P_{disease} \left(1 - P_{Detection}\right) + C_{false} \left(1 - P_{disease}\right) P_{false}$$

  - But How do we Decide the Costs?
- Receiver Operating Characteristic
  - Plot with Threshold as a Parameter
    - Probability of False Alarm (1-specificity) Horizontal
    - Probability of Detection (sensitivity) Vertical
  - Measure Area Under Curve?

ROC Curves

- P_d is probability that signal plus noise is greater than threshold.
- P_{fa} is probability that noise is greater than threshold.
- Goal is upper left corner.
- If it's not good enough, improve the instrument.



Research Opportunities

- NU Directed Studies, Work Study, Volunteer
- NU/UAndes Exchanges
- IRES (Proposed for NU Students)
- Vinculación Internacional (Proposed for UAndes Students)
- Center for Emerging Markets (For NU Students)

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Center for Emerging Markets (NU)

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