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College *of* Engineering

Universidad de los Andes

Biomedical Imaging
X–Rays

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Thanks to Danton Zhao

EECE–4649

Northeastern University

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History

- Röntgen, 1895
- Wide Use by 1930s
  - Military Early Adoptors
  - Bullets *etc.*
  - Commercial Uses in 1950s
- Diagnostic X–Rays
- Therapeutic X–Rays

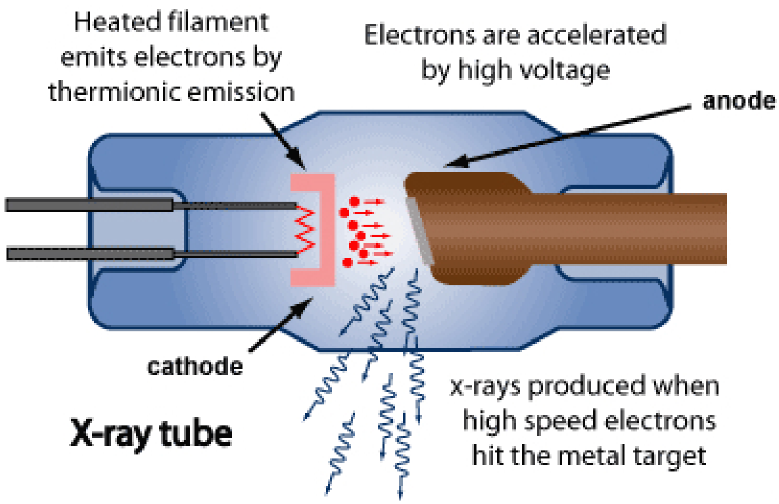
Configurations

- Small Single Source, Small Single Detector
- Plane Wave Source, Array of Small Detectors
- Diverging Point Source, Array of Small Detectors
- Multiple Sources and Detectors
- Many More Ideas

Sources (1)

- X–Ray Tube
  - Cathode (high negative voltage, V_{tube} Typically Many kV)
  - Anode (ground or positive)
  - Electrons Accelerated to Anode
  - Ionization
- Output
  - Characteristic X–Rays (Narrow Band, Depend on Material)
  - Bremstrahlung (Broad Band, Centered near $V_{Tube}/3$)

Sources (2)



http://www.outerspacecentral.com/x_ray_page.html

https://www.flickr.com/photos/received21/27657966/

- Anode Voltage Determines X–Ray Energy
- Anode Current Determines Number/Time

Sources (3)

- Anode Voltage Determines X–Ray Energy, E
- Higher Energy: Higher Frequency $\rightarrow$ Shorter Wavelength
- In X–Rays we usually talk about energy instead of wavelength

$$E = Ve$$

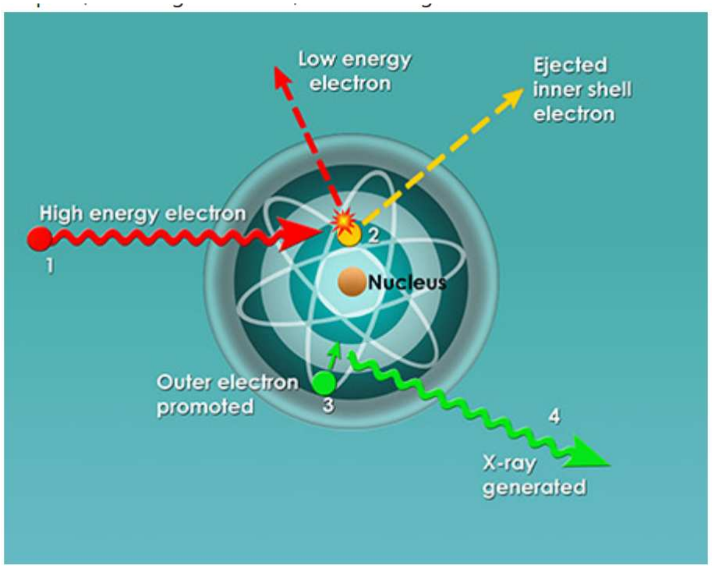
$$E = h\nu = \frac{hc}{\lambda}$$

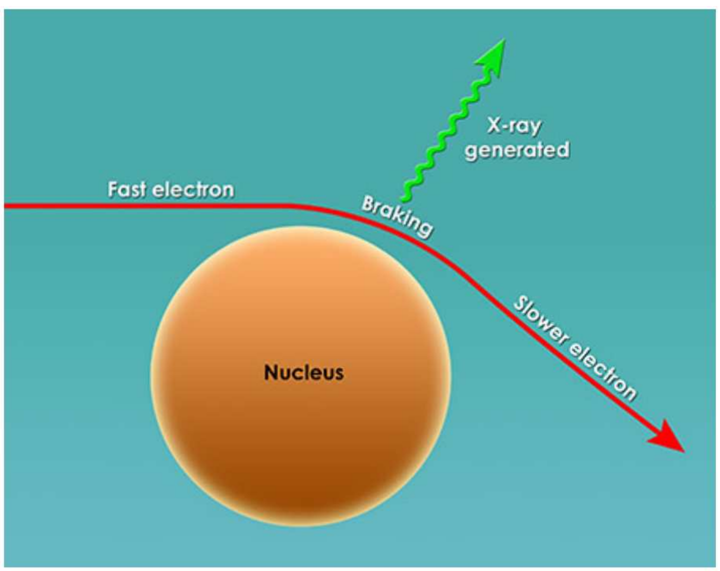
- Anode Current, i, Determines Number/Time
- Higher Current: More Electrons $\rightarrow$ More Photons

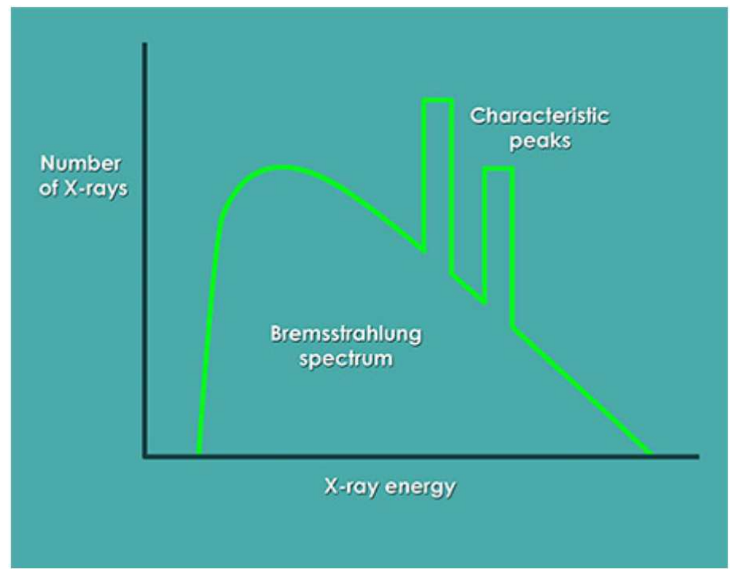
$$i = e\frac{dn}{dt} \qquad \frac{dn}{dt} = \frac{i}{e}$$

- Higher dn/dt $\rightarrow$ Better Image or Shorter Time

Characteristic X–Rays and Bremstrahlung







https://www.radiologymasterclass.co.uk/

Contrast: Atoms & Their Density

- Mostly Absorption by Electrons in Atoms
- Ionization
- Incoherent Addition (Any Consistent Units)

$$\mu_a = \sum_{n=0}^{N} C_n \kappa_n$$

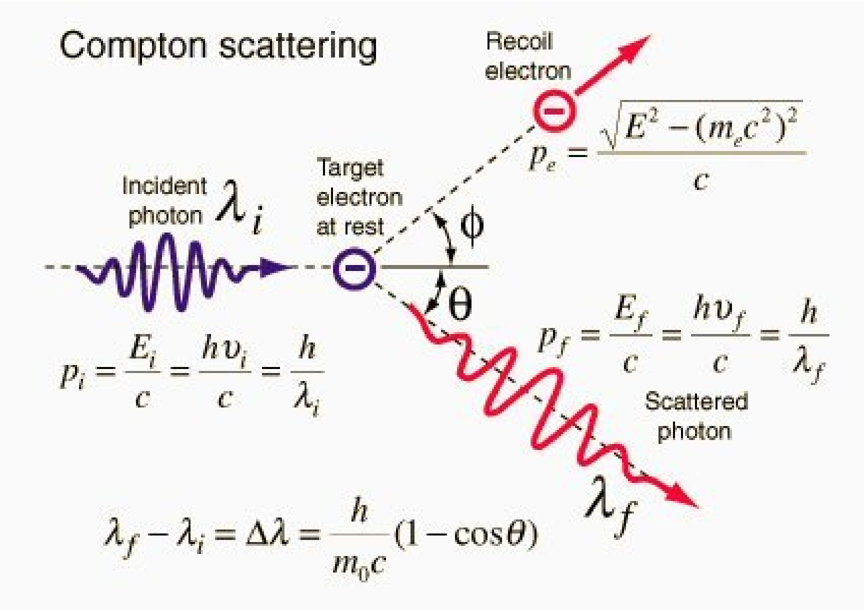
C_n=Concentration: e.g. g/cm^3

κ_n =Specific Absorption cm^2/g

- Little Soft Tissue Contrast (H, C, O)
- Good Contrast for Bone (Ca)
- Also Breast Calcifications
- Safety Issues (Joules/kg = Sieverts)

Scattering

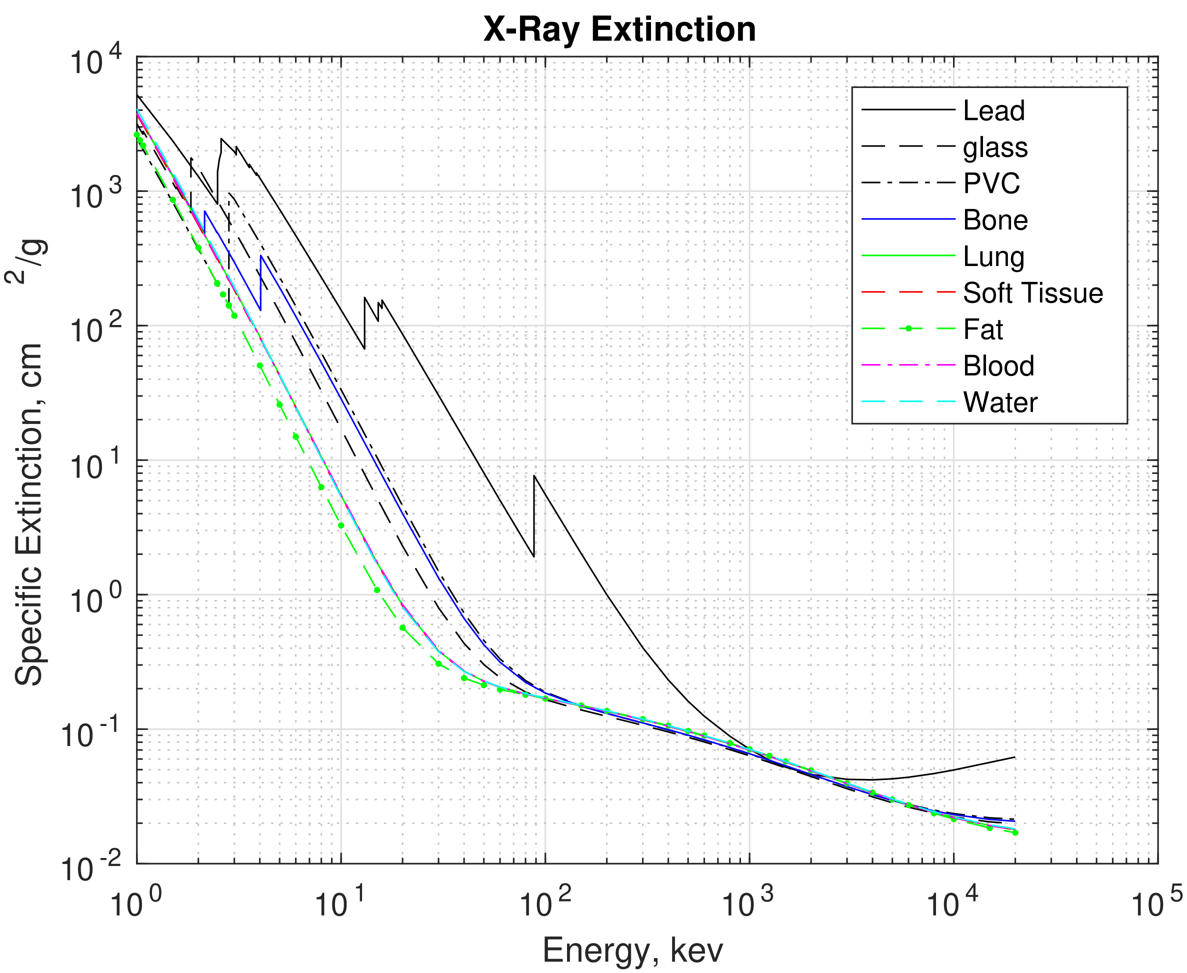
- Compton Scattering
  - Input
    Photon Energy ($h\nu_i$)
    Momentum (hk_i)
  - Output (1)
    Electron Energy
    Momentum
  - Output (2)
    Photon Energy ($h\nu_f$)
    Momentum (hk_f)
- Scattering
  Contributes to Background
  and Reduces Contrast



Detectors

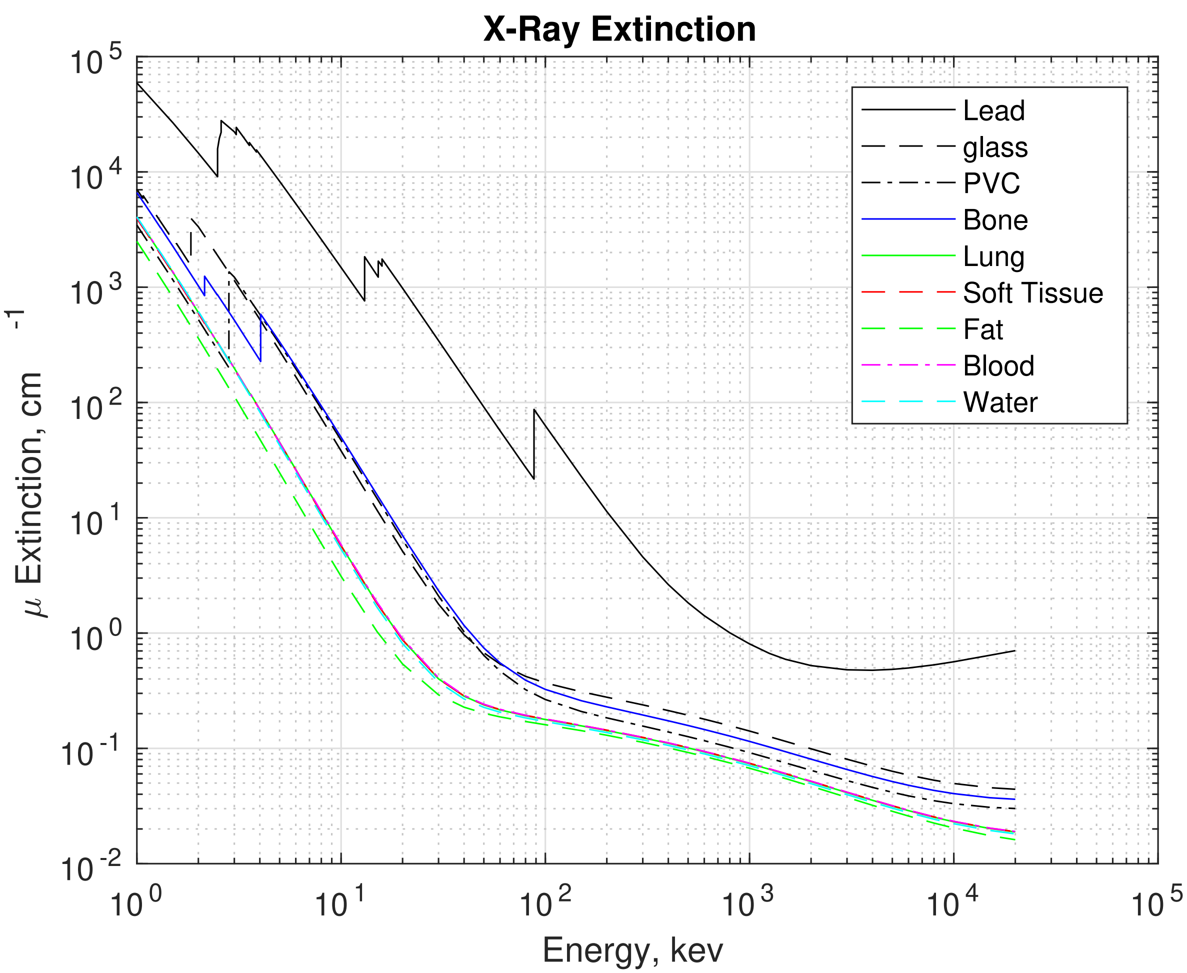
- Film
- Phosphors
- Image Intensifiers
- Arrays
- Fluoroscopy

Specific Extinction (per unit density)



https://www.nist.gov/pml/x-ray-and-gamma-ray-data
http://physics.nist.gov/PhysRefData/XrayMassCoef/tab4.html

Extinction



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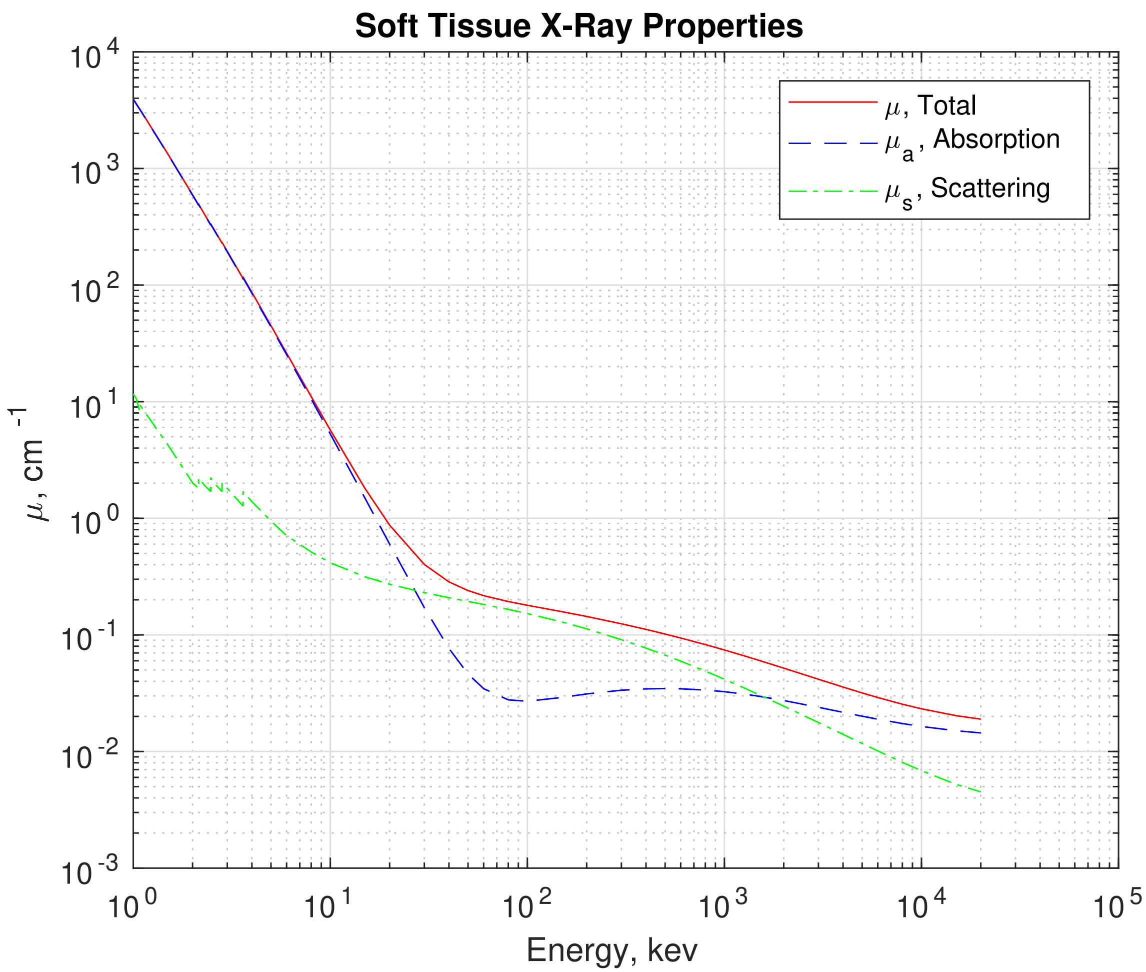
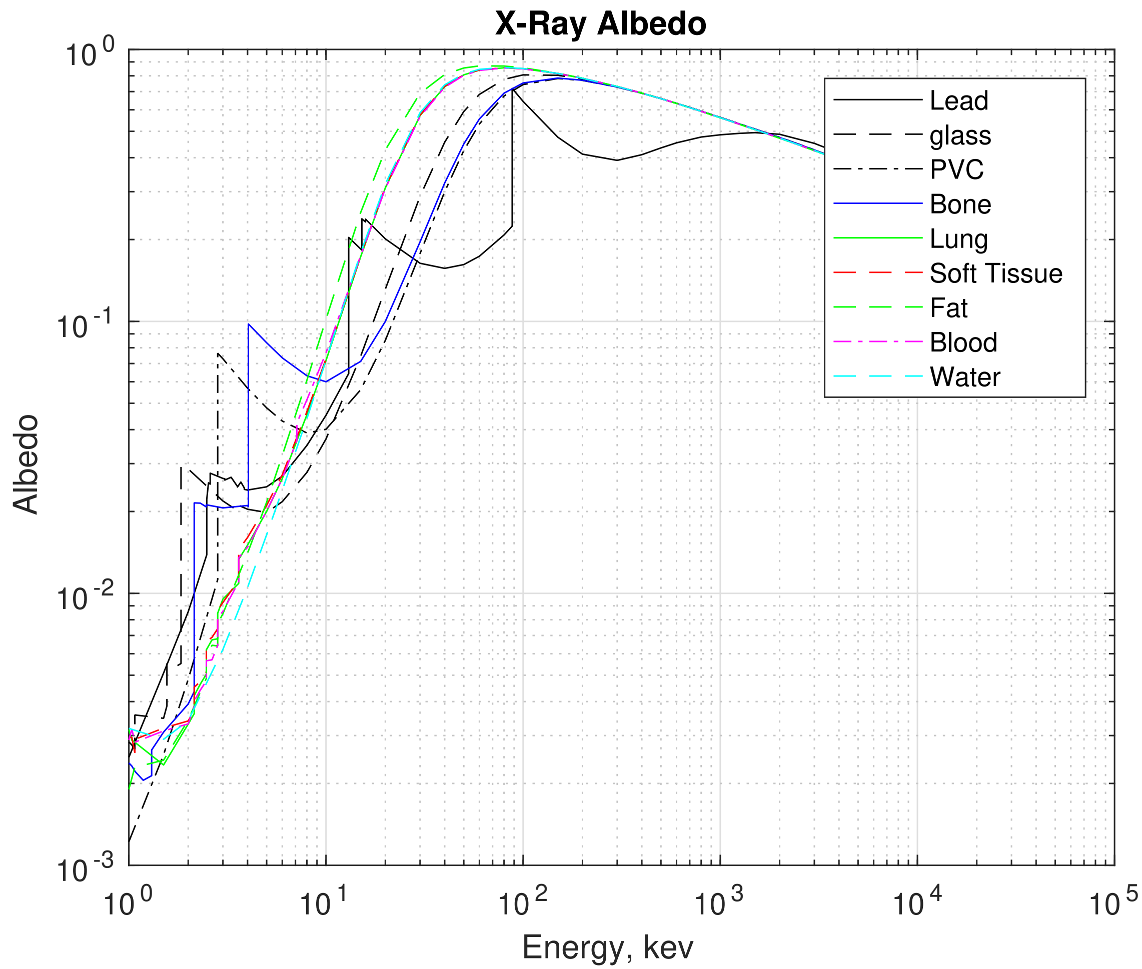
Properties of Soft Tissue



Albedo



Not Normally Used in Medical Imaging

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Wavelength *vs.* Energy

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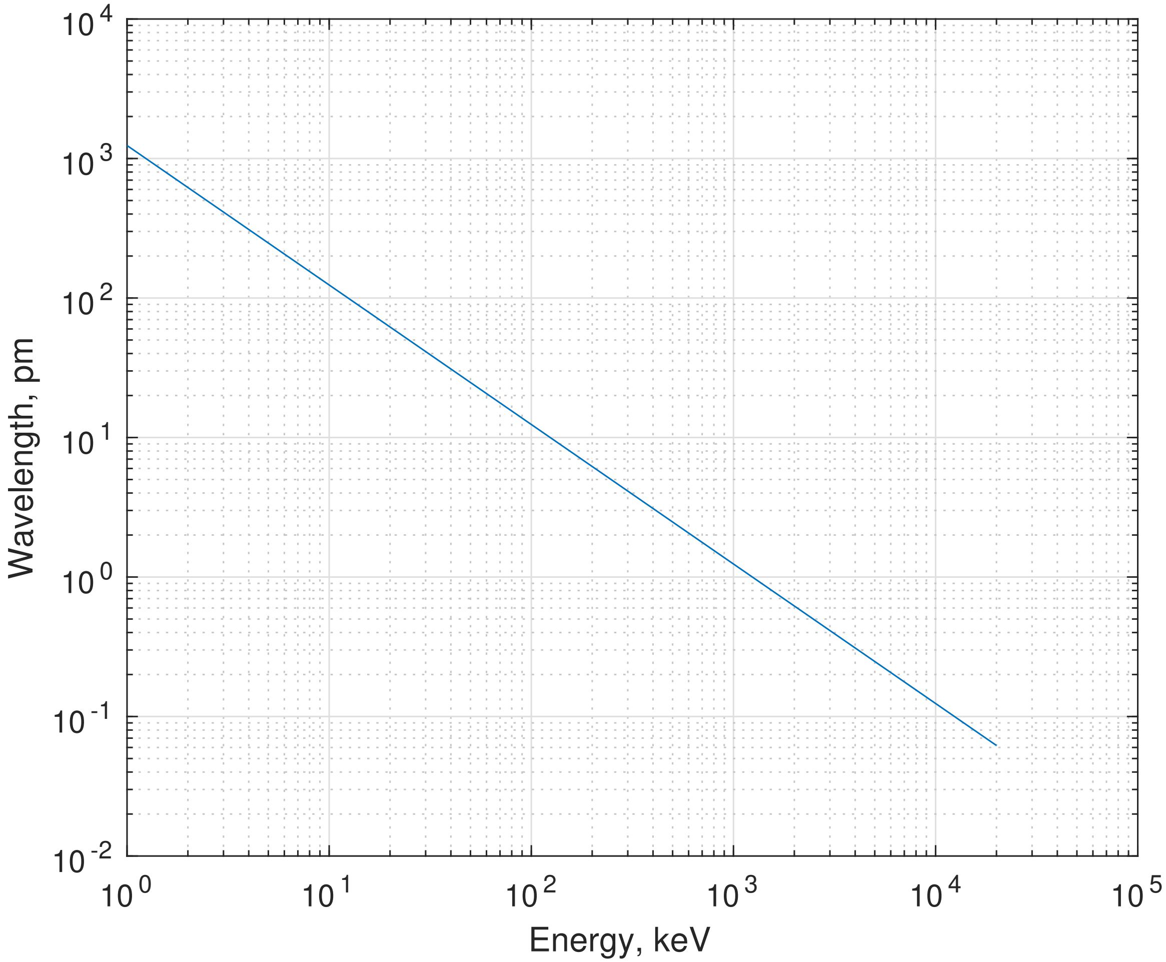

Resolution

- Spot Size, d

$$d = \frac{\lambda}{D} z$$

- Match $d = D$

$$D = \frac{\lambda}{D} z$$

- Solution: Almost no Diffraction Issues

$$D = \sqrt{z\lambda} \approx \sqrt{1 \text{ m} \times 10^{-10} \text{ m}} = 10\ \mu\text{m}$$

- Resolution Often Determined by Sampling

Transmission Image Concept

A	A	A	A	A	A	A	A
A	A	S	S	A	A	A	A
A	A	S	S	S	A	A	A
A	A	S	B	S	A	A	A
A	S	B	B	B	S	A	A
A	S	S	B	B	S	A	A
A	S	S	S	S	S	A	A
A	A	A	S	A	A	A	A
A	A	A	A	A	A	A	A

A = Air

S = Soft Tissue

B = Bone

Sum Each Column
Add μz or Multiply T.

For each column $T = e^{-\int \mu dz} \approx e^{-\sum(\mu_i z_i)}$.
Then do the third dimension

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X–Ray Image (1)

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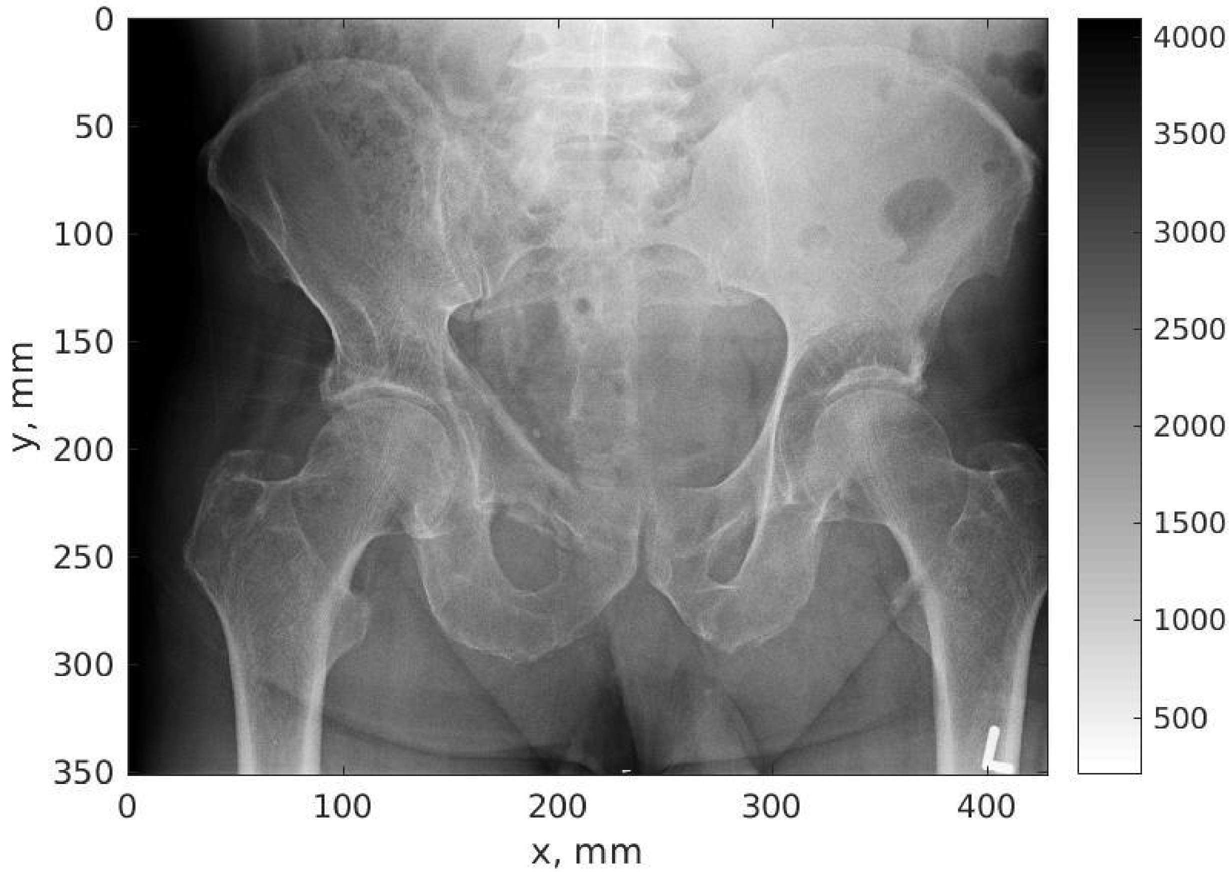
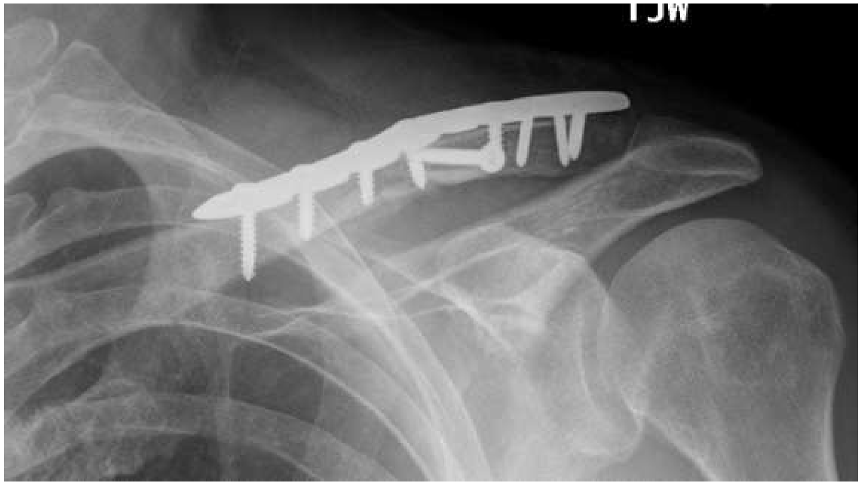


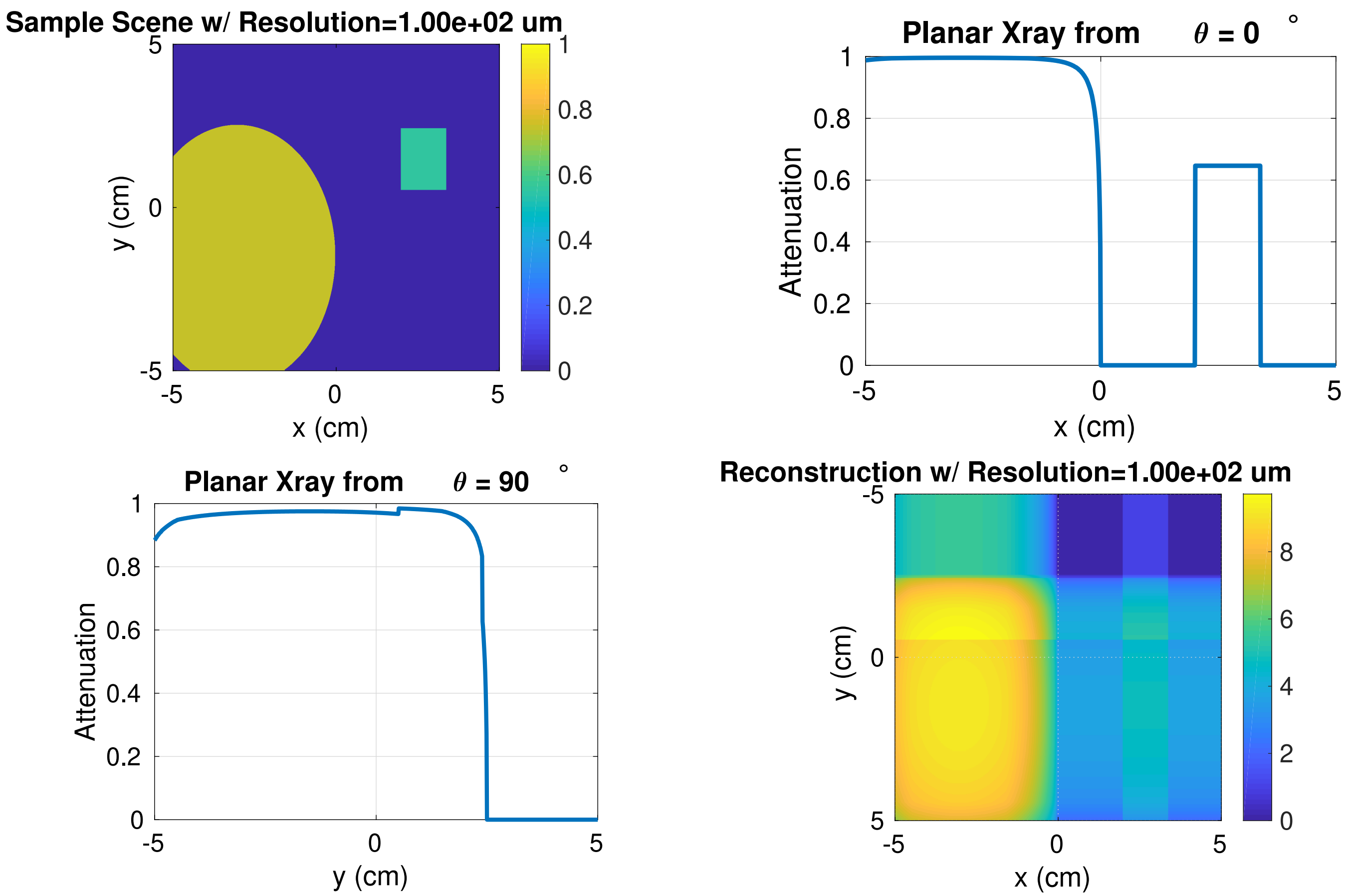
X–Ray Image (2)

X–Ray Image (3)



Thanks to Prof. Mark Niedre, Northeastern

Multiple Views



$$-\ln T = \sum (\mu_i z_i)$$

General Tomography Problems

- Projections
- N Equations and M Unknowns
- Example: Sums of Rows and Columns
- 9 Unknowns and 6 Equations

1	4	5	10
6	-3	3	6
-1	2	4	5
6	3	12	

X–Ray Computed Tomography (1)

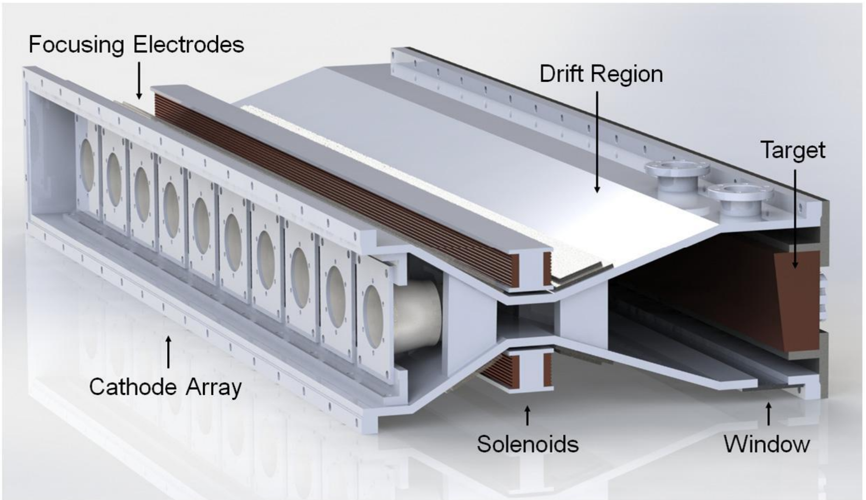
- Concept
  - Multiple 2–Dimensional Projections
  - Reconstructions (2–Dimensional)
  - Slices for 3 Dimensions
- Hard to Do
  - Detector Issues
  - Time Issues
  - Exposure Issues

X–Ray Computed Tomography (2)

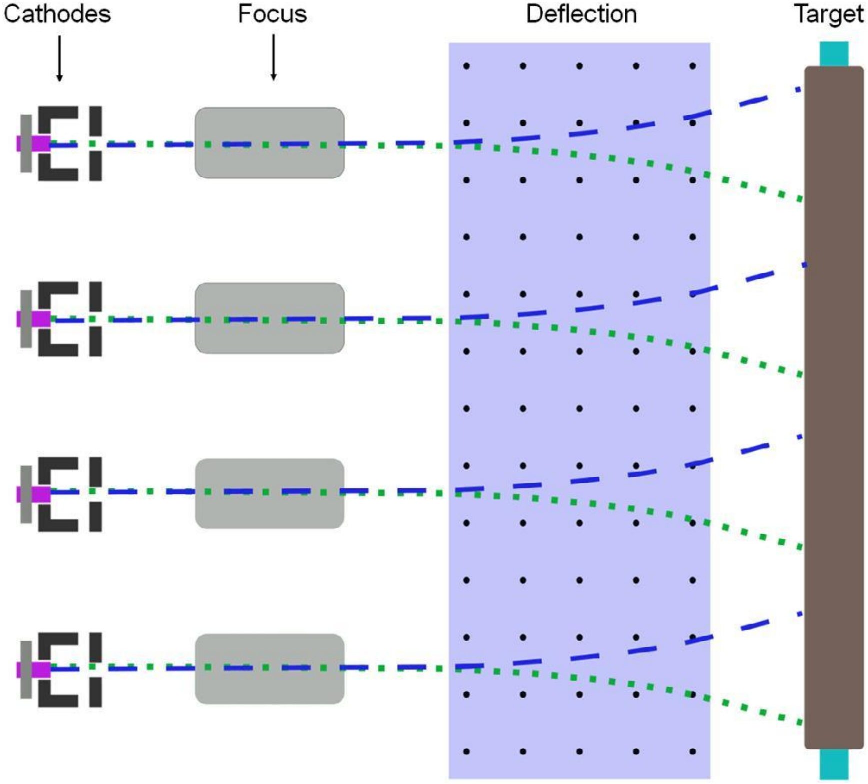
- Generation 1
  - Before Arrays; One Detector
  - Translate and Rotate (Slow)
- Generation 2
  - Multiple Detectors (Faster)
- Generations 3–4
  - Fan Beams
  - More Sources and Detectors (Inverse Fan)
- Move Patient for 3 Dimensions

X–Ray Computed Tomography (3)

The Future?



Walker, *NIMA* 2017



See Video ct-finaldraft4.mp4

Brandon Walker, UW–Madison

CT Issues

- High Exposure
- Slip Rings (low voltages)
- Limited View (Resolution Issues)
- Fluoroscopy
- Stop–and–Go Artifacts
- Helical Scan with Interpolation
- Beam Hardening: T Depends on Frequency (Energy)

$$T(f,x,y) = e^{-\int \mu(f,x,y,z)dz}$$

Forward Model

- Assume Finite Pixels

$$\log T\left(\ell dx, mdy\right) = Y_{\ell,m} = \Sigma_n X dz$$

$$X_{\ell,m,n} = \mu\left(\ell dx, mdy, ndz\right)$$

- Matrix Equation with Noise

$$Y = \mathcal{M}\mathbf{X} + \mathbf{N}$$

- Inverse

$$\hat{\mathbf{X}} = \mathcal{M}^{-1}\mathbf{Y} = \mathcal{M}^{-1}\left(\mathcal{M}\mathbf{X} + \mathbf{N}\right) = \mathbf{X} + \mathcal{M}^{-1}\mathbf{N}$$

- No Noise, No Problem (Well; One Less Problem Anyway)