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Biomedical Imaging
Magnetic Resonance Imaging

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Thanks to Eric Kercher, Danton Zhao
EECE–4649 Dialogues of Civilization
Northeastern University
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Background and History

- Measurement of Nuclear Spins
  - Widely used in physics/chemistry labs (Absorption)
  - First Medical applications in the 1980s (Wiggles)
  - Improvement over Decades with Computer Technology
- NMR = Nuclear Magnetic Resonance
  - But you can't say "Nuclear" to Patients!
  - Not about ionization
  - Not about bombs
- Marketable name: Magnetic Resonance Imaging

Larmor Precession

- An object with magnetic moment μ is placed in an external magnetic field $\mathbf{B}$. Torque τ is applied on the object:

$$\tau = \mu \times \mathbf{B} \tag{1}$$

- Torque causes the object to rotate at a frequency proportional to the applied field, i.e., the Larmor frequency

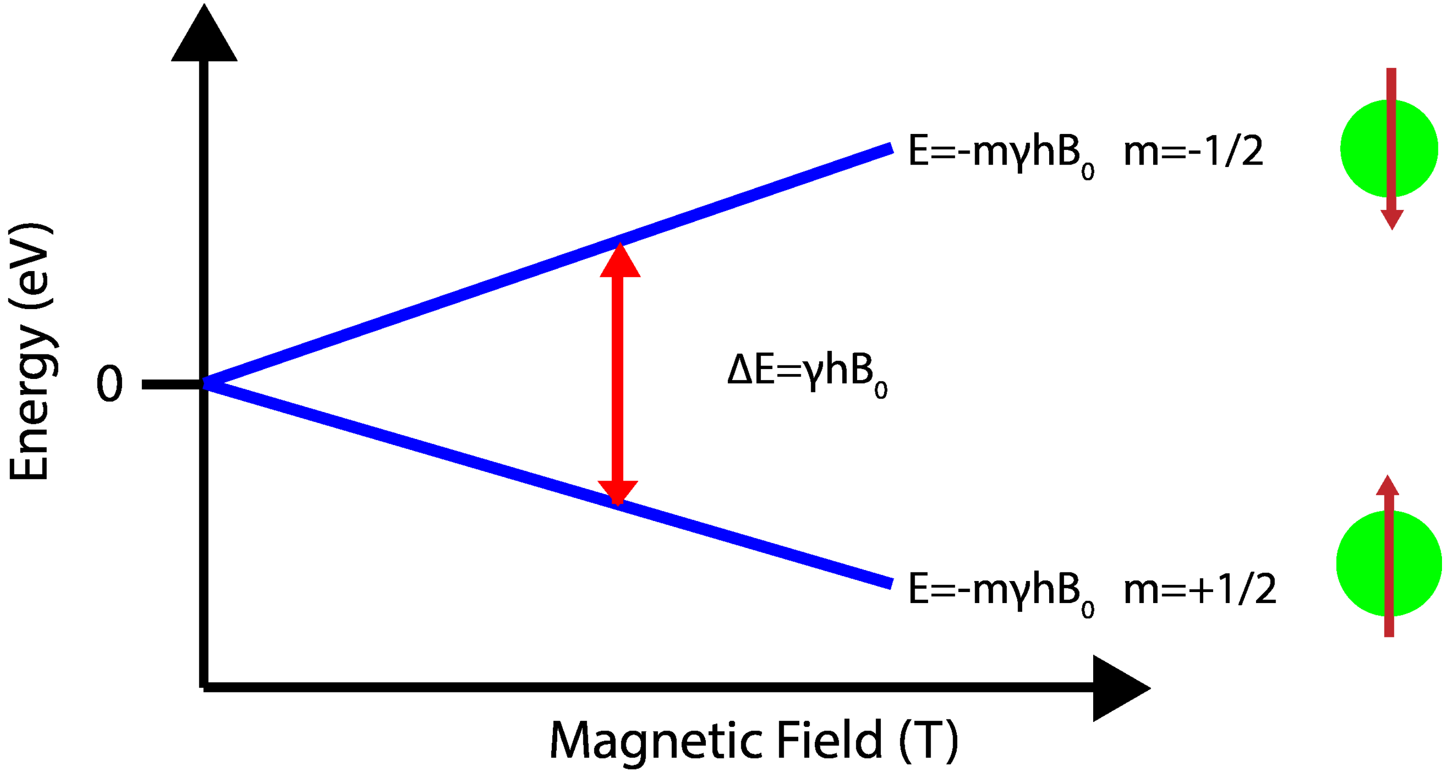
$$\omega = \gamma B \tag{2}$$

- γ is the gyromagnetic ratio, which depends on the properties of the object

$$\gamma = \frac{|e|}{2m} g \tag{3}$$

Zeeman Effect

- Spin-state energy levels "split" under an applied magnetic field



Magnetization

- Convenient to talk about bulk material properties.

- Imagine a material with many objects ”spinning” in random directions...

- Result of a external magnetic field is two-fold:
  - Torque causes precession at $\omega = \gamma B$ around the B field.
  - Two spin states ”appear”; spin up $(+1/2)$ and spin down $(-1/2)$. These are also aligned with the B field.

- The material now has a net magnetization $\mathbf{M} = \sum_i m_i$.

Population Difference

- Spin states populated in a Boltzmann distribution. Most spins will align with B field (low energy state), but some will be anti-aligned!

- Fields in a few Teslas, Larmor frequencies in Tens of MHz.

- Photon Energies $\approx 10^{-26}$ Joules
  (Below μEv)

$$N_{upper}/N_{lower} \approx e^{-hf/kT} = 1 - 10^{-5}$$

- but $N \approx N_A/$ cm^3

$$N_{lower} - N_{upper} \approx 10^{18}/\text{ cm}^3$$

MRI Imaging

- "Excite" spins into the higher energy state.
  - Use RF pulses to "Flip" $\mathbf{M}$
  - If half the spins flip → $\mathbf{M}$ rotates 90 degrees
  - If most of the spins flip → $\mathbf{M}$ rotates 180 degrees
- Let spins relax back to equilibrium. $\mathbf{M(x, y, z, t)}$ is 4D!
  - M_z: Longitudinal relaxation
  - M_x, M_y: Transverse relaxation
- Reconstruct image from collected signals.

Bloch Equations

$$\frac{dM_{x'}}{dt} = (\omega_0 - \omega)\, M_{y'} - \frac{M_{x'}}{T_2}$$

$$\frac{dM_{y'}}{dt} = -(\omega_0 - \omega)\, M_{x'} - \frac{M_{y'}}{T_2} + 2\pi\gamma B_1 M_z$$

$$\frac{dM_z}{dt} = -\frac{M_z - M_{z0}}{T_1} - 2\pi\gamma B_1 M_y$$

Green Terms are Rotation “Error”

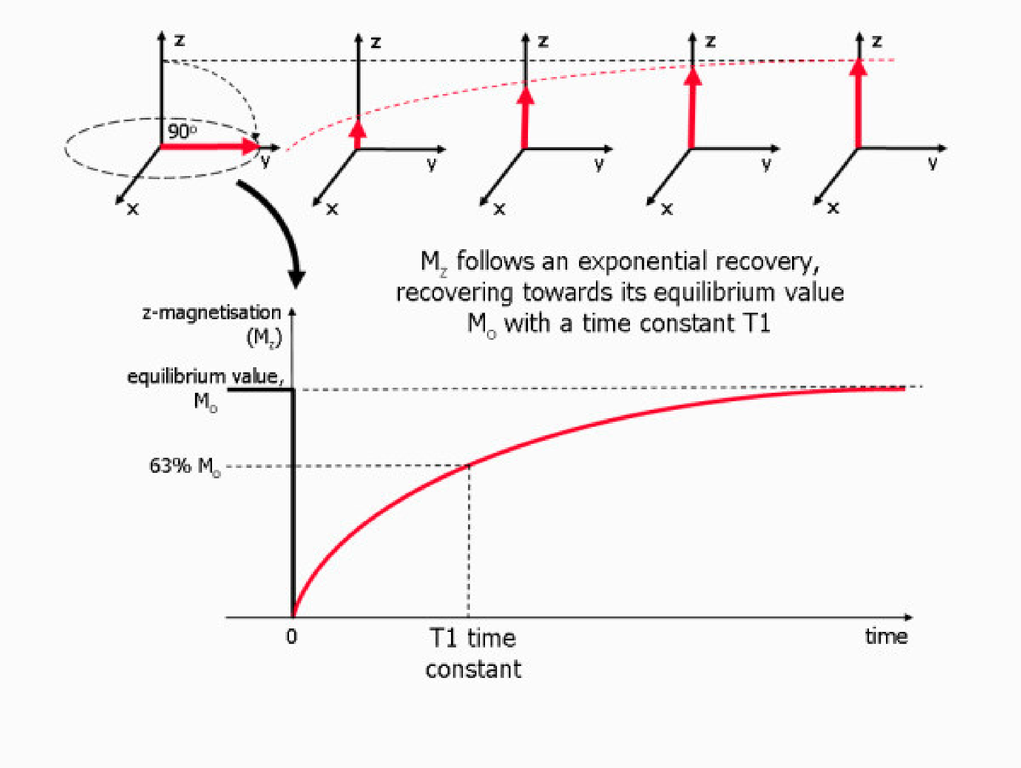
Red Term is Decay

B_1 is RF field parallel to $\hat{x}$

Blue Terms are Dephasing

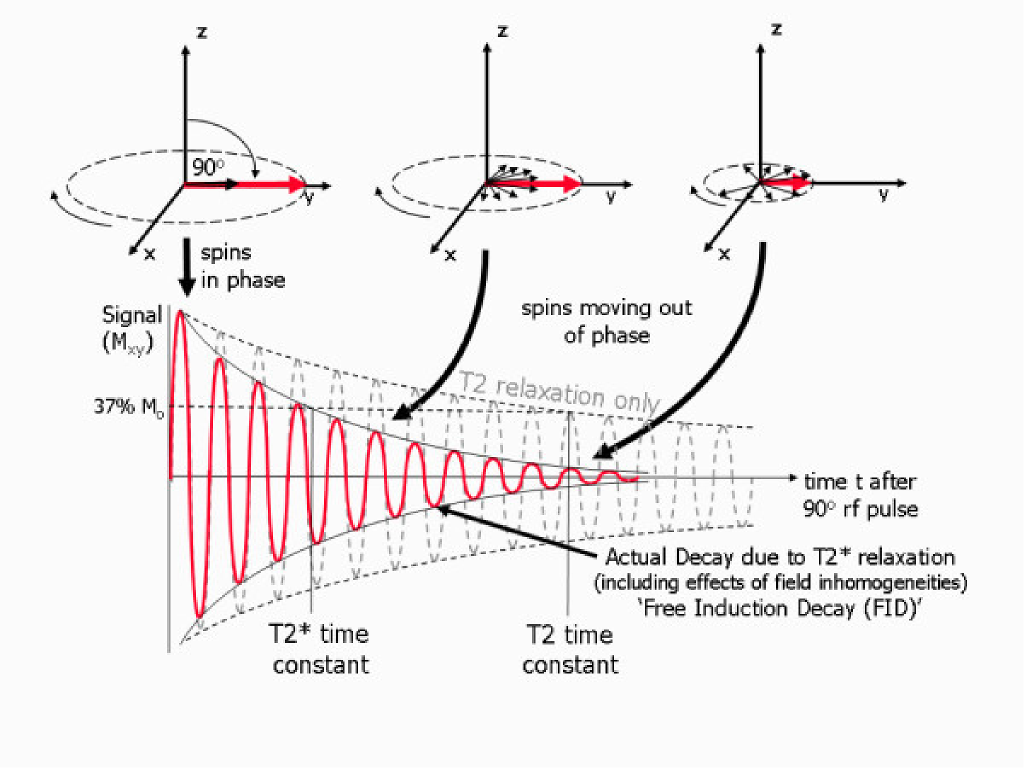
Longitudinal Relaxation - T1

- AKA Spin-Lattice relaxation, applies to the z-component of $\mathbf{M}$. Natural decay from spins flipping back to low energy state (thermal decay).

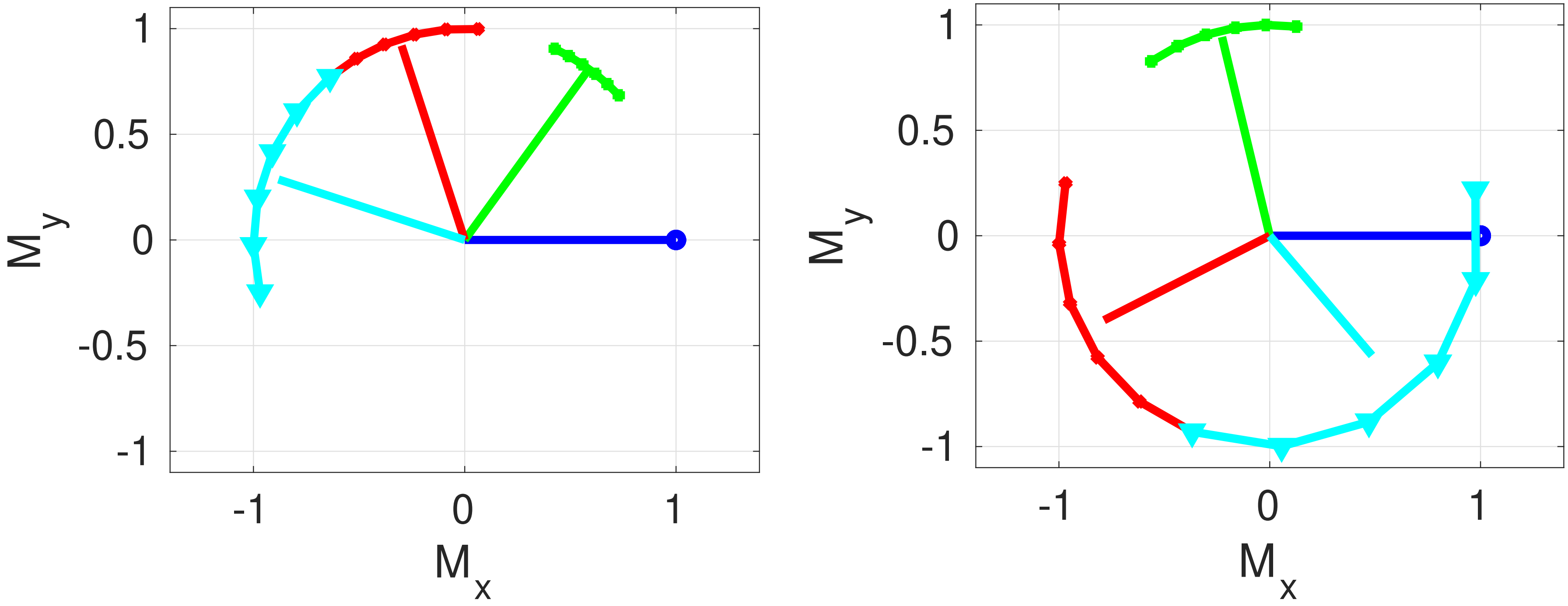


Transverse Relaxation - T2

- AKA Spin-Spin relaxation, applies to the xy-components of $\mathbf{M}$. Spins in phase create coherent M_{xy} vector (rotating at ω). Signal decays as spins de-phase. Local field imhomogeneities cause faster-than-expected decay $\rightarrow$ T2*.



Dephasing



Slow: Large T_2

Fast: Small T_2

Non–Random Dephasing Caused by Field Inhomogeneities is Reversible

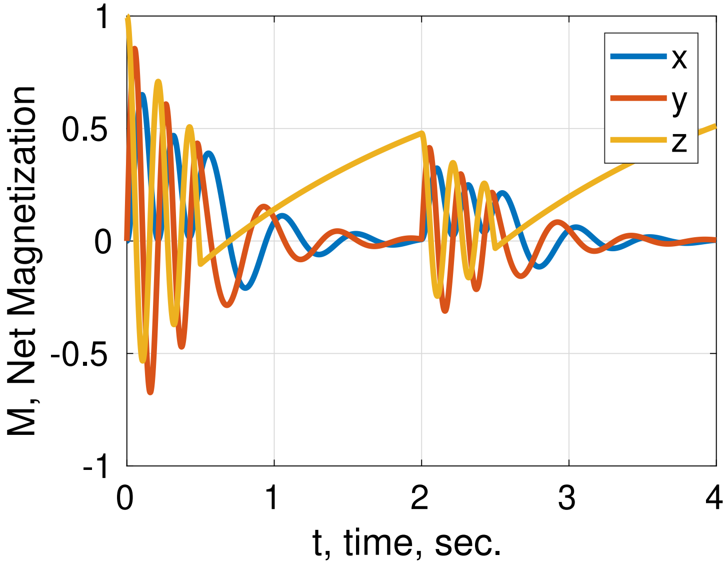
Random Dephasing Caused by Material is Not.

The Material One is the One We Want.

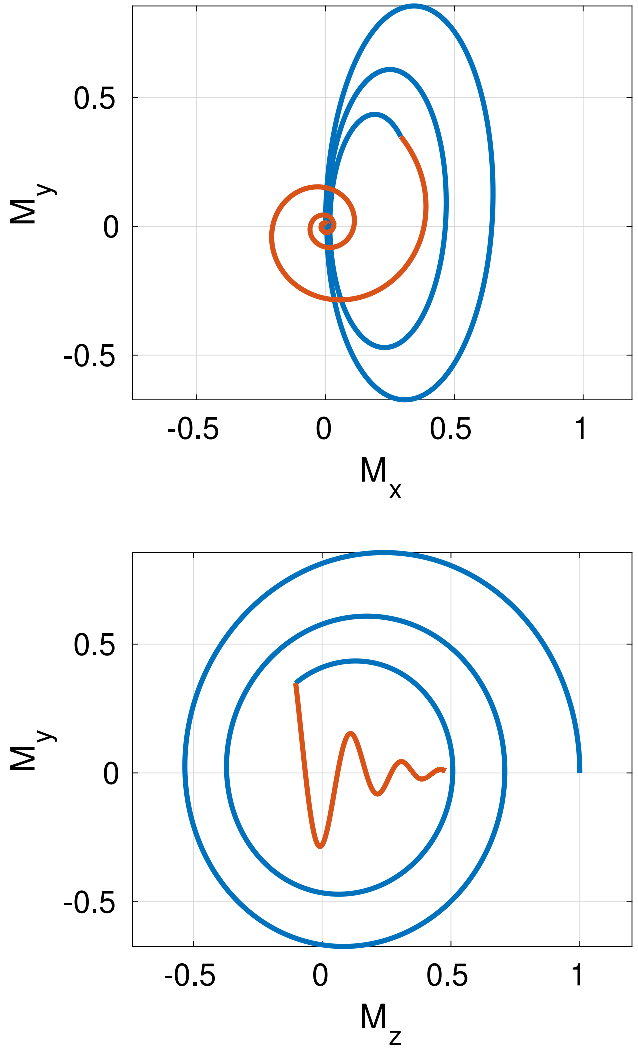
Example: Long RF Pulse

RF B_1 in $\hat{x}$ direction
0.5 Sec Pulses
Repetition Time of 2 Sec

Note T_1 and T_2 in Graph



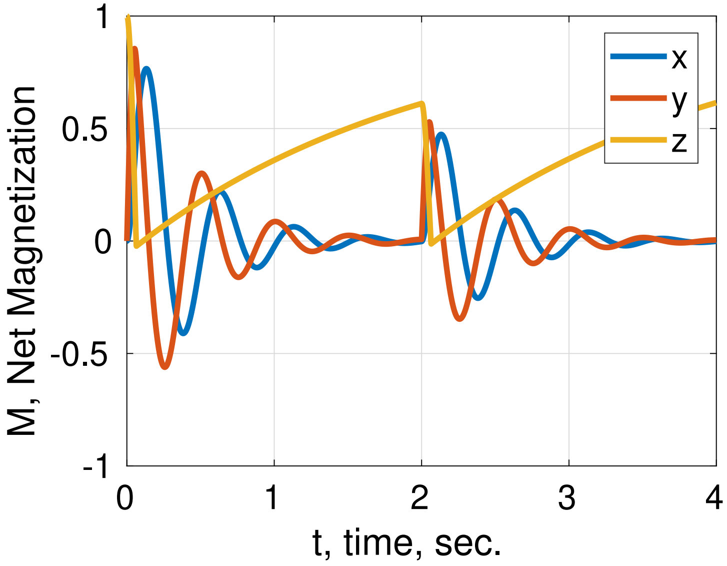
Blue During Pulse, Red After



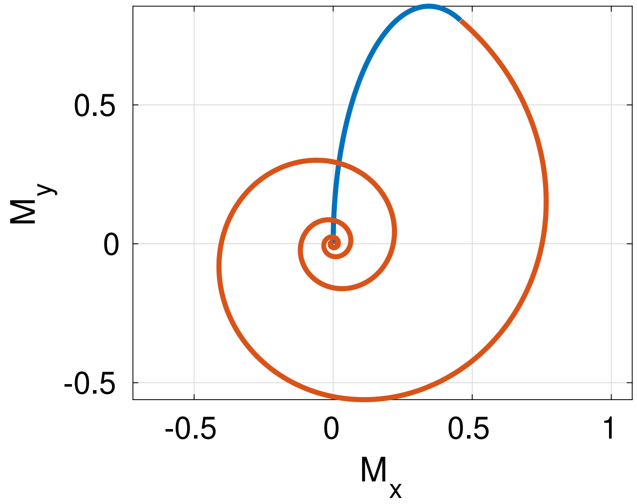
Example: 90–Degree Pulse

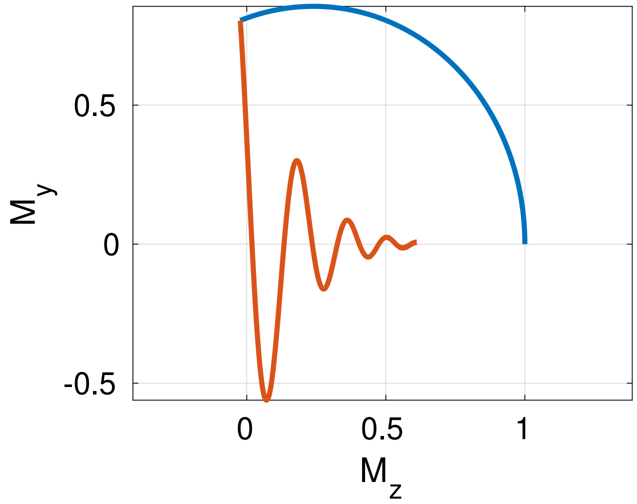
RF B_1 in $\hat{x}$ direction
0.0643 Sec Pulses
Repetition Time of 2 Sec

Note T_1 and T_2 in Graph



Blue During Pulse, Red After

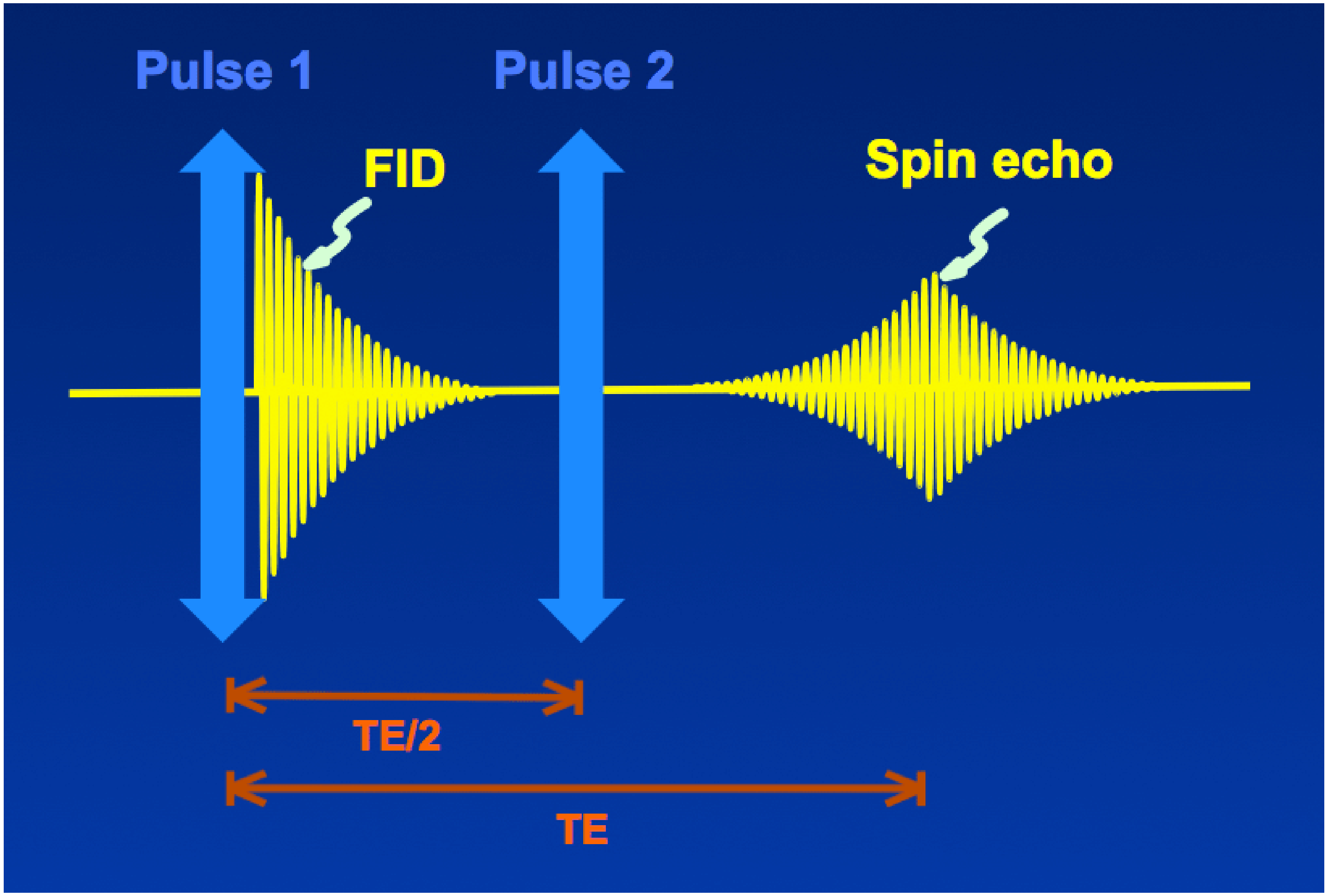




Spin Echo

- T_2 Random Dephasing (Material Dependent)
- Dephasing due to inhomogeneous B (Non–Random, Instrument Dependent)
- T_2^* Combines Both
- Spin Echo: Flip the Spin
- Rewind Inhomogeneous Field Dephasing
- Still Have T_2 Random Dephasing for Signal Decay (Material Dependent)

Spin Echo



http://www.mri-q.com/uploads/3/4/5/7/34572113/_7707793_orig.gif

Measureing Decay Times

- T_R is Pulse Repetition Time
- T_E is Echo Time
- T1 and T2 decay happen simultaneously. Put together:

$$S = k\rho \left(1 - e^{-T_R/T_1}\right) e^{-T_E/T_2}$$

- Rule: $T_1 > T_2$.

Parameter to Which Signal is Sensitive

	T_R Long	T_R Med	T_R Short
T_E Long	0	0	0
T_E Med	T_2	All	0
T_E Short	ρ	T_1	0

Measuring Decay Times

- Equation: $S = k\rho\left(1 - e^{-T_R/T_1}\right)e^{-T_E/T_2}$
- T_R Longer than T_1: $S = k\rho\,(1)\,e^{-T_E/T_2}$
  Let Higher State Decay Completely for Big Signal
- T_R Shorter than T_1: $S = k\rho\,(0)\,e^{-T_E/T_2}$
  Try to Pump to Higher State before Return to Lower State
  Nothing to Pump so No Signal
- T_E Longer than T_2: $S = k\rho\left(1 - e^{-T_R/T_1}\right)0$
  T_2 Transverse Decay Goes to Zero; No Signal
- T_E Shorter than T_2: $S = k\rho\left(1 - e^{-T_R/T_1}\right)1$
  Strong Transverse Signal to Measure T_1
- T_R, T_E Both Moderate: Sensitive to Everything.

Decay Times

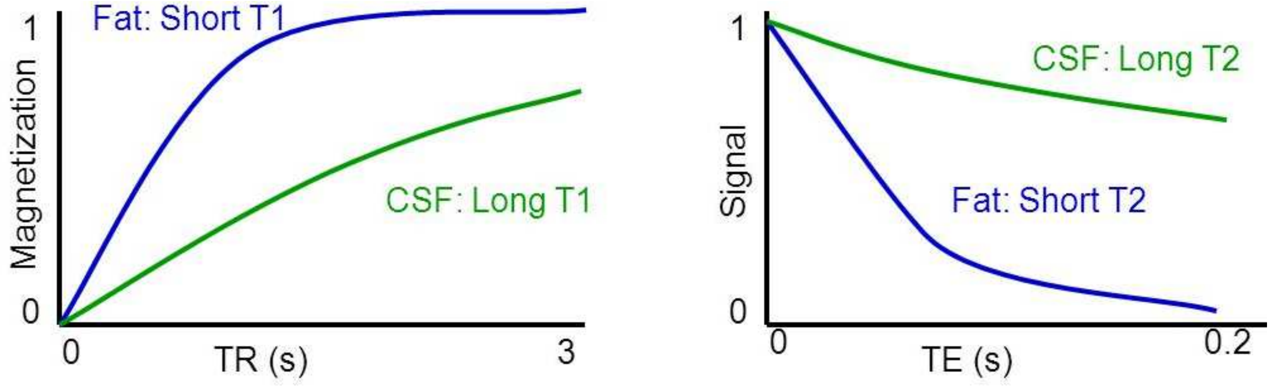
Material	T1 (ms)	T2 (ms)
Gray	921	101
Fat	259	84
Bone Marrow	752	106
Muscle	868	47

Fractions of a Second: MRI Is Slow

Contrast

- Endogenous contrast comes from differences in bulk tissue properties:
  - Water, fat: Lots of $^1H \rightarrow$ High signal (Most of body)
  - Bone: Not as much signal

- Tissues have varying T1 and T2. Compare Fat and CSF:

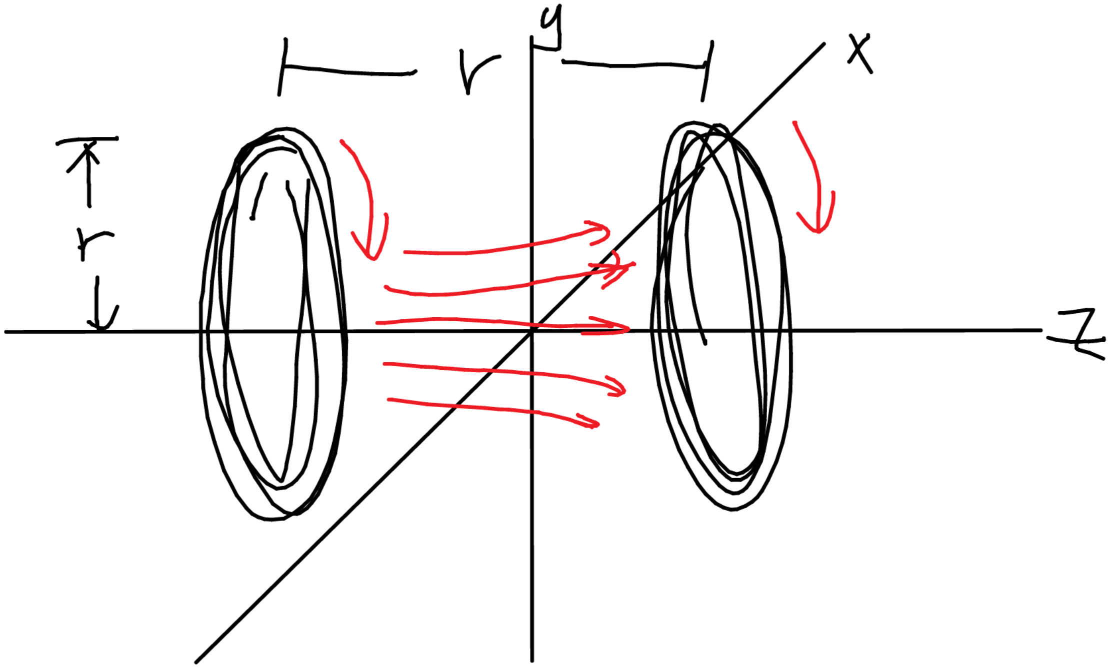


Contrast Agents

- Exogenous contrast alters T1 and T2 to boost contrast
- Gadolinium
  -
  -
- Iron Oixide Nanoparticles (Ferumoxytol)
  -
  -

Helmholtz Coils

Large, Uniform, DC Magnetic Field



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Anti–Helmholtz Coils

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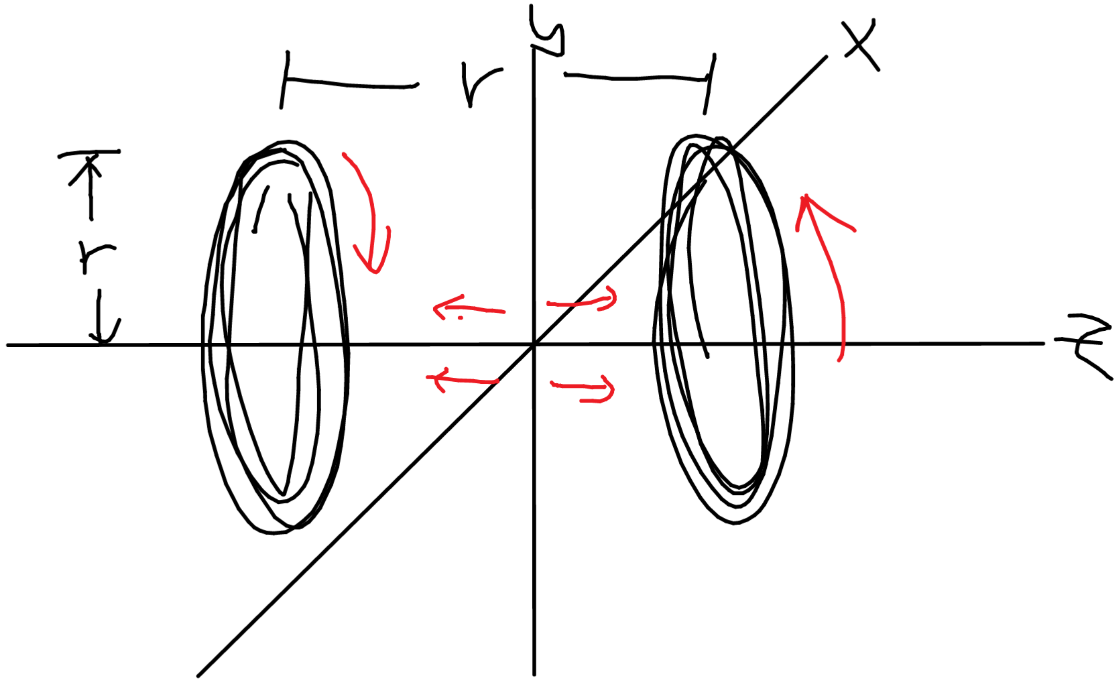
Moderate Field Gradient (More is Better)



Big Fields: Big Problems

- Large Coils for Uniform Field
- High Current for High Field
- Superconductors
- Liquid Helium
- High Cost
- B Field Hazards →
- dB/dt: Loud Noise
- Start/Stop Challenges

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Slice Selection: Excitation Frequency

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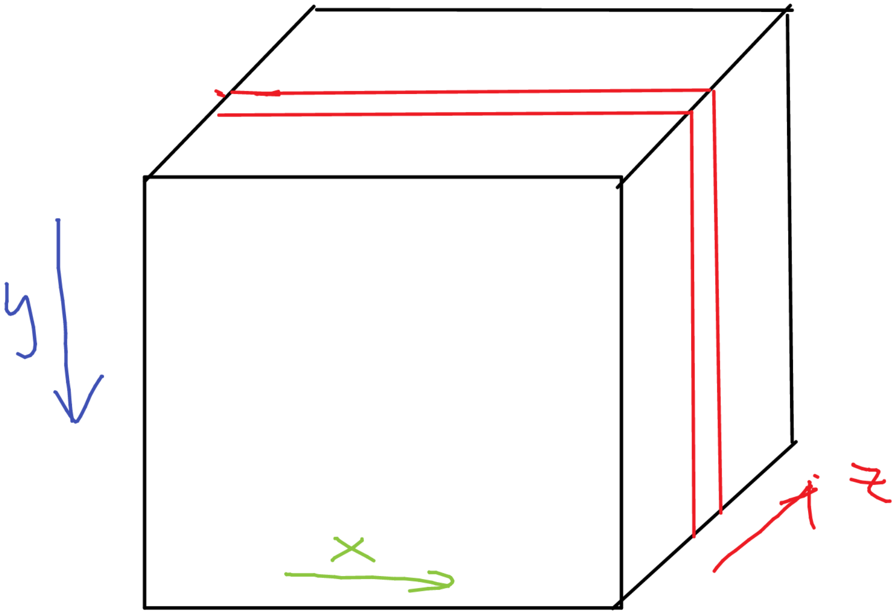
Excite with Narrow–Band RF Signal, $B = B_0 + G_z z$

$$\omega = \gamma B_0 + \gamma G_z z$$

Different B for Every z Slice; Excite Only One Slice

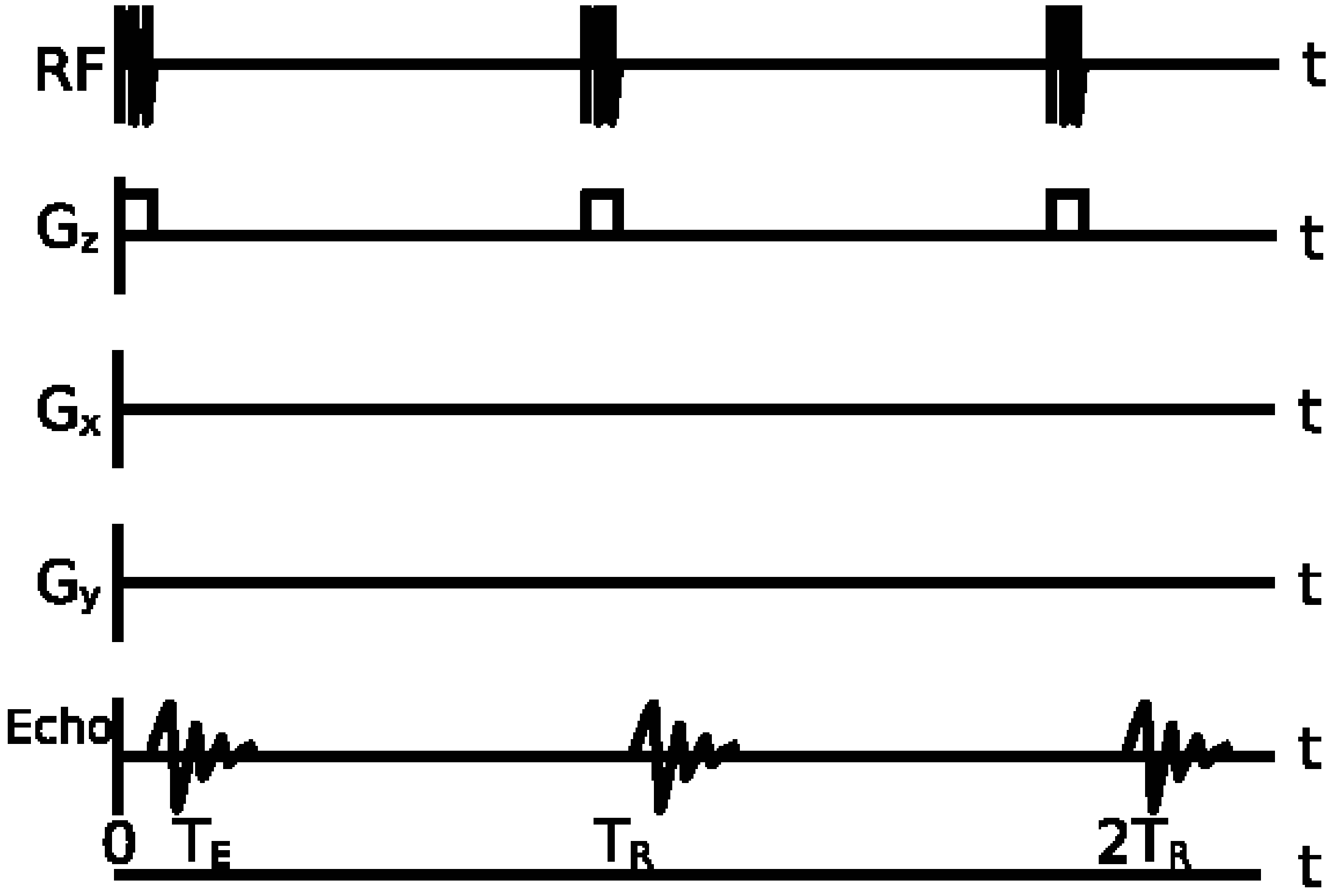


Slice Selection

Match the Resonant Frequency

Free–Induction Decay

Timing Diagram

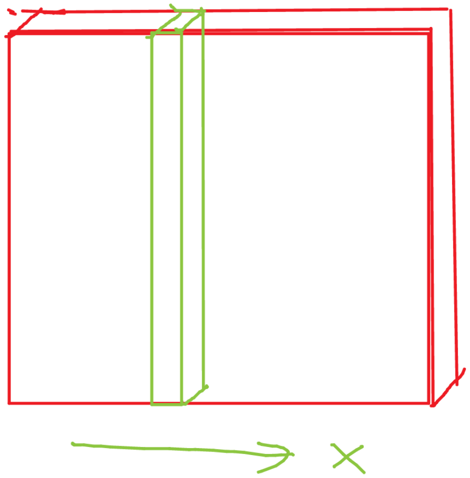


Column Measurement: Detection Frequency

Sort Detected Signal by Frequency, $B = B_0 + G_x x$

$$\omega = \gamma B_0 + \gamma G_x x$$

Each Column Emits at a Different Frequency

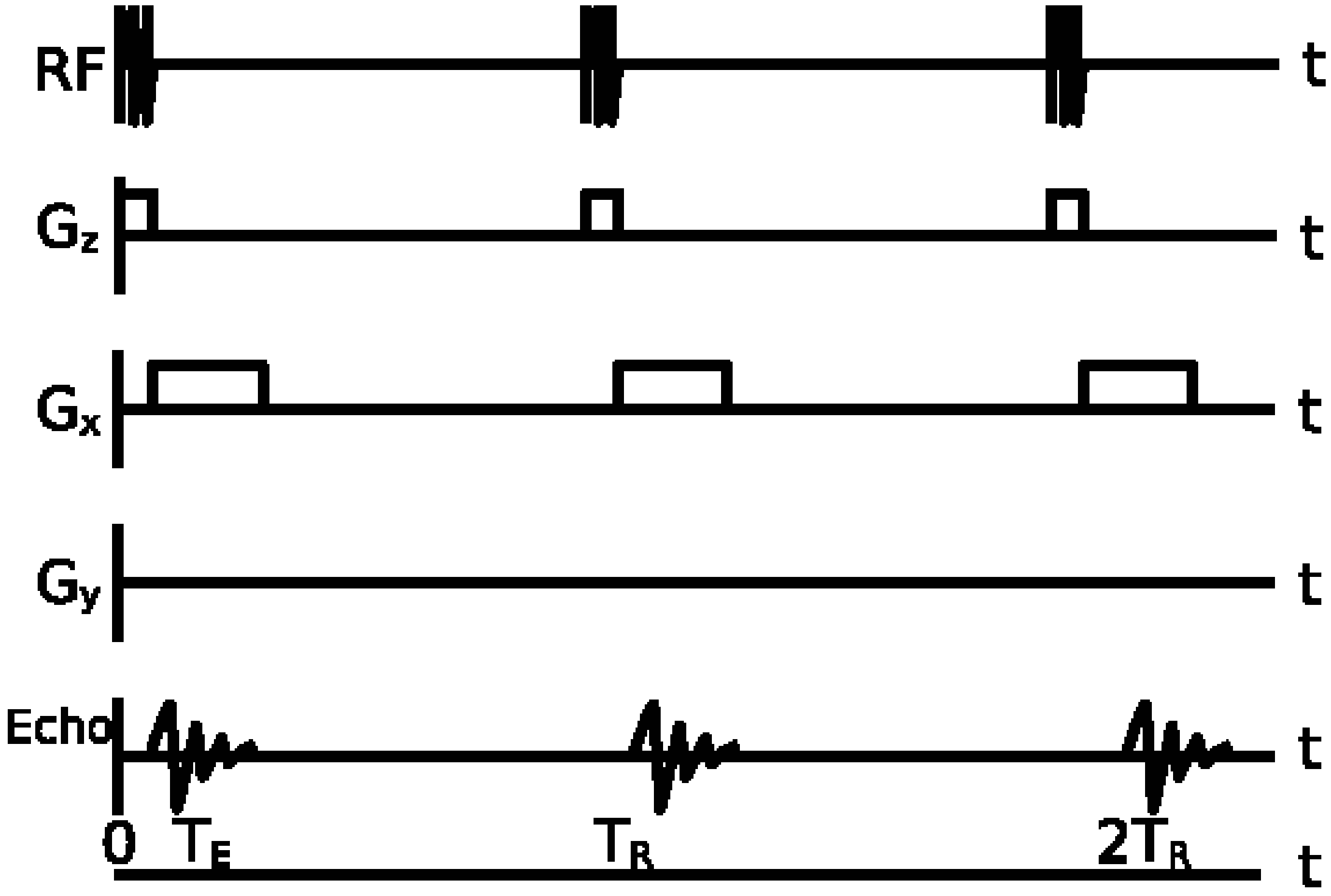


Example: $dB/dx = 2$ mT/m:

$$df = \gamma \frac{dB}{dx} dx = 42.58 \text{ MHz} \times 2 \text{ mT/m} \times 1 \text{ mm} \approx 80Hz$$

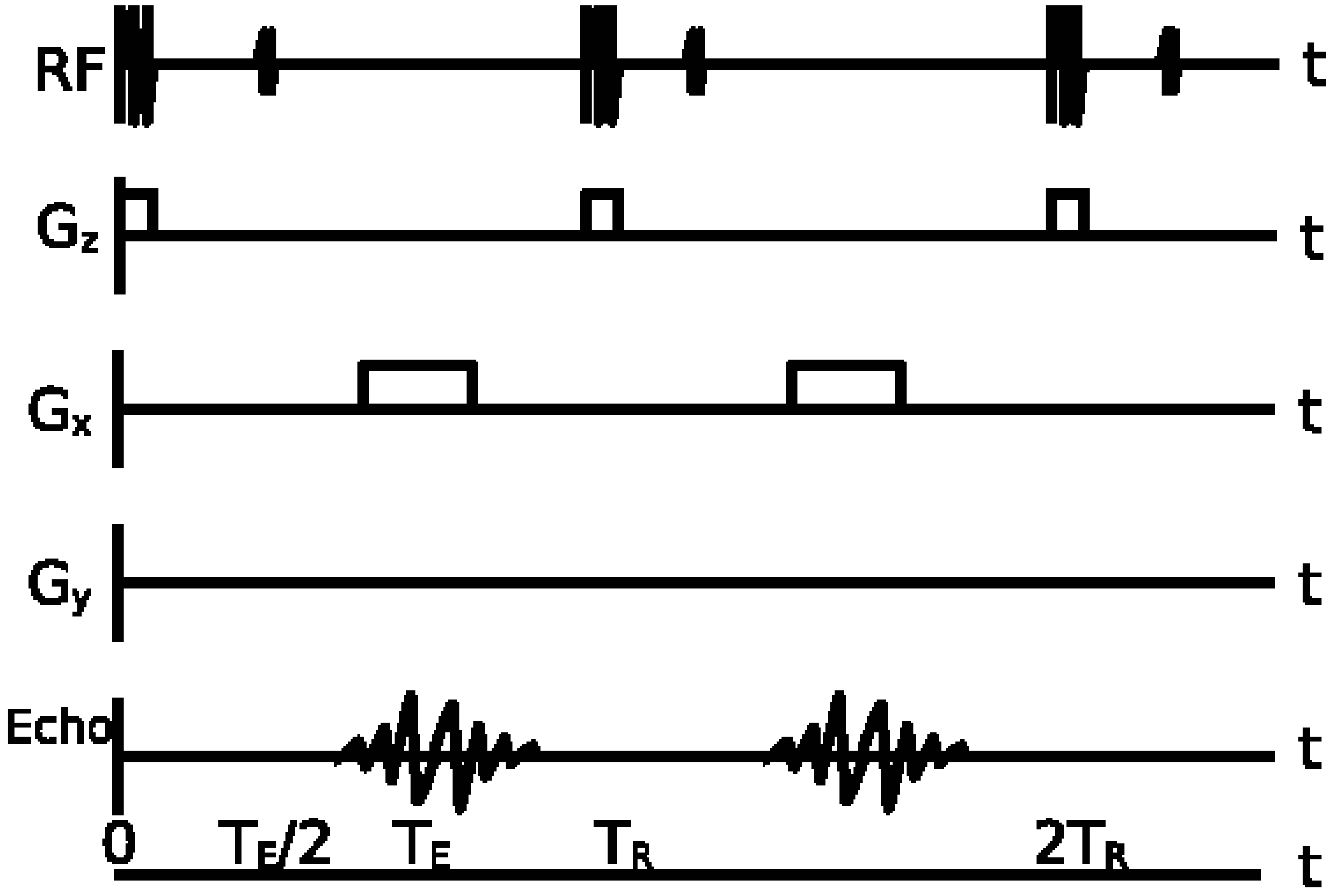
Column Measurement

Timing Diagram



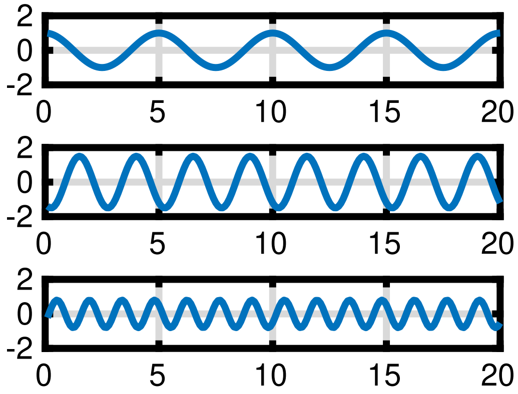
Use Spin Echo

Timing Diagram

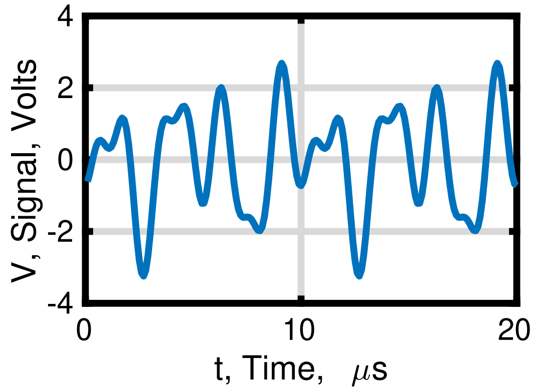


Fourier Transforms

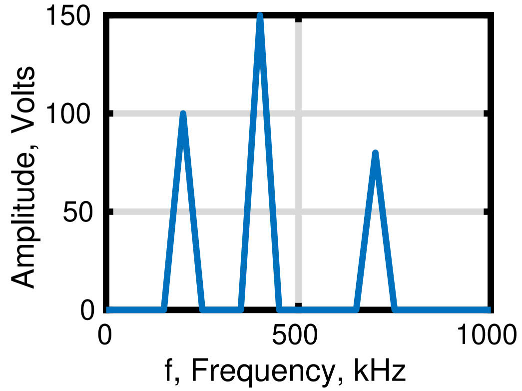
Signals *vs.* Time



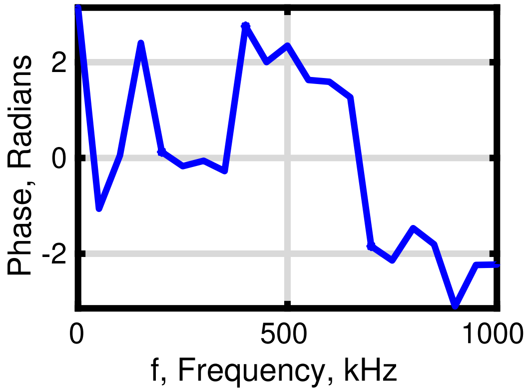
Sum Signal *vs.* Time



IFT of Sum (Amplitude)



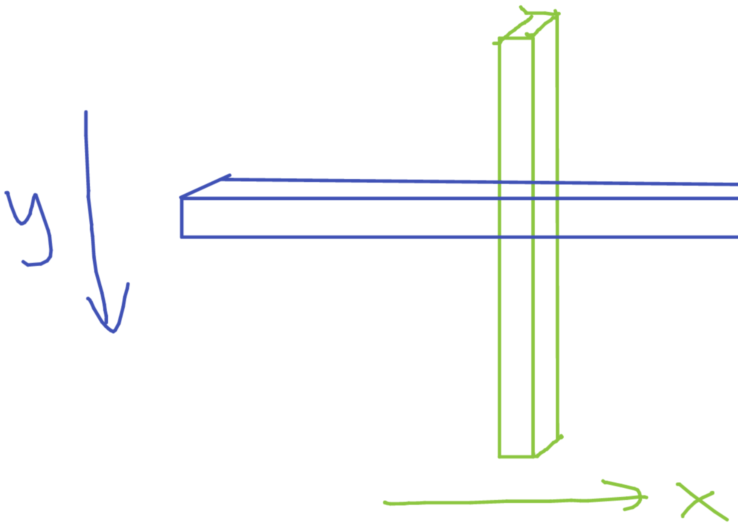
IFT Transform of Sum (Phase)



Row Measurement

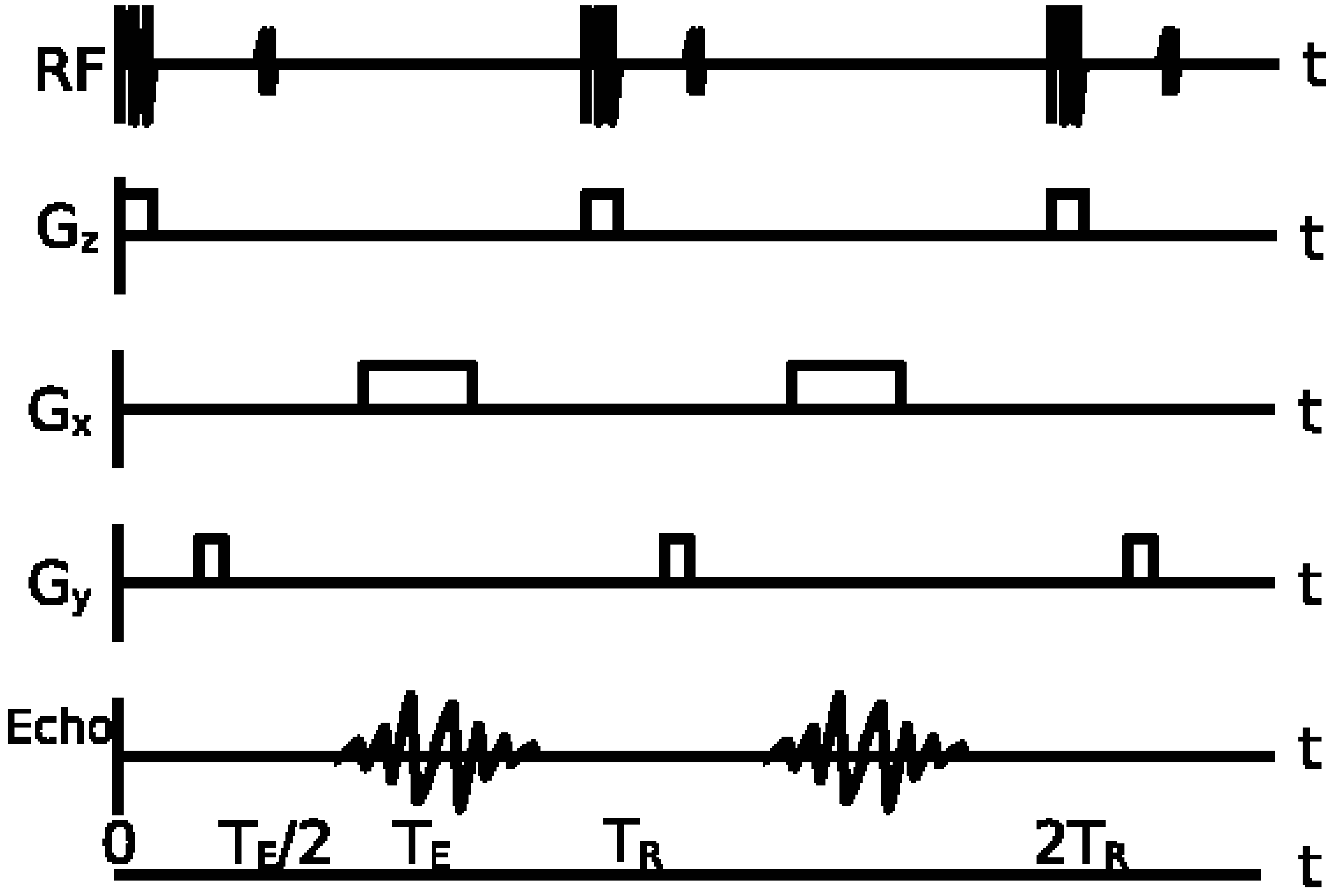
Sort Detected Signal by Phase, $B = B_0 + G_y y$

$$\omega = \gamma B_0 + \gamma G_y y$$



Row Measurement

Timing Diagram



Measurement Time

- $\approx$ 10 ms or more per measurement
- 10 cm cube with 1 mm resolution: 10^6 voxels
- Column Detection in parallel: 10^4 measurements
- 10^4 measurements takes at least 100 s
- Is $(10\text{ cm})^3$ Enough?

Resolution

- Typically 0.5 to 1.0 mm
- Ultimately Depends on Field Gradient
- May Depend on Time and Field of View
- Theoretically Better is Possible
  - Bigger/Better Magnets
  - 0.1 mm Estimated at $250 Million*

Vedrine, *IEEE Trans. Superconductivity*, 2008

More Information

- https://www.cis.rit.edu/htbooks/mri/inside.htm
- https://www.imaios.com/en/e-Courses/e-MRI/
- https://www.youtube.com/watch?v=EDyxBWXp6IU
- https://www.youtube.com/watch?v=1jph1A0hP3U
- Lots of other websites