

# Biomedical Imaging Ultrasound

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# Ultrasound Agenda

- Ultrasound Waves
- Interactions with Materials
- Pulses and Transit Time
- A-Scans
- B-Scans
- More Scans
- Sources and Detectors
- Doppler Ultrasound
- Mixed Modalities: PAT, UOT, *etc.*
- Ultrasound Therapy

# A Wave is a Wave...

- Pressure Difference Causes Acceleration

$$-\nabla P = \rho a = \rho \frac{\partial \mathbf{v}}{\partial t}$$

- Convergence Increases Pressure

$$K \nabla \cdot \mathbf{v} = \frac{\partial P}{\partial t}$$

- Solve for Pressure

$$\nabla^2 P = \frac{\rho}{K} \frac{\partial^2 P}{\partial t^2}$$

- Plane Wave Solution

$$P = P_0 e^{-j(\omega t - kz)}$$

# Plane Waves

- Plane Wave (Previous Page)

$$P = P_0 e^{-j(\omega t - kz)}$$

- Speed

$$c = \sqrt{\frac{K}{\rho}}$$

- Impedance

$$Z = \rho c$$

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$$R = \frac{\frac{\rho'}{\rho} \cos \theta - \frac{n'}{n} \sqrt{1 - \frac{\sin^2 \theta}{(c/c')^2}}}{\frac{\rho'}{\rho} \cos \theta + \frac{n'}{n} \sqrt{1 - \frac{\sin^2 \theta}{(c/c')^2}}} = \frac{\cos \theta - \frac{Z}{Z'} \sqrt{1 - \frac{\sin^2 \theta}{(c/c')^2}}}{\cos \theta + \frac{Z}{Z'} \sqrt{1 - \frac{\sin^2 \theta}{(c/c')^2}}}$$

# Speed

- In Air:  $c = 344$  m/s
- In Water:  $c = 1482$  m/s
- $f\lambda = c$

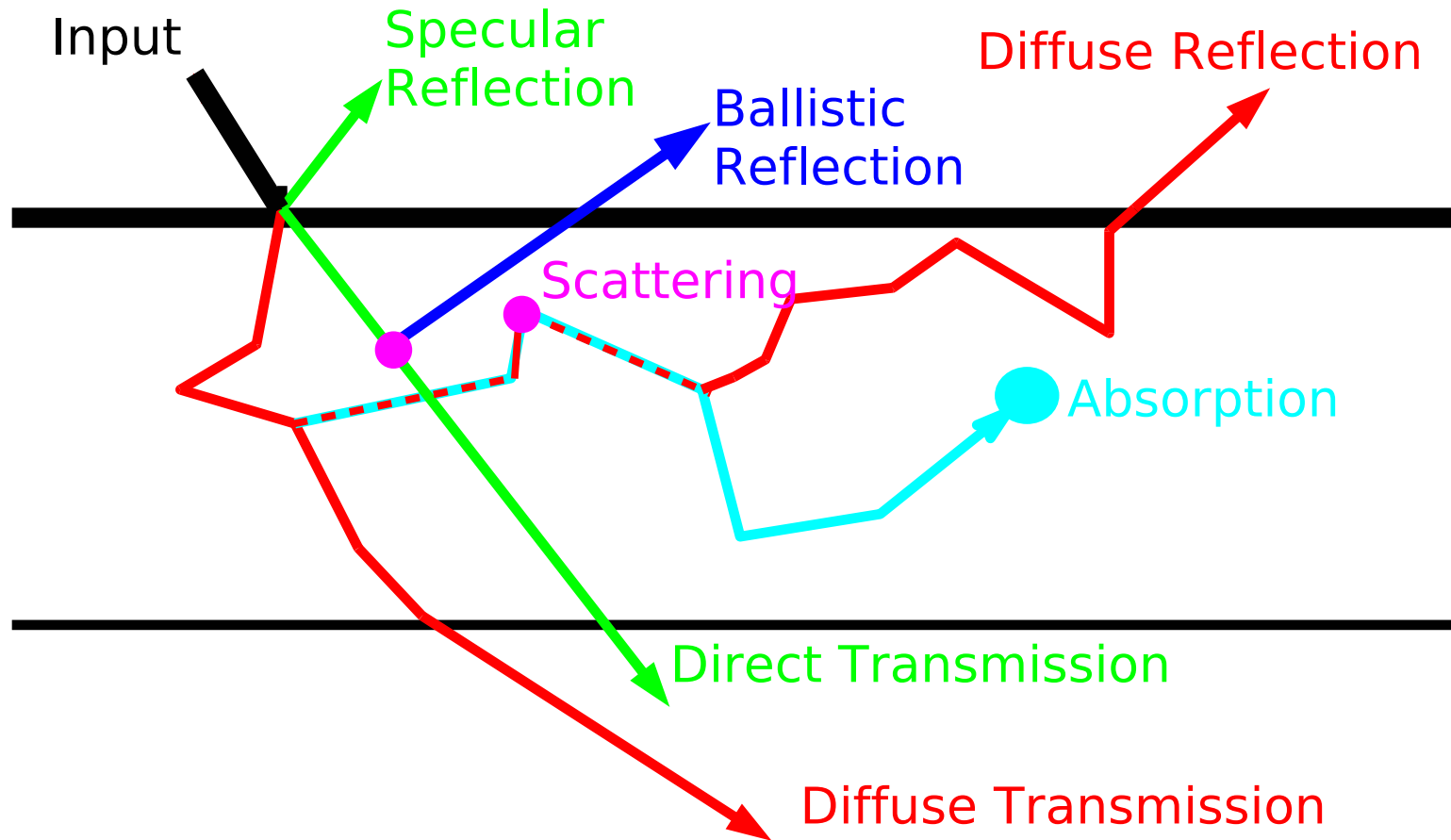
1	MHz	1500	$\mu\text{m}$
2		740	
5		300	
10		150	
100		15	

- Slow Enough for Time-of-Flight
  - 1482 m/s
  - 1482 mm/ms
  - 1482  $\mu\text{m}/\mu\text{s}$
- Round Trip:  $2z = ct$

# Echo from $\rho c$ Contrast



# Waves Interactions



(and reverberation, cavitation)

# Impedance Matching

- Reflection Equation

$$R = \frac{\cos \theta - \frac{Z}{Z'} \sqrt{1 - \frac{\sin^2 \theta}{(c/c')^2}}}{\cos \theta + \frac{Z}{Z'} \sqrt{1 - \frac{\sin^2 \theta}{(c/c')^2}}}$$

$$Z = \rho c \quad \frac{\text{kg}}{\text{m}^2\text{s}} = \text{Rayl}$$

- Match speed and density
- Ultrasound Gel
- $\rho c$  (Rosy!) Rubber
- Acrylamide Gel (Optical Match Too)



# Impedance

## Sound Speed and Impedance

Material	Velocity (mm/ $\mu$ s)	Impedance(MRayl)
Water	1.48	1.48
Blood	1.57	1.61
Liver	1.55	1.65
Kidney	1.56	1.62
Muscle	1.58	1.70
Fat	1.45	1.40
Soft tissue	1.54	1.63
Dense bone	4.10	7.8
Air	0.33	0.0004

S. A. Goss, et al. J. Acoust. Soc. Am. 64(2):423–457, 1978.

S. A. Goss, et al. J. Acoust. Soc. Am. . 68(1):93–108, 1980.

F. A. Duck, *Physical Properties of Tissue (Academic, New York, 1990)*.

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Note: Air to Soft Tissue,  $R = 0.999$

Thanks to Robin Cleveland, Oxford

## Attenuation

For tissue it is common to assume attenuation increases linearly with frequency  $\bar{\alpha} = \alpha f$

Report  $\alpha$  in dB/cm measured at 1 MHz and then extrapolate

$$1 \text{ Np/m} = 0.0869 \text{ dB/cm}$$

Typical **attenuation** in soft tissue:

Kidney	0.32 dB/cm/MHz
Fat	0.63 dB/cm/MHz
Muscle	1.3 dB/cm/MHz
Skin	3.3 dB/cm/MHz
“Average”	0.5 dB/cm/MHz

FDA derating: 0.3 dB/cm/MHz

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Beer's Law;  $e^{-\mu z} = 10^{-\alpha z/10}$  where  $\mu = 10 \log 10\alpha$

Thanks to Robin Cleveland, Oxford

# Attenuation, Frequency, Depth

Attenuation - range decreases with higher frequency

Freq (MHz)	$\lambda$ (mm)	Att. coeff. (dB/cm)	Imaging depth (cm)
2.0	0.75	1.0	15
3.5	0.45	1.8	8
5	0.30	2.5	6
7.5	0.20	3.8	4
10	0.15	5	3

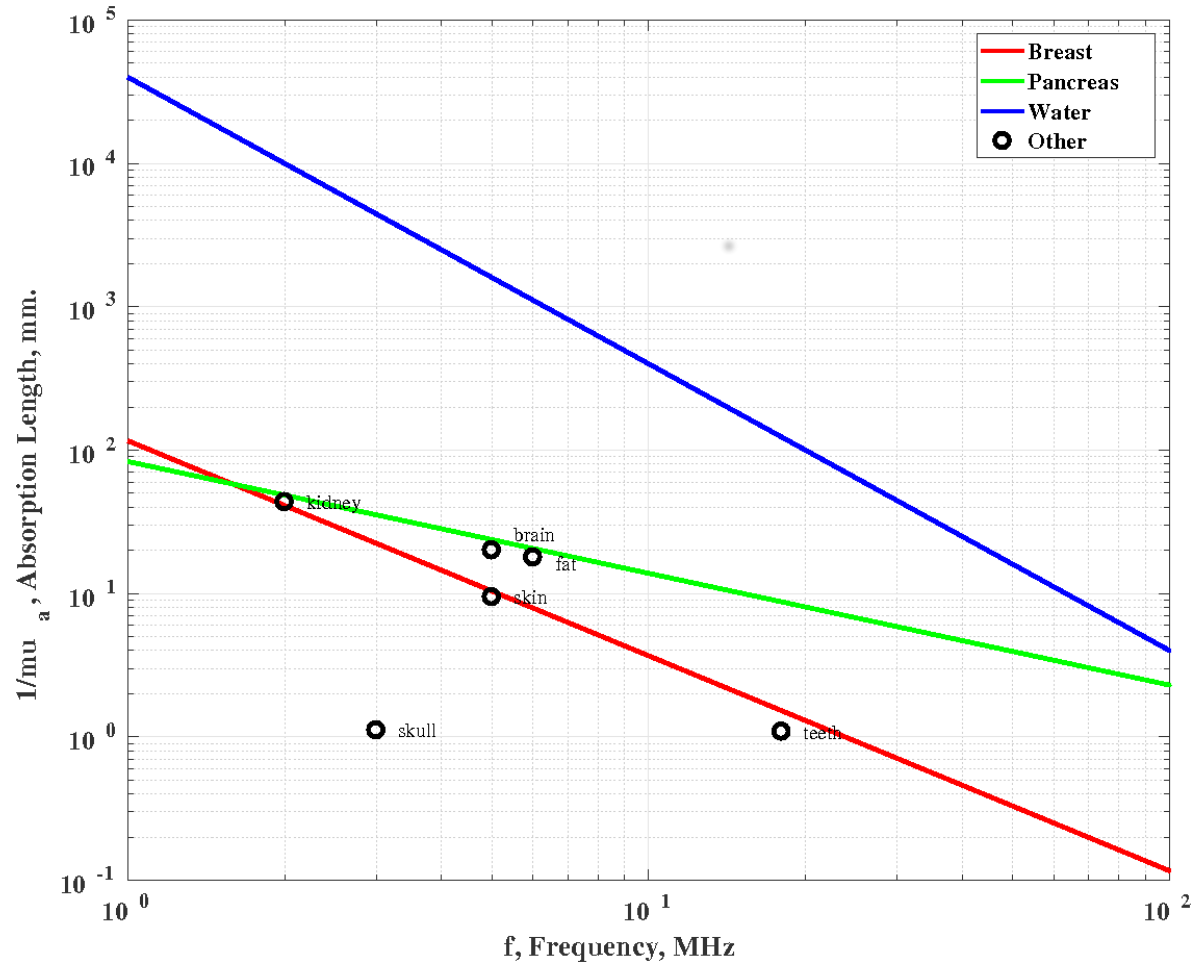
Wavelength:

$$\lambda = \frac{c}{f}$$

Frequency  ↔ Attenuation  ↔ Imaging depth 

Imaging depth is usually on the order of **400 wavelengths** (~ -30dB)

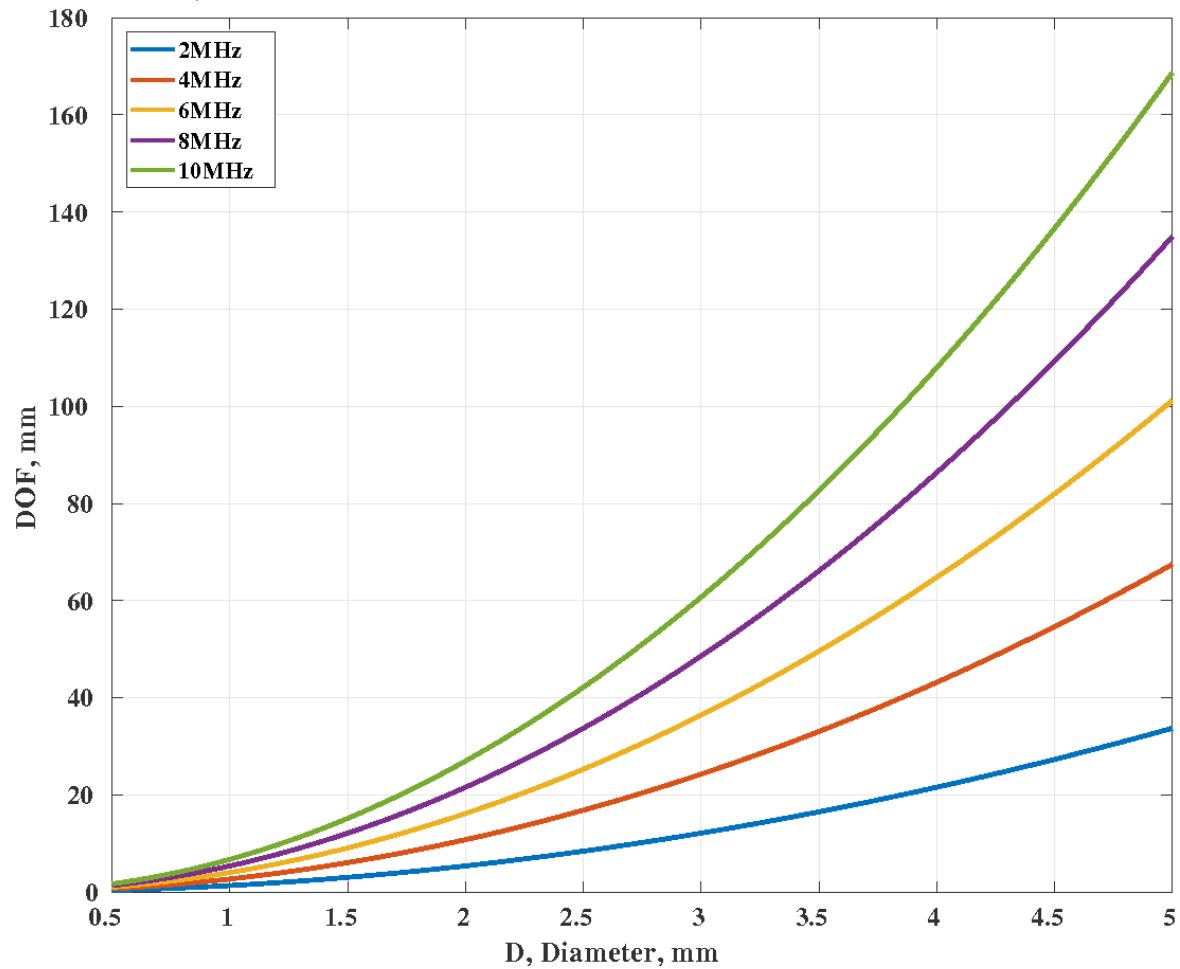
# Extinction



# Resolution & Depth of Focus

Resolution:  $\lambda/NA$

Depth of Focus:  $\lambda/NA^2$



# A-Scan

- One Pulse
- Signal vs. Depth
- Assume Known  $c$

$$2z = ct$$

- Transverse Resolution  $\lambda/NA$
- Axial Resolution for Pulse Length  $\tau$

$$2z = c\tau$$

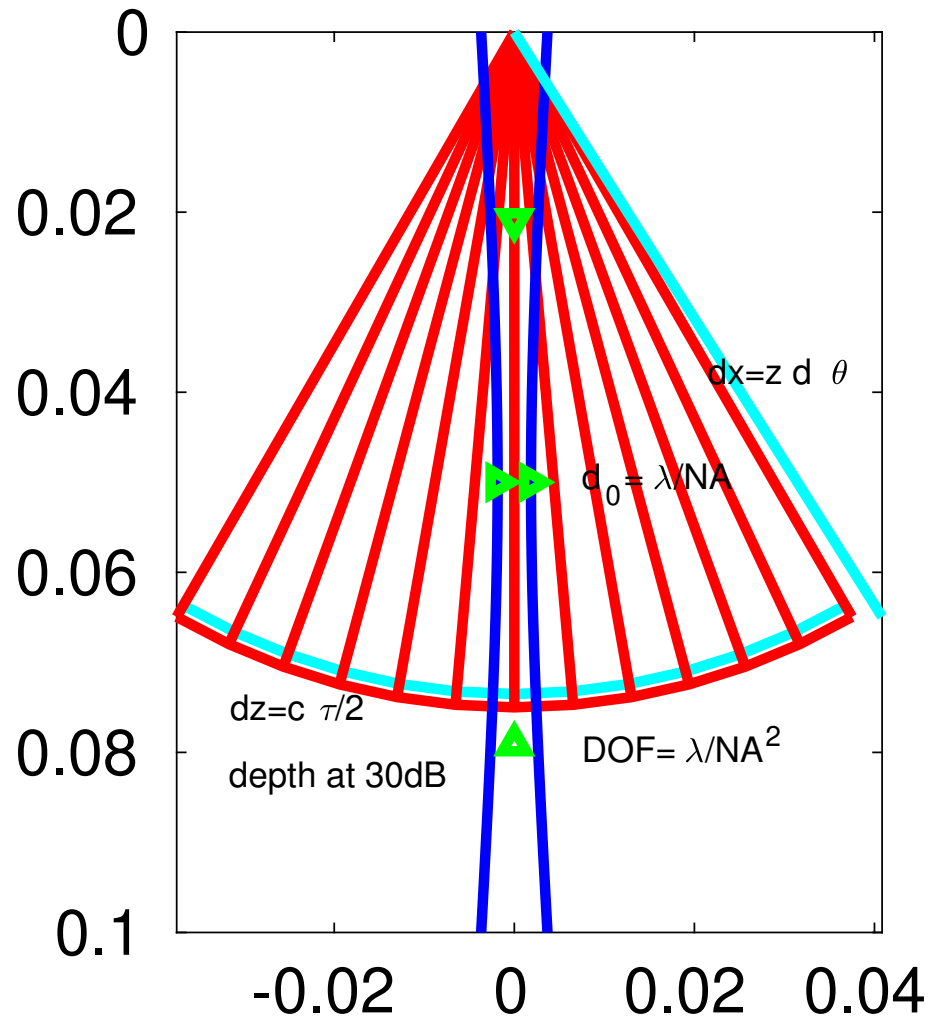
- Avoid Ambiguity (Pulse Repetition Frequency)

# B-Scan



Thanks to Robin Cleveland, Oxford

# Ultrasound Distances





- Piezoelectric Transducer (PZT)
  - Usually Resonant, Moderate  $Q$
  - Focused or Not
  - Arrays or Not
- Transmit / Receive Switch
- Maybe Dynamic Focus on Receiver

# Doppler Ultrasound

- “Color Doppler”
- Principles

$$f_{doppler} = \frac{2v_{\parallel}}{\lambda}$$

- 100s to 1000s of Hz.
- Pulsed or CW? (Resolution and Ambiguity?)

- C-Scans
- Microbubbles
- Elastography
- Photoacoustic Tomography
- Ultrasound Modulated Optical Tomography

# Speckle

- Interference Effect
- Most Noticable with Highly Coherent Sources
  - Ultrasound
  - Optical Imaging with Laser Sources
- Random, High-Contrast Pattern
- Normally Unwanted



# Image Time

- A-Scan Limited by Depth (wait for return)
- B-Scan Limited by Transverse Resolution Requirement
- Example
  - 20-cm Depth ( $260 \mu\text{s}$  for A-Scan)
  - 128 A-Scans per B-Scan
  - Total 24 ms (29 Hz Frame Rate)

# Safety


- Mechanical Index  
Cavitation
- Thermal Index  
Heating

# High Power Focused Ultrasound (HIFU)

**Lesion**

**US beam direction**

Heat Diffusion Equation      Heat Deposition

$$\rho C \frac{\partial T}{\partial t} = \kappa \nabla^2 T + 2\alpha I$$


**Beef Liver**

The Institute of Cancer Research

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Thanks to Robin Cleveland, Oxford

## Applications of HIFU

- Ophthalmology
  - FDA approval 1985
- Cancer
  - Liver, kidney, prostate, breast, brain, skin...
- Non Cancer
  - Uterine fibroids, liver surgery, BPH, ...
- Trauma Care
  - Acoustic hemostasis through vessel occlusion
    - Transcutaneous
    - Intraoperative



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