Disjoint versus Independent Events:

"Disjoint" and "Mutually Exclusive" are equivalent terms

Def: Disjoint Events

Two events, say A and B, are defined as being disjoint if the occurrence of one precludes the occurrence of the other; that is, they have no common outcome. Mathematically: $AB = \{\}$.

NOTE: It is tempting to consider this to mean that the events are independent…. Why is this not the case? Lets look at the definition of independent events:

Def: Independent Events

Two events, say A and B, are defined as being statistically independent if the occurrence of one event has no effect on the probability of the occurrence of the other; assuming that $P\{A\}$ and $P\{B\}$ are non-zero: $P\{A \mid B\} = P\{A\}$ and $P\{B \mid A\} = P\{B\}$.

OBSERVE:
$P\{AB\} = P\{A|B\}P\{B\} = P\{A\}P\{B\}$
$P\{A\} = P\{AB\}/P\{B\}$
If the events were disjoint, then $P\{AB\}$ would be zero—hence, $P\{A\}$ would have to be zero (which contradicts the assumption that $P\{A\}$ is non-zero…

CONCLUSION: The occurrence of an event that is disjoint from some other event actually tells you something about the probability of occurrence of the other; **hence disjoint events cannot be independent**: $P\{A\}P\{B\} = P\{AB\}$: independence; whereas $P\{AB\} = 0$: disjoint…

Collective versus Pairwise Indendent Events:

Consider a set of events: A_1, A_2, … A_n.

Pairwise indepence is a less rigid requirement than collective indepence… The set of events are considered to be pairwise independent if $P\{A_iA_j\} = P\{A_i\}P\{A_j\}$ for (i,j) pairs.

NOTE: Pairwise independence does not imply collective independence over a set of events… Why?

Collective independence means that none of the events individually or collectively will affect the probability of occurrence of any other event; hence, for three events: $P\{A_1A_2A_3\} = P\{A_1\}P\{A_2\} P\{A_3\}$

Counter Example from homework #2…