ECE 3484 - Combinatorial Optimization - Problem Set #2

Solve the following problems. Proofs should be as precise and concise as possible. Clearly state any results that your arguments depend on.

1. (a) Prove that if A_1, A_2, and A_3 are decision problems, $A_1 \leq A_2$ and $A_2 \leq A_3$, then $A_1 \leq A_3$, where $\leq$ represents polynomial reduction.

   (b) Prove that for decision problems A_1 and A_2, if $A_1 \leq A_2$, $A_1 \in NPC$, and $A_2 \in NP$, then $A_2 \in NPC$.

   (c) Why is it considered unlikely that any NP-complete problem is tractable? Use the definition of NPC to give a precise argument.

   Prove that the following decision problems are NP-complete using the indicated reduction. Don't forget to show that each problem is $\in$ NP:

2. Up-To-4-Satisfiability: Given an expression E consisting of a set of clauses C_i each containing up to 4 literals, does a satisfying truth assignment exist? (use a transformation from 3SAT)

3. 4-Satisfiability: Given an expression E consisting of a set of clauses C_i each containing exactly 4 literals, does a satisfying truth assignment exist? (use a transformation from 3SAT)

4. Clique: Given a graph G(V,E), does G contain a clique of size k? A *clique* of size k is a subset of the vertices V of size k, such that each vertex in the clique is directly connected to every other vertex in the clique. (use the transformation from 3SAT given in Cormen)