ECE 3484 - Combinatorial Optimization - Problem Set #3

Solve the following problems. Proofs should be as precise and concise as possible. Clearly state any results that your arguments depend on.

1. Prove that all maximal subsets of a matroid are the same size. Explain why this makes intuitive sense for the cyclic, head-partition, and partition matroids.

2. Argue that activity selection either can or cannot be formulated as an optimization problem over a matroid.

3. Demonstrate that not all maximal subsets are optimal within the partition matroid.

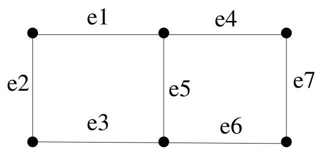


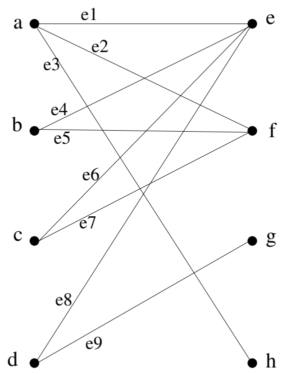
Figure 3.

4. For the Maximum Matching problem applied to the graph in Figure 3, does the greedy algorithm always find an optimal solution? Are all maximal subsets the same size? Give a proof or a counter example.

5. For the problem of finding a Maximum Independent Subset of the set $E = \{a, b, c, d, e, f, g\}$, given partition $\Pi = \{\{a, b, c\}, \{d, e\}, \{f, g\}\}$, answer the questions in Problem 4.

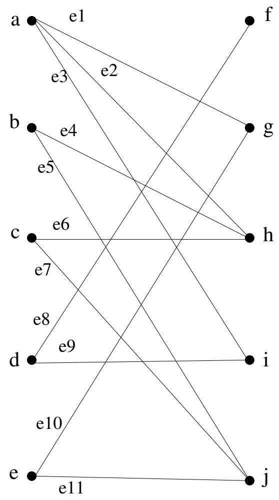
6. For the problem of finding a Maximum Matching for the bipartite graph (V, U, E) in Figure 4:
   (a) find two maximal independent subsets of E that are not the same size - what does their existence imply?
   (b) show how this instance can be represented as the intersection of two partition matroids.

7. Consider the problem of finding a Maximum Independent Subset of the set $E = \{a, b, c, d, e, f, g\}$, given partitions $\Pi_1 = \{\{a, b, c\}, \{d, e\}, \{f, g\}\}$ and $\Pi_2 = \{\{a, f\}, \{d, g\}, \{b, c, e\}\}$. In this case to be independent a set must be independent in both Π_1 and Π_2. Show how this instance can be represented as the intersection of two partition matroids.



A = {e1,e5,e9}

Figure 4.



A = {e1,e5,e6,e9}

Figure 5.

8. For the Maximum Matching problem applied to the bipartite graphs in Figures 4 and 5:
   (a) given the matching A in each case, give augmenting paths containing three and five edges, if they exist,
   (b) given the matching A in each case, show the vertices visited in the breadth-first search for an augmenting path,
   (c) give the corresponding improved matching,
9. For the matroid intersections shown in Figure 8.3 of Cook:
   (a) explain the construction of the auxiliary graphs,
   (b) give the improved independent set for every augmenting path.