

Blackbody Distribution Function

The radiant power emitted per unit area by an object at temperature T at wavelengths between λ and $\lambda+d\lambda$ is given by the Planck law, where $\epsilon(\lambda, T)$ is the object emissivity:

$$M_{e,\lambda}(\lambda, T)d\lambda = \epsilon(\lambda, T) \frac{2\pi hc^2 d\lambda}{\lambda^5 [e^{hc/\lambda kT} - 1]}$$

- a. Plot $M_{e,\lambda}$ as a function of λ for $10\text{nm} < \lambda < 10000\text{ nm}$ for an object with $\epsilon(\lambda, T)=1$ (“blackbody”) at $T=5000\text{K}$. Use semilogx and loglog plot.
- b. What is the wavelength of maximum energy emission?
- c. How much total power is emitted between $\lambda=550\text{nm}$ and $\lambda=560\text{nm}$ from a 1cm^2 area of this object?
- d. Find an expression for $M_{p,\lambda}d\lambda$, the number of photons emitted per unit area between λ and $\lambda+d\lambda$ by an object at temperature T with emissivity $\epsilon(\lambda, T)$.
- e. Plot $M_{p,\lambda}$ as a function of λ for $10\text{nm} < \lambda < 10000\text{ nm}$ for an object with $\epsilon(\lambda, T)=1$ (“blackbody”) at $T=5000\text{K}$. Use semilogx and loglog plots.
- f. What is the wavelength at which the maximum number of photons are emitted.
- g. How many photons/second are emitted between $\lambda=550\text{nm}$ and $\lambda=560\text{nm}$ from a 1cm^2 area of this object?
- h. Find an expression for $M_{e,\nu}$ where $M_{e,\nu}d\nu$ is the radiant power emitted per unit area by an object at temperature T between frequency ν and $\nu+d\nu$.
- i. Plot $M_{e,\nu}$ as a function of ν from $\nu_1=c/10000\text{nm}$ to $\nu_2=c/100\text{nm}$ for an object with $\epsilon(\lambda, T)=1$ (“blackbody”) at $T=5000\text{K}$. Use semilogx and loglog plots.
- j. What is the frequency of maximum energy emission? What wavelength does this frequency correspond to.
- k. How much total power is emitted between $\nu=c/550\text{nm}$ and $\nu=c/560\text{nm}$ from a 1cm^2 area of this object?
- l. Find an expression for $M_{p,\nu}d\nu$, the number of photons emitter per unit area between ν and

$\nu+d\nu$ by an object at temperature T with emissivity $\epsilon(\nu,T)$.

m. Plot $M_{p,\nu}$ as a function of ν from $\nu_1=c/10000\text{nm}$ to $\nu_2=c/100\text{nm}$ for an object with $\epsilon(\nu,T)=1$ ("blackbody") at $T=5000\text{K}$. Use semilogx and loglog plots.

n. At what frequency is the maximum number of photons emitted? What wavelength does this frequency correspond to?

o. How many photons/second are emitted between $\nu=c/550\text{nm}$ and $\nu=c/560\text{nm}$ from a 1cm^2 area of this object?

p. It is often stated that the human eye, with a maximum sensitivity at around 550nm , has evolved to be most sensitive at the wavelength of the maximum output of the sun (modeled as a blackbody of $T=5000\text{K}$). Based on the results of this exercise, would you have any misgivings about this statement?