[1]

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

NORTHEASTERN UNIVERSITY

ECE G287 **OPTICAL DETECTION** **Spring 2004**

Homework Set 1, Problem 2

Here we explore the assumptions and results associated with the Poisson distribution. Start with an unknown distribution function;

$$P(n, T),$$

which is the probability of detecting n photons in a time interval T. Now, assume a short additional time, dT, during which we can expect either one or no photons. The probability of detecting one photon during this time is αdt.

(a) Write an expression for

$$P(n+1, T+dT),$$

in terms of $P(n,t)$ and α.

(b) Now arrange this expression as a differential equation with everything involving $P(n+1, ?)$ on the left and $P(n, ?)$ on the right.

(c) Now, if you are feeling courageous, solve this differential equation. If not, substitute in

$$P(n,t) = e^{-\bar{n}} \frac{\bar{n}^n}{n!},$$

and find an equation for $\bar{n}$.

(d) Plot the distribution for different values of $\bar{n}$, in powers of 10, from 10^{-2} to 10^3. You may wish to use logarithmic scales on some of these plots.

(e) Show that

$$\sigma_n = \sqrt{n}.$$

[1] 10478/poisson.tex — 15 January 2004 at 12:07