ECEG333 Midterm Project
Multiple Dwell Synchronization for Multipath Fading Channels.

1 Overview

In this project you will design and investigate the performance of a synchronizer in the presence of multipath fading. To assist you in this investigation, I will provide the structure of the detector which explores each candidate timing phase, the statistics of the decision variable at any candidate timing phase, and some underlying assumptions which will make your effort a little easier.

The written report for this project will be graded as the midterm plus two homeworks for the course. The report must address (at least) a specific set of points at the end of this document. It may be written by yourself, or with at most one other student. If you work with another student, you may not submit a single-author report. In that case, both authors must be listed on the single report. Further, you may not consult anyone outside the author list besides myself about this project (i.e., do not brainstorm with experts in your field, other students, etc.) In short, I want the ideas in each report to be the sole possession of the authors of the report.

This report should be submitted to me in pdf format no later than 5pm on Monday, March 8, 2004.

2 Multipath Fading Model

In our course material, we have assumed that the received baseband signal is affected only by attenuation and additive white Gaussian noise,

$$r(t) = \sqrt{P}c(t - T_d) + n(t).$$

In some applications, there are multiple received copies of the signal which are presented to the receiver at slightly different delays. We represent the baseband signal as the convolution of a complex impulse response $h(\tau)$ and the above signal, where

$$h(\tau) = \sum_{m=1}^{M} h_M u_0(\tau - \tau_m),$$

where the delays are ordered $0 = \tau_1 < \tau_m < \tau_{m+1}$, $u_o(\tau)$ is the Dirac delta function, and h_m are complex random variables. Note that M, the total number of paths, is usually unknown. The equivalent received signal from the output of this multipath channel is

$$r(t) = \sum_{m=1}^{M} h_m c(t - \tau_m - T_d) + n(t), \tag{1}$$

where we have included the effects of signal attenuation into the multipath channel.

It is clear that the presence of multiple copies of the signal will lead to acquisition problems, namely false declarations of the propagation delay $\hat{T}_d$ to a value $T_d + \tau_m$, even in the absence of additive noise. The statistics of the random variable sequence $\{h_i\}$ will be very helpful in this regard: we will assume that the random variables are mutually independent, and that the real random variable $|h_i|^2$ is an exponential random variable having a probability density function

$$f_i(h) = \begin{cases} \alpha_i \mathrm{e}^{-\alpha_i t}, & t > 0, \\ 0 & t < 0. \end{cases}$$

For this project will assume that we know the set $\{\alpha_i\}$, at least, as a function of the delay τ_i. For two known values of T_m and α_0, the relationship is

$$\frac{1}{\alpha_i(\tau_i)} = \frac{1}{\alpha_0} \mathrm{e}^{-\tau_i/T_m}. \tag{2}$$

In practice, the value of T_m is related to the delay spread of the multipath channel, and the value of α_0 may be determined from knowledge of the average received power.

3 Detector Model

Each candidate timing phase will be explored for a user-specified period of time, with a response of either hit (1) or miss (0) at the end of this period. The circuit of Figure 5.8 in the text will be the one used for all such explorations. This circuit required two parameters from the user: the integration period T_i (which we assume to be an integer number of periods of the spreading waveform $c(t)$, and a decision threshold v_i.

While the circuit is the same as for a single received signal, the method of threshold setting is completely different, due to the random nature of the multipath amplitudes. In this section, we present the method of threshold setting that you should use.

We begin with equation 5-63 in the text, which provides an expression for the output of the integrator in the detector circuit. We will make the following simplifying assumptions with regards to this expression. We will assume that for each candidate timing phase, the integrator output corresponds to 5-63, by replacing $\sqrt{P}\cos(\theta)$ by $real(h_i)$ and $\sqrt{P}\sin(\theta)$ by $imag(h_i)$ for some i. Equivalently, we assume that adjacent paths have separations greater than T_c, the chip period, and that the delays $T_d - \tau_m$ fall exactly on the grid of candidate delays. Thus, we assume that each attempted timing phase is exactly $T_d - \tau_m$ for some m and the detector output is not effected by all other paths, or the detector output is not affected by any paths.

If the output of the detector is effected by the i^{th} path, then the decision statistic is a chi-squared random variable with $2BT_i$ degrees of freedom, multiplied by a constant. Here the constant is $1 + \frac{1}{2\alpha_i(\tau_i)}$. A chi-squared random variable with n-degrees of freedom has a mean value of n, a variance of $2n$, and may be approximated by the Gaussian distribution if n exceeds 10. The probability density function of the chi-squared random variable is given in the text by equation 5-61. For the setting of v_i, equation 5-67b may be used to relate P_{fa} to the voltage level. The detection probability P_d may be related to the voltage v_i by using equation 5-67b as well, only replacing v_i by $\frac{v_i 2\alpha_i(\tau_i)}{1+2\alpha_i(\tau_i)}$ in that expression. Note that we are using v_i here to denote the decision threshold, while equation 5-67b uses V_T.

4 Acquisition Model

In this section, we will state the assumptions which you will use to model the acquisition process. Denote the total number of candidate delays by C, as in the text. We will assume an unusually coarse delay grid with increment of T_c, in order to satisfy previous assumptions. We will also assume that C coincides with the period of the spreading sequence (N in the text), so that we span a full period of delays. If we assume that the candidate delays are indexed by the integers 1 through C, then we shall denote the correct delay by D such that $1 \leq D \leq C$. We shall assume that the delay increases with indexing. With this ordering, index $D+1$ corresponds to the cell having a path at lag T_c, and index $D+n$ corresponds to the cell having the path at lag nT_c. We will also assume that $T_m/T_c << C$, so that the number of cells with significant multipath will be a small fraction of C. If $D = C$ for example, the cells with indices $1, 2, \ldots$ will have significant multipath content, since $c(t)$ is a periodic function, with period CT_c.

The decision variable at index D will be a chi-squared random variable, multiplied by a scale factor of $1 + \frac{1}{2\alpha_0}$, and the decision variable at index $D+n$ will be a chi-squared random variable, multiplied by a scale factor of $1 + \frac{1}{2\alpha_0 \mathrm{e}^{-(n-1)T_c/T_m}}$. You may make the simplifying assumption that if cell index n, $n \neq D$ is selected as the estimated delay, then the time penalty to discover this false alarm is $T_{fa} = 40CT_c$, for any $n \neq D$. Note, however, that the probability of making this false alarm should depend on the index n.

5 Tasks

In this project, you will design and analyze a multiple-dwell detector to acquire the correct delay index D. Your design will use knowledge of T_m, N_0, B, α_0, C, and T_c. Your design goal will be to minimize the average acquisition time. You will do this by choosing T_i and v_i for each detection stage of your multiple dwell detector. You will not have to design the tracking loop portion of the synchronization, however. If your acquisition system enters the tracking loop with an incorrect timing estimate, simply account for the timing penalty as T_{fa} defined above. If the tracking loop is passed the correct timing estimate, then there is no time penalty.

You must provide convincing analysis of the average acquisition time for your proposed system. One method of doing this is to use state flow diagrams, as presented in the 2/18/04 and 2/23/04 lectures.

6 Report Format

At a minimum, your report should address each of the following points:

1. A flowchart indicating the number of stages of acquisition, with 'hit' and 'miss' directed arcs emanating from each stage.

2. A clearly presented method of setting T_i and v_i for each stage. Examples include a graph or equation, relating T_i and v_i to the independent variables described earlier. For this analysis, you may assume $C = 31$, $B = 2/T_c$, $T_m = 4T_c$. You should vary the other quantities to show a wide range of acquisition performance for your system.

3. The improvement in acquisition time for your system over a single-dwell acquisition system.