Hence, as $E_s/N_0 \to 0$, the upper and lower bounds for P_1 become

$$\text{Upper bound}: P_1 \le \frac{\pi}{4}\frac{\cos\left(\frac{\pi}{2M}\right)}{\sqrt{\cos(\pi/M)}}$$

$$\text{Lower bound}: P_1 \ge 0$$

In the limit as $E_s/N_0 \to \infty$, The upper and lower bounds for P_1 become

$$\text{Upper bound}: P_1 \le \frac{\pi}{2}\frac{\cos\left(\frac{\pi}{2M}\right)}{\sqrt{\cos(\pi/M)}}\frac{\exp\left[-2\left(\frac{E_s}{N_0}\right)\sin^2\left(\frac{\pi}{2M}\right)\right]}{2\sqrt{2\pi\frac{E_s}{N_0}\sin\left(\frac{\pi}{2M}\right)}}$$

$$\text{Lower bound}: P_1 \ge \frac{1}{2}\frac{\cos\left(\frac{\pi}{2M}\right)}{\sqrt{\cos(\pi/M)}}\frac{\exp\left[-2\left(\frac{E_s}{N_0}\right)\sin^2\left(\frac{\pi}{2M}\right)\right]}{2\sqrt{2\pi\frac{E_s}{N_0}\sin\left(\frac{\pi}{2M}\right)}}$$

Problem 1-5

A Mathcad solve function for finding the argument of the Q-function to give a desired value is given on the next page. There it is seen that $Q(x) = 10^{-5}$ for $x = 4.276$. The bit error probability for BPSK is given by

$$P_{E,\text{BPSK}} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Thus $E_b/N_0 = d^2/2 = PT_b/N_0$. Solving for P and putting in values, we find that

$$P_{\text{BPSK}} = \frac{E_b}{N_0}\frac{1}{T_b}N_0 = \frac{d^2}{2}R_b N_0 = 0.091 \text{ watts}$$

where R_b is the data rate. For coherent FSK

$$P_{E,\text{CFPSK}} = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

so $E_b/N_0 = d^2 = PT_b/N_0$ and putting in values we obtain $P = 0.018$ watts. For coherent ASK we have the same relationship to E_b/N_0. Putting in values we get 5.49×10^{-4} watts.

Increasing the data rate by a factor of 10 increases the power requirements by a factor of 10, and decreasing the noise power spectral density by a factor of 10 decreases the power requirements by a factor of 10.

Q-function definition:

$p := .2316419$ $\quad b_1 := .31938153$ $\quad b_2 := -.356563782$ $\quad b_3 := 1.781477937$

$b_4 := -1.821255978$ $\quad b_5 := 1.330274429$

$Z(x) := \frac{e^{-\frac{x^2}{2}}}{\sqrt{2 \cdot \pi}}$ $\qquad t(x) := \frac{1}{(1 + p \cdot x)}$

$Q0C(x) := Z(x) \cdot \left(b_1 \cdot t(x) + b_2 \cdot t(x)^2 + b_3 \cdot t(x)^3 + b_4 \cdot t(x)^4 + b_5 \cdot t(x)^5 \right)$ $\qquad Q0(x) := \frac{Z(x)}{x}$

$Q1(x) := \text{if}(|x| > 4, Q0(x), Q0C(x))$ $\qquad Q2(x) := 1 - Q1(|x|)$

$Q(x) := \text{if}(x > 0, Q1(x), Q2(x))$

$x := 1$ $\qquad P_0 := 10^{-5}$

Given

$Q(x) = P_0$

$d := \text{Find}(x)$

$d = 4.276$

Problem 1-6

(a) For ASK and BPSK, the bandwidth requirement in terms of data rate, from (1-15), is

$$B_{\text{BPSK, ASK}} = 2R_b$$

so the maximum data rate that can be supported is 5 kbps. For CFSK, (1-16) gives

$$B_{\text{CFSK}} = 2.5R_b$$

so the maximum data rate that can be supported is 10/2.5 = 4 kbps.

(b) This can be solved similarly to Problem 1-5 except with different data rates and noise power spectral density. The results are given in the table on the next page.

4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101
10	1010	1111
11	1011	1110
12	1100	1010
13	1101	1011
14	1110	1001
15	1111	1000

Problem 1-10

Normalize $s_1(t)$ for the first basis function:

$$\int_0^{\infty} |s_1(t)|^2 dt = \frac{1}{2}$$

and

$$\phi_1(t) = \sqrt{2}\, e^{-t} u(t)$$

Now form

$$v_2(t) = s_2(t) - (s_2, \phi_1)\phi_1(t) = \left[e^{-2t} - \int_0^{\infty} e^{-2t} \sqrt{2}\, e^{-t} dt \sqrt{2}\, e^{-t} \right] u(t) = \left[e^{-2t} - \frac{2}{3} e^{-t} \right] u(t)$$

Find the normalization for this second basis function:

$$\int_0^{\infty} |v_2(t)|^2 dt = \frac{1}{36}$$

Therefore

$$\phi_2(t) = 6\left[e^{-2t} - \frac{2}{3}e^{-t}\right]u(t)$$

Finally, form

$$v_3(t) = s_3(t) - (s_3,\phi_2)\phi_2(t) - (s_3,\phi_1)\phi_1(t)$$

$$= \left[e^{-2t} - \int_0^\infty e^{-3t}6\left(e^{-2t} - \frac{2}{3}e^{-t}\right)dt\, 6\left(e^{-2t} - \frac{2}{3}e^{-t}\right) - \int_0^\infty e^{-2t}\sqrt{2}\,e^{-t}dt\,\sqrt{2}e^{-t}\right]u(t)$$

$$= \left[e^{-3t} - \frac{6}{5}e^{-2t} + \frac{3}{10}e^{-t}\right]u(t)$$

The normalization for this third basis function is found from

$$\int_0^\infty |v_3(t)|^2 dt = \frac{1}{600}$$

which gives

$$\phi_3(t) = \sqrt{6}\left[10\,e^{-3t} - 12\,e^{-2t} + 3\,e^{-t}\right]u(t)$$

Problem 1-11

First, a derivation of (1-41) is provided. Consider the average of the square of (1-39):

$$E_s = \overline{\int_0^{T_s} s_i^2(t)\,dt} = \overline{\frac{2}{T_s}\int_0^{T_s}(A_i\cos\omega_c t + B_i\sin\omega_c t)^2 dt} = \frac{2}{T_s}\left(\frac{\overline{A_i^2}}{2} + \frac{\overline{B_i^2}}{2}\right) = \overline{A_i^2} + \overline{B_i^2}$$

Assume each value of A_i and B_i to be independent and equally likely where the number of signal points is $M = 4^n = 2^{2n}$. Then

$$\overline{A_i^2} = \overline{B_i^2} = 2\frac{a^2}{2^n}\sum_{i=1}^{2^{n-1}}(2i-1)^2 = \frac{a^2}{2^{n-1}}\left[4\sum_{i=1}^{2^{n-1}} i^2 - 4\sum_{i=1}^{2^{n-1}} i + \sum_{i=1}^{2^{n-1}} 1\right]$$

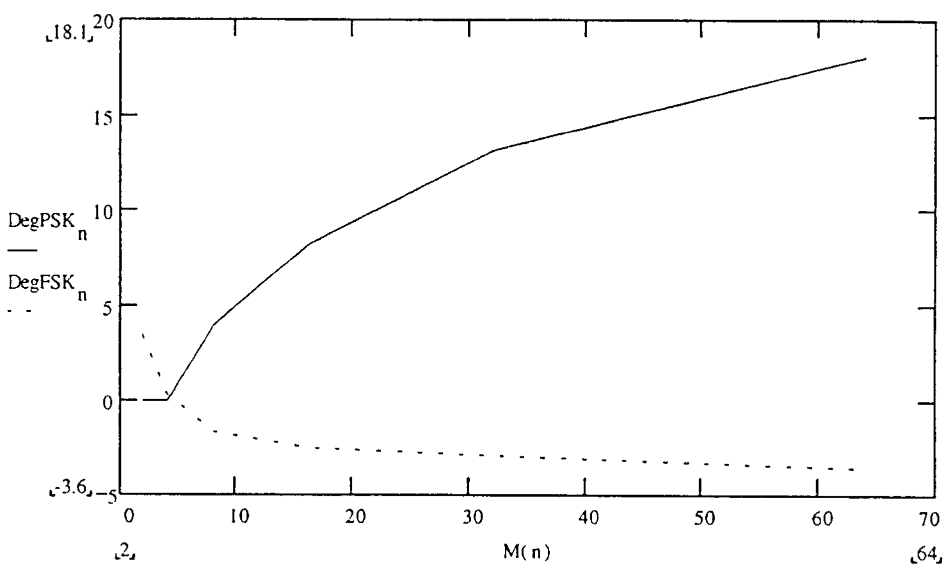
Make use of the summations

$$\sum_{i=1}^{m} i = \frac{m(m+1)}{2} \quad\text{and}\quad \sum_{i=1}^{m} i^2 = \frac{m(m+1)(2m+1)}{6}$$

Problem 1-20

Plots only for M-ary PSK and M-ary CFSK will be given here. From the curves of Figures 1-16 and 1-17, the following data is obtained. Other results can be obtained from the curves of Figures 1-18 -1-20.

M	MPSK; dB degrad. from QPSK	CFSK; dB degrad. from QPSK
2	0	3.4
4	0	0.3
8	3.8	-1.7
16	8.1	-2.5
32	13.1	-3.0
64	18.1	-3.6



Problem 1-21

(a) Note that $d(t) = \pm 1$, and write

$$s(t) = A \sin[\omega_c t \pm \cos^{-1}(a)]$$
$$= A \{ \cos[\pm\cos^{-1}(a)] \sin \omega_c t + \sin[\pm\cos^{-1}(a)] \sin \omega_c t \}$$
$$= A \{ \cos[\cos^{-1}(a)] \sin \omega_c t \pm \sin[\cos^{-1}(a)] \sin \omega_c t \}$$

where the last equation follows because sine is odd and cosine is even. Now substitute

$$\cos[\cos^{-1}(a)] = a \text{ and } \sin[\cos^{-1}(a)] = \sqrt{1 - a^2}$$

to obtain

$$s(t) = Aa \sin \omega_c t + d(t)A\sqrt{1 - a^2} \cos \omega_c t$$

where the ± 1 has been replaced by $d(t)$. The first term is the carrier component and the second term is the modulation component.

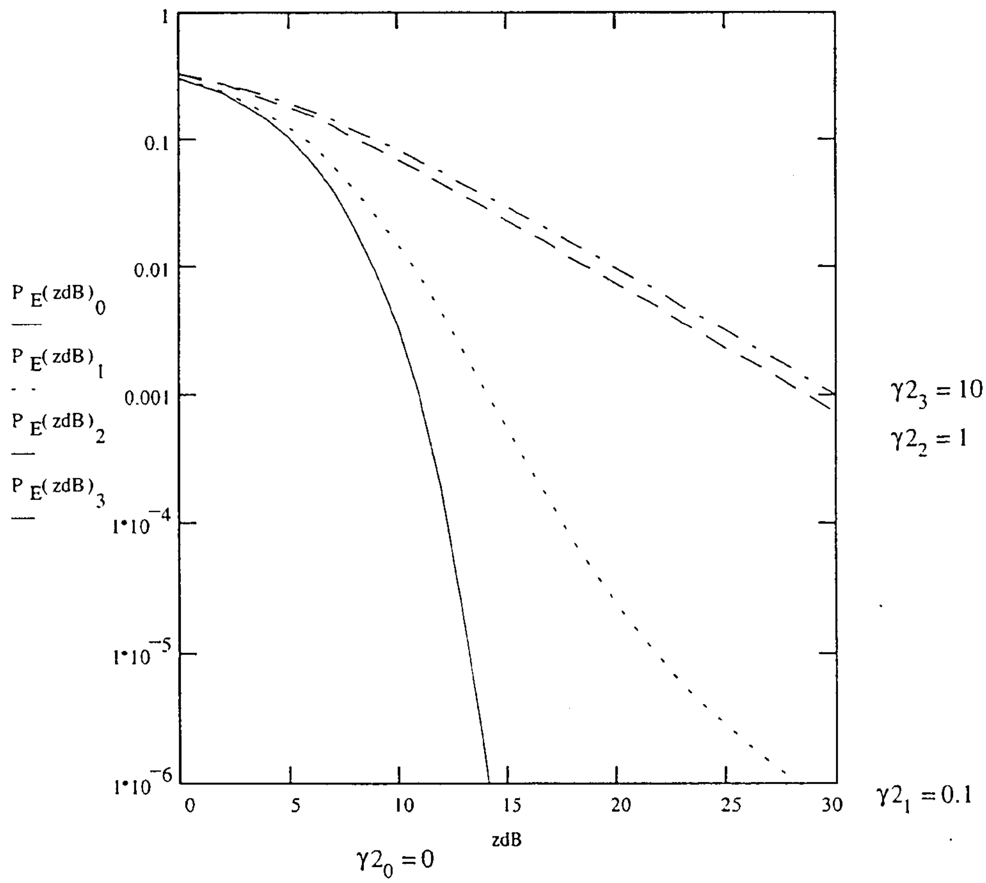
(b) Make use of the fact that the power in a sinusoid is 1/2 the amplitude squared to obtain the given result.

(c) The bit error probability is

$$P_E = Q[\sqrt{z(1 - a^2)}]$$

Problem 1-22

From the plot on the next page, it is seen that the degradations at $P_E = 10^{-3}$ for $\gamma^2 = 0.1$, 1, and 10 are approximately 3.3 dB, 18.9 dB, and 19.7 dB, respectively. The case for $\gamma^2 = 10$ is very close to the Rayleigh fading case.



Problem 1-23

A Mathcad program is given on the next page for computing these degradations. The results are summarized in the table below. Only the solve block for computing the additional dB in signal-to-noise ratio for BPSK is shown. Similar solve blocks can be used for the remaining cases.

Modulation scheme	E_b/N_0 in dB, nonfading, $P_E = 10^{-3}$	E_b/N_0 in dB, fading, $P_E = 10^{-3}$	Additional E_b/N_0 in dB, $P_E = 10^{-3}$
BPSK	6.79	23.97	17.18
DPSK	7.93	26.98	19.05
CFSK	9.8	26.98	17.18
NFSK	10.94	29.99	19.05