CHAPTER 7
PERFORMANCE OF SPREAD-SPECTRUM SYSTEMS WITH FORWARD ERROR CORRECTION

Problem 7-1

There are a total of 15 codewords generated. Because of the properties of the m-sequences, each initial register load generates a different phase of the same m-sequence. Thus, all codewords (except the all zeros word) are cyclic phase shifts of each other. The first four codeword symbols are the information symbols. The code is specified by thelist below.

codeword	message	distance from received sequence
000000000000000	0	7
100011110101100	1	9
010001111010110	2	9
110010001111010	3	7
001000111101011	4	7
101011001000111	5	9
011001000111101	6	9
111010110010001	7	7
000111101011001	8	5
100100011110101	9	7
010110010001111	10	11
110101100100011	11	9
001111010110010	12	9
101100100011110	13	3
011110101100100	14	7
111101011001000	15	5

The closest codeword is number 13 with distance 3.

Problem 7-2

The modulator maps zeros and ones into ±1 according to 0→ 1 and 1→ -1. The decoder calculates the square of the Euclidian distance between the received sequence and all codewords. It chooses its output estimate to be the codeword and corresponding message which has the minimum squared distance from the received sequence.

Calculate the contribution to the squared distance for each received symbol and for a codeword with either a +1 or -1 in the corresponding position. For example, the first symbol is $y_1 = 0.8$. The squared distance from a codeword +1 is 0.04 and the squared distance from a codeword -1 is 3.24. For each received symbol, the squared distance is:

y_i	squared +1 (0)	distance -1 (1)	y_i	squared +1 (0)	distance -1 (1)
0.8	0.04	3.24	0.87	0.02	3.5
-0.1	1.21	0.81	-0.01	1.02	0.98
1.1	0.01	4.41	0.2	0.64	1.44
0.25	0.56	1.56	1.0	0	4
1.0	0	4	1.7	0.49	7.29
-1.0	4	0	-0.82	3.31	0.03
-0.35	1.82	0.42	0.13	0.76	1.28
0.95	0.003	3.8			

The squared distance of any codeword from the received sequence is the sum of the distances calculated above. For example, the codeword 100011110101100 the squared distance is 3.24 + 1.21 + 0.01 + 0.56 + 4 + 0 + 0.42 + 3.8 + 0.02 + 0.98 + 0.64 + 4 + 7.29 + 3.31 + 0.76 = 30.24. The remaining squared distances are:

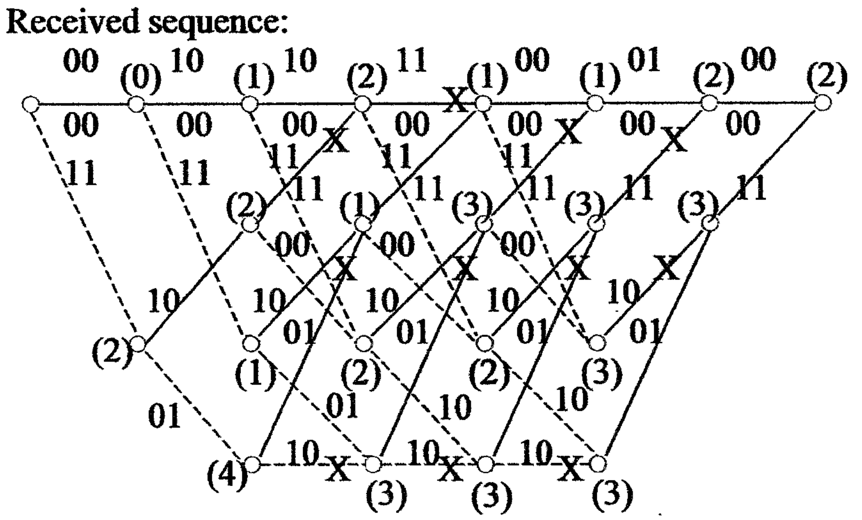
message number	squared distance	message number	squared distance
0	13.88	8	22.28
1	30.24	9	33.44
2	19.68	10	30.32
3	25.64	11	9.48
4	25.36	12	20.56
5	19.00	13	29.40
6	25.96	14	31.72

7	28.80	15	29.36

The minimum squared distance occurs for codeword 11 so $\hat{m} = 11$.

Problem 7-3

The decoding trellis is shown below. The Hamming distance between the received sequence and the winning path into each node is shown in parentheses. The losing path into a node is eliminated by an "X". At the last node on the right, the maximum likelihood path is established. Tracing a reverse path from the rightmost node to the starting node using all winning branches yields the output codeword estimate, which is 00111011000000.

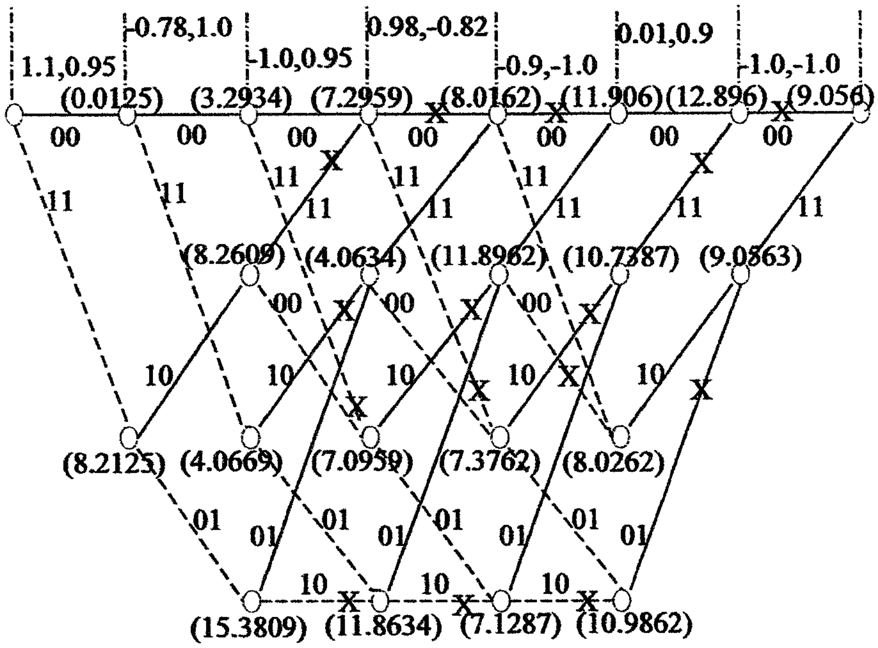


Problem 7-4

The decoding trellis is shown on the next page. The squared Euclidian distance between the received sequence and the winning path into each node is shown in parentheses near the node. The squared Euclidian distance is calculated by squaring the difference between the received sequence and the transmitted symbol. For example, the squared distance for the branch with label 00 leaving the starting node is

$$d^2 = (1.1 - 1.0)^2 + (0.95 - 1.0)^2 = 0.0125$$

Recall that a 0 codeword symbol corresponds to a +1 transmitted. The losing path into each node is eliminated from further consideration by an "X". The winning path through the trellis is found by backing through the trellis starting at the end node. The winning codeword is 001110111111011.



Problem 7-5

(a) The BCH code bit error probability is given by (7-36):

$$P_b \le \frac{1}{15}\sum_{i=4}^{15} i \binom{15}{i} p^i (1-p)^{15-i}$$

This result can be easily solved by trial and error to find the BSC crossover probability corresponding to $P_b = 10^{-5}$. The result is $p = 0.0115$.
(b) The crossover probability is given by (6-55) where

$$\frac{W}{3R} = \frac{2\times 100\times 10^6}{3\times 10^6} = 66.7$$

where the $3R$ factor includes the code rate of 1/3. Thus

$$p = \begin{cases} \dfrac{Q(1/\sqrt{K})}{133\,(P/J)}, & 7.5\times 10^{-3} < \dfrac{P}{J} \\ Q\left(\sqrt{\dfrac{133}{K}\dfrac{P}{J}}\right), & \dfrac{P}{J} \le 7.5\times 10^{-3} \end{cases}$$