ECEU574 Wireless Communication Circuits | Spring, 04
Assignment 6 | Prof. Brady

1. Equivalent Impedance. Consider an ideal transmission line having length ℓ and characteristic impedance Z_0. At the load end of the line, a load with impedance Z_ℓ is attached. At the source end of the line, it is desired to find an equivalent input impedance for this combination. You may assume that the source has an input impedance of Z_0.

a. Find an expression for the total voltage at the source end, V_{in}.

Solution. If the source has a voltage phasor V_0, then the input voltage is

$$\begin{aligned} V_{in} &= \tfrac{V_0}{2}\left[1+\rho e^{-j\frac{4\pi\ell}{\lambda}}\right] \\ \rho &= \tfrac{Z_\ell - Z_0}{Z_\ell + Z_0}. \end{aligned} \tag{1}$$

b. Find an expression for the total input current at the source end I_{in}.

Solution.

$$I_{in} = \tfrac{V_0}{Z_0 2}\left[1-\rho e^{-j\frac{4\pi\ell}{\lambda}}\right] \tag{2}$$

c. Find an expression for the input impedance of the line/load combination, Z_{in}.

Solution.

$$Z_{in} = Z_0 \frac{\left[1+\rho e^{-j\frac{4\pi\ell}{\lambda}}\right]}{\left[1-\rho e^{-j\frac{4\pi\ell}{\lambda}}\right]} \tag{3}$$

d. For the special case $Z_l = \infty$, is it possible to make $Z_{in} = Z_0$? Explain your reasoning clearly, using the results from part c.

Solution. In this case $\rho = 1$ in (**??**), and it is not possible.

e. For the special case of $Z_l = 0$, is it possible to make $Z_{in} = Z_0$? Explain your reasoning clearly, using the results from part c.

Solution. No, since $\rho = -1$ in this case, the fraction in (**??**) cannot be forced to unity.

f. For the special case of $Z_l = Z_0$, is it possible to make $Z_{in} = Z_0$? Explain your reasoning clearly, using the results from part c.

Solution. Since $\rho = 0$ in this case, the fraction in (**??**) is unity, and $Z_{in} = Z_0$ here.

2. Matching Stub. Consider the transmission line combination in Figure 1. A transmission line with characteristic impedance Z_0 has a load impedance Z_l, as shown in Figure 1. At a length L_2 before the load end, a second transmission line, having characteristic impedance Z_0 and shorted at its load end, is attached in parallel as shown. The length of this shorted line is L_1. In this problem, you may assume that that Z_0 is real, and that $Z_l = Z_0 + jX$ for some known reactance X.

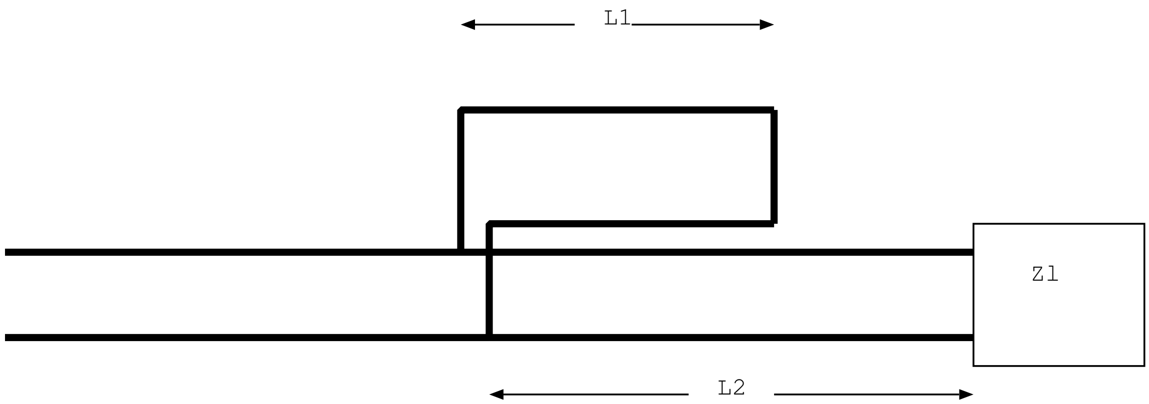

Figure 1: Figure for problem 2.

a. Using the results from problem 1, find the equivalent *admittance* of the transmission line segment of length L_2 and load impedance Z_l at the junction point of the two lines.

Solution.

$$\begin{aligned} \frac{1}{Z_2} &= \frac{1}{Z_0}\frac{\left[1-\rho_2 e^{-j\frac{4\pi\ell}{\lambda}}\right]}{\left[1+\rho_2 e^{-j\frac{4\pi\ell}{\lambda}}\right]}, \\ \rho_2 &= \frac{Z_\ell - Z_0}{Z_\ell + Z_0}. \end{aligned} \tag{4}$$

b. Using the results from problem 1, find the equivalent *admittance* of the shorted transmission line segment of length L_1 at the junction point.

Solution.

$$\begin{aligned} \frac{1}{Z_1} &= \frac{1}{Z_0}\frac{\left[1+e^{-j\frac{4\pi\ell}{\lambda}}\right]}{\left[1-e^{-j\frac{4\pi\ell}{\lambda}}\right]} \\ &= \frac{1}{Z_0} e^{j\frac{2\pi\ell}{\lambda}} \frac{(-j)}{\tan\left(\frac{2\pi\ell}{\lambda}\right)} \end{aligned} \tag{5}$$

Note that the admittance of the shorted line is purely susceptive, that is, there is not conductance. This is the point to keep in mind: a matching stub cancels the load susceptance, translated to the junction point.

c. Find the equivalent admittance of the combination of these two transmission line segments at the junction point.

Solution. Admittances of parallel devices add.

d. Find a relationship between L_1, L_2, and X for which the equivalent admittance is real. Specify the value of this admittance.

Solution.

e. What is the relationship in part d. for the special case when $L_2 = 0$?

Solution. See additional sheets.

f. Why is the L_1-length transmission line segment referred to as a 'matching stub'?

Solution. See (2.b).