

Compound Lens Example

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1 Thin Lens

To better understand the concept of principal planes, we consider the compound lens of two elements shown in Figure 1. Initially, assume that we have designed this system assuming thin lenses. We can locate the image by applying the lens equation twice. For the first lens, the object distance is

$$s_1 = 8cm, \quad (1)$$

the focal length is

$$f_1 = 10cm, \quad (2)$$

and the image distance after the first lens, s'_1 is given by

$$\frac{1}{s'_1} = \frac{1}{f_1} - \frac{1}{s_1} \quad s'_1 = -40cm. \quad (3)$$

This image is virtual, and is to the left of the lens. The magnification is

$$m_1 = \frac{-s'_1}{s_1} = 5, \quad (4)$$

and the image of an object $1cm$ high is $5cm$ high.

Treating this image as an object for the second lens, the object distance is

$$s_2 = z_{12} - s'_1 = 47cm, \quad (5)$$

the focal length is

$$f_2 = 5cm, \quad (6)$$

and the image distance, s'_2 is given by

$$\frac{1}{s'_2} = \frac{1}{f_2} - \frac{1}{s_2} \quad s'_2 = 5.60cm. \quad (7)$$

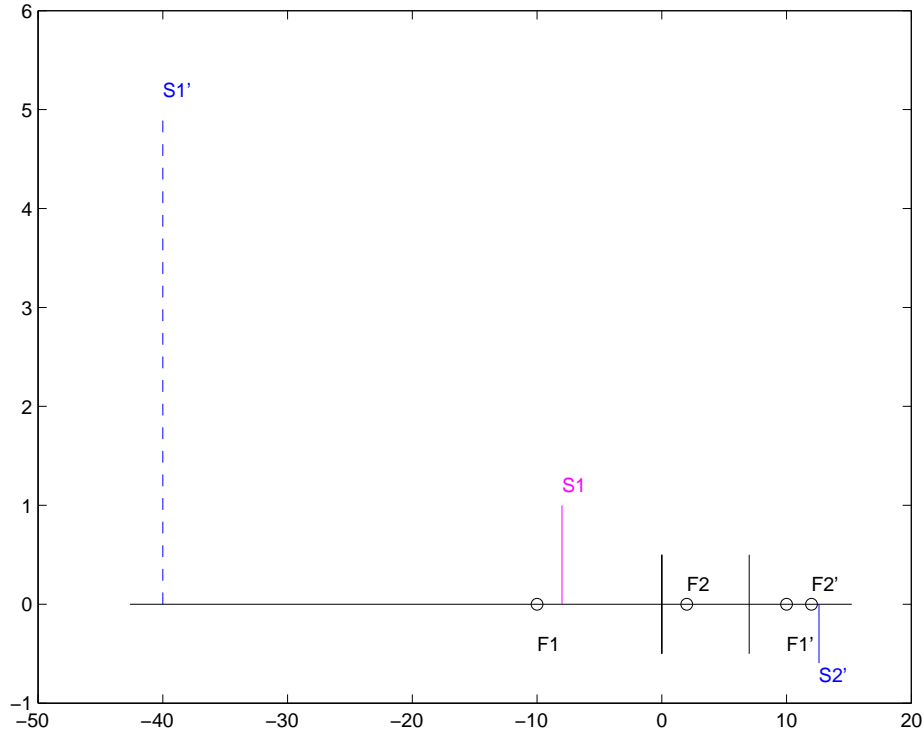


Figure 1: COMPOUND LENS. This lens consists of two glass elements, each represented by a black vertical line, assumed to be biconvex lenses of focal lengths, $f_1 = 10\text{cm}$ and $f_2 = 5\text{cm}$ respectively, separated by 7cm . An object (magenta line) will be placed 8cm in front of the first lens. Each lens is 6mm thick, but we will assume thin lenses for the first approximation.

This image is real, and is to the right of the lens. The magnification is

$$m_2 = \frac{-s_2'}{s_2} = -0.119, \quad (8)$$

and the image of the original object 1cm high is given by

$$m = m_1 m_2 \quad x_2' = m x_1 = -0.595\text{cm}, \quad (9)$$

so the image is smaller and inverted with respect to the original object.

Objects are placed on the drawing using absolute coordinates, in which we have arbitrarily chosen the first vertex of the first lens as the origin. As each of the above equations is used, the result is converted to this coordinate system. On the drawing, capital letters are used to represent points or planes illustrated by the examples, S_1 is the location of the object, H_1' is the location of the back principal plane of the first lens, and V_2 is the front vertex of the second lens. It is convenient in writing computer code to use a variable name beginning with a lower-case z for the locations. Thus in these examples, the locations are zS_1 , $zH_1\text{prime}$ and zV_2 respectively.

2 Thick Lens

Next, we choose to refine our design, accounting for lens thickness. Given that the lenses are biconvex, we will apply the “one-third” rule to locate the principal planes. As shown in Figure 2, the first principal plane of each lens is one third the thickness, or $2mm$ inside the lens from the first vertex. The second principal plane is an additional $2mm$ in, and there is, of course, another $2mm$ to the second vertex.

To keep the performance the same as in the simpler design of Figure 1, we want Equations 3 and 7 to have the same numerical values, but for a thick lens, measurements are made from the principal planes. Thus, the object, S_1 is moved to the right by $2mm$ and the inner vertices of the two simple lenses are spaced

$$z_{v_2} - z_{v'_1} = z_{12} + h'_1 + h_2 = z_{12} - 0.4cm = 6.6cm. \quad (10)$$

Now, we can simply apply the same equations we used with the thin lenses, but with the understanding that the results are also with respect to principal planes. Specifically, the final image distance, s'_2 is measured from the back principal plane of the second lens.

3 Matrix Formulation

The matrix for the compound lens can be found from matrix multiplication;

$$\mathcal{L}_1 = \begin{pmatrix} 1 & 0 \\ -1/f_1 & 1 \end{pmatrix}. \quad (11)$$

$$\mathcal{L}_2 = \begin{pmatrix} 1 & 0 \\ -1/f_2 & 1 \end{pmatrix}. \quad (12)$$

$$\mathcal{T}_{12} = \begin{pmatrix} 1 & z_{12} \\ 0 & 1 \end{pmatrix}. \quad (13)$$

$$\mathcal{M}_{H_1:H_2'} = \begin{pmatrix} 0.3 & 7cm \\ -0.16/cm & -0.4 \end{pmatrix} = \begin{pmatrix} m_{11} & m_{21} \\ m_{12} & m_{22} \end{pmatrix}. \quad (14)$$

The focal length is obtained from the lower left element,

$$f = -\frac{1}{m_{21}} = 6.25cm, \quad (15)$$

and the principal plane distances are

$$h = \frac{1 - m_{22}}{m_{21}} = -8.75cm, \quad (16)$$

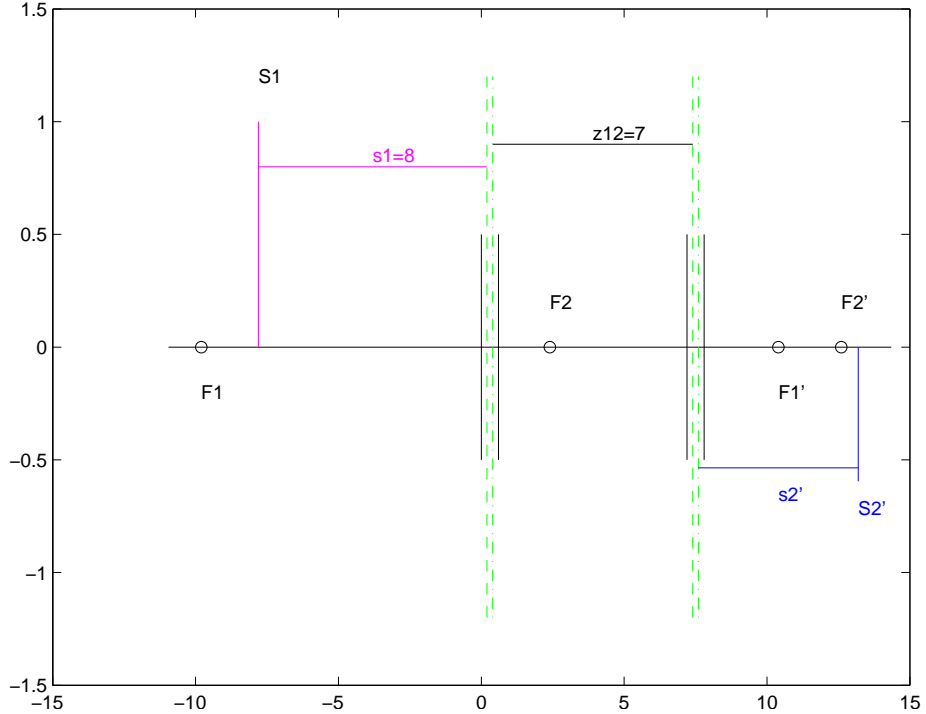


Figure 2: COMPOUND LENS WITH PRINCIPAL PLANES. The principal planes are located here using the “one-third” rule. The goal is to have the lens behave the same as the one in Figure 1, so the object and lens have been moved to make s_1 and z_{12} be numerically the same as in that figure, but with the measurements taken relative to principal planes.

and

$$h' = \frac{1 - m_{11}}{m_{21}} = -4.38\text{cm}. \quad (17)$$

We could have used the matrix formulation to find the focal length of the compound lens assuming thin lens elements. The distances h and h' would be measured from Lens 1 and Lens 2 respectively. However, we will go directly to the thick lens. Then h is measured from the front principal plane of the first lens, H_1 , to locate H and h' is measured from H'_2 to H' . This is because the matrix (Equation 14) we used to calculate h (Equation 16) and h' (Equation 17) was from H_1 to H'_2 .

The locations of these principal planes are shown in Figure 3. Note that the principal planes have crossed over each other. The “back” principal plane is before the “front” one. The front focal point, F , is quite close to the back principal plane, and *vice versa*, but this is just coincidental.

We now find the object distance as the distance from the object to the

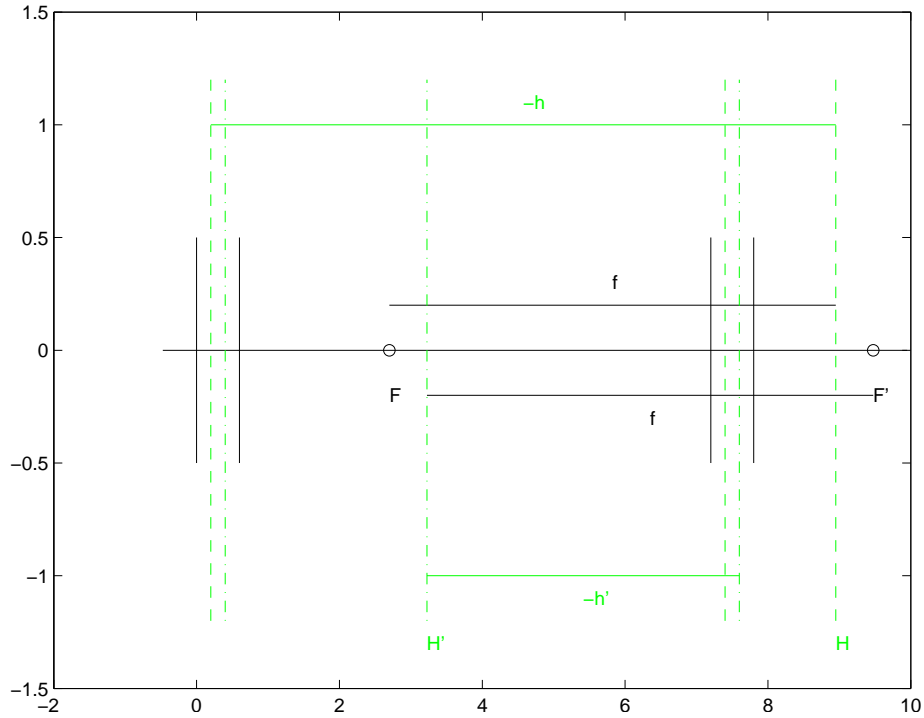


Figure 3: PRINCIPAL PLANES AND FOCAL LENGTHS: The compound lens can be characterized by its focal length, 6.25cm , and the locations of its principal planes.

first principal plane as shown in Figure 4,

$$s = 16.7\text{cm}, \quad (18)$$

using $f = 6.25\text{cm}$. The lens equation yields

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} \quad s'_1 = 9.97\text{cm}, \quad (19)$$

which is measured from the back principal plane.

4 Apertures in Object Space

We want to find all the apertures as seen in object space. Viewing the system from the object, looking through Lens 1, only rays that pass through a circle corresponding to the lens diameter will arrive at the image. Looking through Lens 1, we see an image of Lens 2, and only rays that pass through a circle of this diameter at this location will arrive at the image. Thus these two circles limit the propagation of light through the compound lens. Figure 5 shows the image of the second lens seen through the first. The

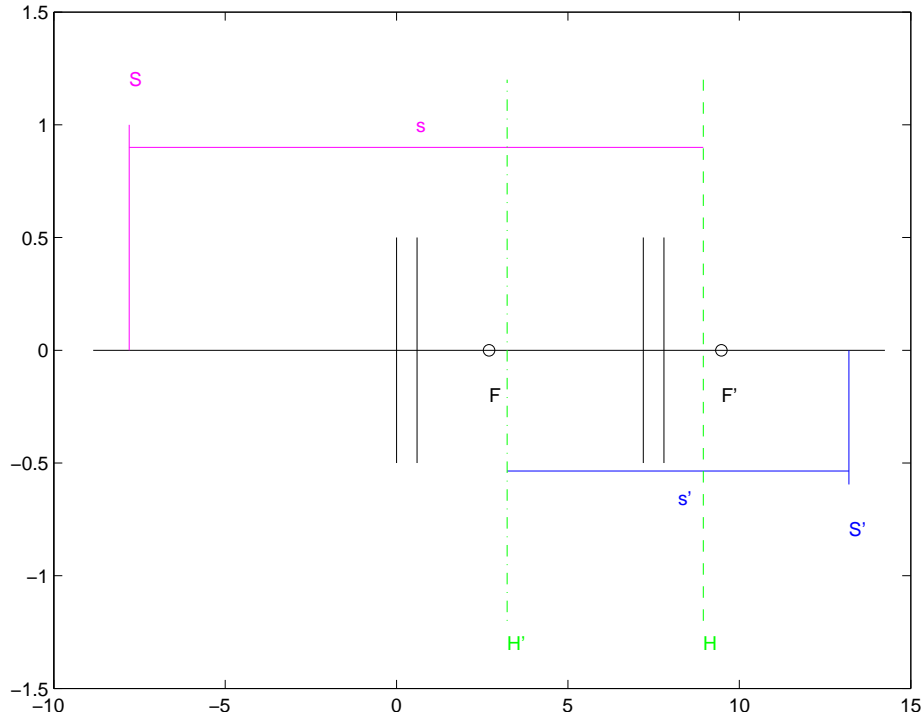


Figure 4: IMAGING WITH THE COMPOUND LENS: The lens equation can be used, provided that distances are measured from the principal planes. The vertices of the lenses are shown, so that the user can see if they will be in conflict with any hardware associated with the object or image.

lens equation is applied, thinking of light traveling from right to left in this case. The object distance is a bit ambiguous; Where exactly are rays cut off in going through the lens? More detailed information would require a raytrace. We will chose the “object” to be at the front vertex of the second lens, $s_{2,1} = z_{V2} - z_{H1'} = 6.8cm$, and apply the lens equation.

$$\frac{1}{s'_{2,1}} = \frac{1}{f_1} - \frac{1}{s_{2,1}} \quad s'_1 = -21.5cm. \quad (20)$$

Now we need to remember that we are working “backward” in this diagram, so s' is positive to the left. Therefore the image is $21.5cm$ to the right of $H1$. The magnification is

$$m_{2,1} = \frac{-s'_{2,1}}{s_{2,1}} = 3.12, \quad (21)$$

so the diameter is $3.12cm$.

Now, we can consider only the object and the apertures as viewed in object space, as shown in Figure 6. To locate the entrance pupil, we construct rays from the base of the object to the boundary of each aperture. We

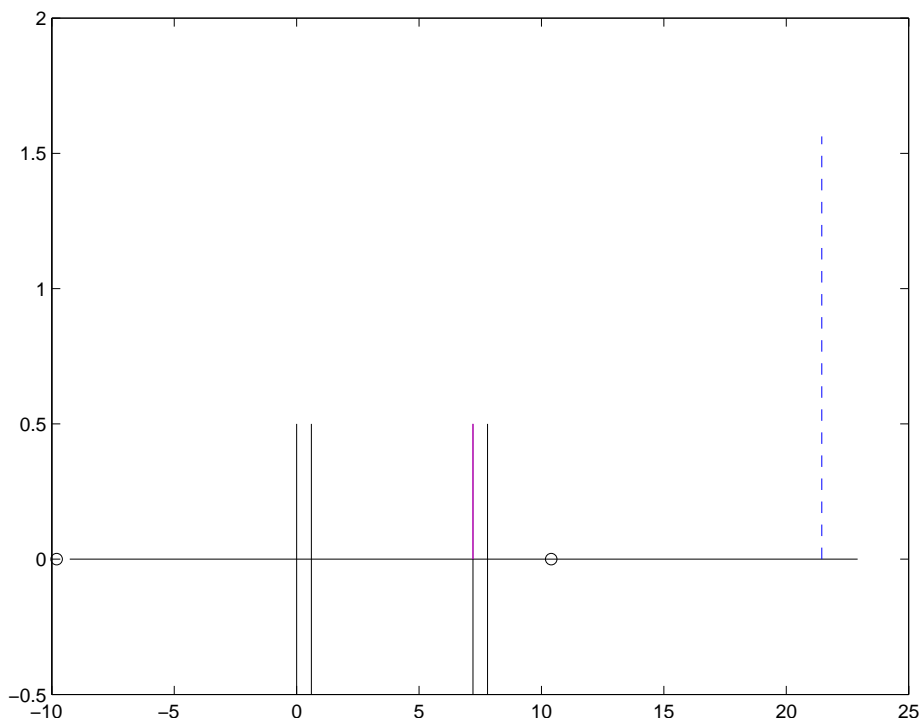


Figure 5: THE IMAGE OF LENS 2 SEEN THROUGH LENS 1: All apertures are located in object space to determine the location of the entrance pupil and entrance window.

note that the image of Lens 2 subtends the smaller angle, so it becomes the entrance pupil. The tangent of the half-cone angle is the radius divided by the distance from the object to the image of Lens 2. The numerical aperture is the sine of this angle, or approximately 0.533. The solid red line in the figure shows the limiting aperture, and the dotted red line shows the other one.

The entrance window subtends the smallest angle as measured from the center of the entrance pupil. In this case, obviously it is Lens 1. The tangent of this angle is about 0.0233 so the full field of view is 0.0466 radians, as shown by the cyan line. The cyan line is extended to the object plane to show the size of the largest object which can be imaged through this compound lens.

It is important to note that one must consider all apertures regardless of whether they are to the left or the right in object space. Also, if there are more than two apertures, only two of them play the roles of pupil and window. The others are irrelevant, and in fact could be made smaller.

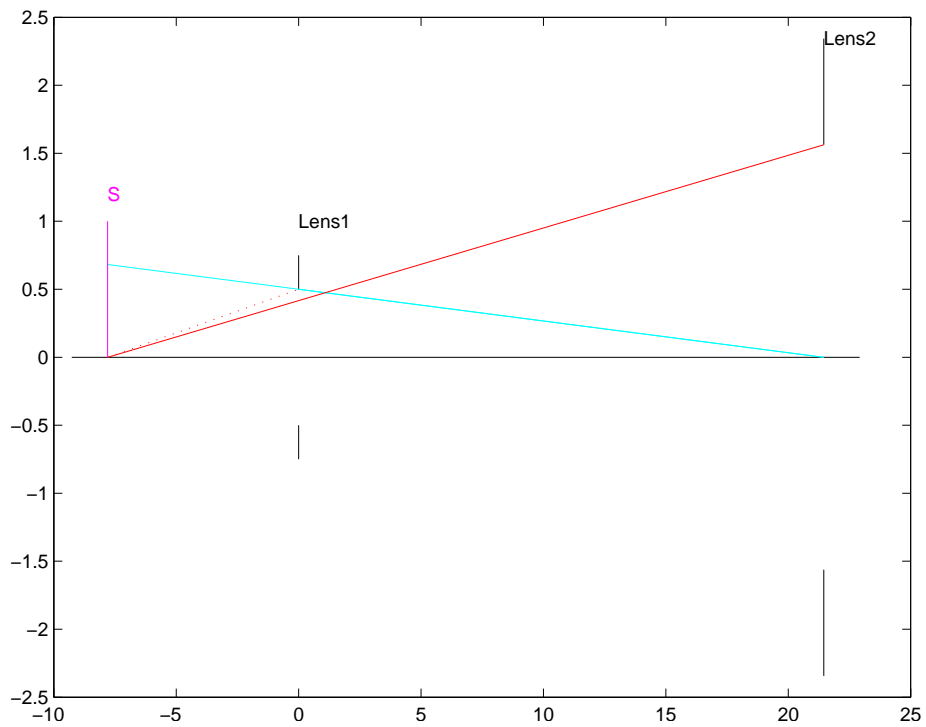


Figure 6: OBJECT-SPACE REPRESENTATION: In object space, we show the object, Lens 1, and the image of Lens 2 as seen through Lens 1. The entrance pupil is the aperture which subtends the smallest angle seen from the object. The entrance window is the one which subtends the smallest angle seen from the pupil.