DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

NORTHEASTERN UNIVERSITY

EECE 2150 **Circuits and Signals: Biomedical Application** Fall 2017

Homework 9
Fourier Transforms and Frequency Response

Problem 1

A system can be modeled by the following differential equation:

$$0.01y'(t) + 5y(t) = 50x(t)$$

where the $'$ indicates differentiation in time.

(a) What is the frequency response of the system? Use the differentiation property of the Fourier Transform to derive it.

(b) Is this a low-pass, high-pass, or some other kind of filter ? Explain your reasoning.

(c) At what frequency will the output be attenuated by $1/\sqrt{2}$ from its maximum?

(d) Design a circuit that implements this differential equation in the steady state. You may find it easier to do a little algebra on the expressin for $H(\omega)$ to get it into the "standard form" we have been using in class. You can choose any values you like for the components as long as the circuit does as desired and the components are in reasonable ranges for the ones we have been using in the lab.

(e) Give an expression for the Fourier Transform of the output, $Y(\omega)$, if the input is $x(t) = 2e^{-3t}$ for $t \geq 0$ and 0 for $t < 0$. You can leave this expression in whatever form you find convenient; no need to do a lot of algebra here.

Problem 2

Compute the Fourier Transform of the following signal

$$x(t) = e^{-2(t-3)}, \quad t \geq 0 \tag{1}$$

$$x(t) = 0, \quad t < 0. \tag{2}$$

Problem 3

Suppose you want to sample a signal $x(t)$ whose Fourier Transform is non-zero at all frequencies. This implies, based on the Sampling Theorem, that you cannot sample it "perfectly".

What is usually done in practice is to put the signal through a continuous-time ("analog") filter to ensure that the frequency content of the filter output is attenuated "sufficiently" above a desired frequency, and then use that frequency to choose the sampling rate. Thus there will be some aliasing but it will be small enough to safely neglect. This kind of filtering is called "anti-aliasing".

To make this concrete, work out the following problem: Suppose that you want to sample a signal of the type described in the first paragraph of this problem, and that you believe that you can safely neglect any frequency content in that signal at frequencies beyond 2000 Hz.

(a) Let's use a filter whose frequency response is of the form

$$H(\omega) = \frac{A}{1 + j\omega\alpha},$$

where A and α are real numbers and ω is frequency in radians per second. Choose a value for α so that for frequencies greater that 2000 Hz the output of this filter is attenuated to below .01 of its maximum amplitude. You can use any reasonable approximations to simplify the algebra as long as you state what they are. Be careful to use the right units for frequency.

(b) Choose A so that the maximum gain of this system is 10.

(c) Design a first-order OpAmp filter of the type we have been discussing in class that meets these specifications. Note that A can be negative if that is convenient.

(d) If you choose A to be negative, how would you "fix" that so that the sampled signal that you saved had the same polarity as the original signal ?

Problem 4

Compute the Fourier Transform of a singal $x(t) = 1$, $0 \leq t \leq 2$ and $x(t) = 0$ otherwise. Use the derivation of the Fourier Transform of a similar pulse that is symmetric around $t = 0$ as a guide to help you.

Problem 5

A system is composed of two subsystems connected in series (which means that the output of the first subsystem is connected to the input of the second subsystem).

The first subsystem is characterized by the following differential equation:

$$y''(t) - y'(t) + 6y(t) = x'(t) + 2x(t).$$

The second subsystem has a frequency response $H_2(\omega)$ that is 1 for $|\omega| < 3$ rad/s and 0 for $|\omega| > 3$rad/s. (Note that this is an idealized and not physically realizable system but lets just take it as given for this problem.)

(a) What is the frequency response $H_1(\omega)$ of the first system? (Hint: use the derivative property of the Fourier Transform.)

(b) If the input to the overall system is the signal $3 + 2\cos(2t + \pi/3) + 4\sin(4t + \pi/6)$, what is the output of the overall system? Your answer should be time-domain signals and all values should be real, not complex. You may leave your answer in terms of the magnitude and angle of complex-valued quantities, since both the magnitude and angle of a complex number are real numbers, as long as you do so carefully.