Circuits and Signals: Biomedical Applications
Week 2

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Week 2 Agenda

- Better Source Models
- Kirchoff's Current Law
- Kirchoff's Voltage Law
- Resistors in Series
- Resistors in Parallel
- AC Circuits
- Diodes, Briefly

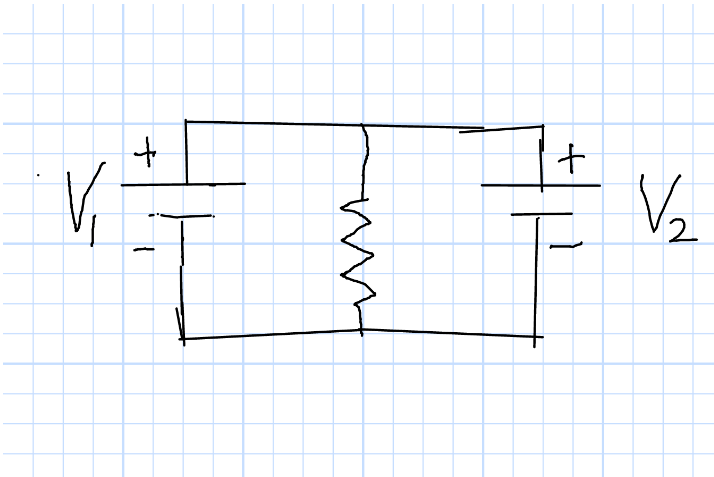
Models

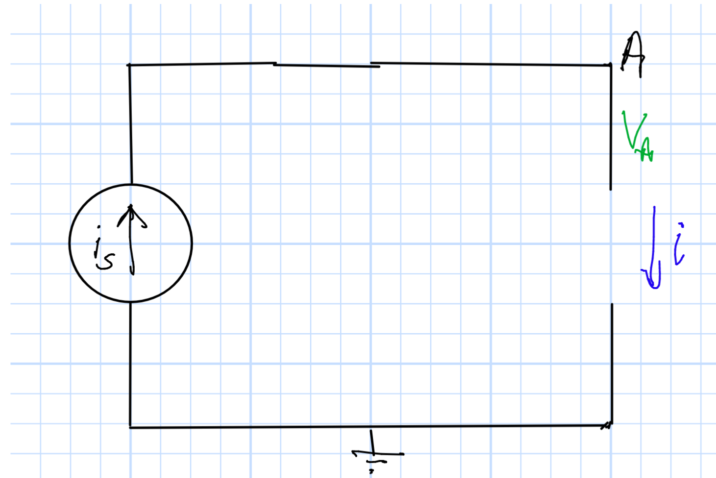
All Models Are Approximations

- The Earth is Flat
- The Earth is a Sphere
- The Earth is Very Complicated
- Sources: Perfect Voltage or Current Sources
- Conductors: $R = 0$

What if a 5–amp source is connected to an infinite resistance?

Contradictory Circuits



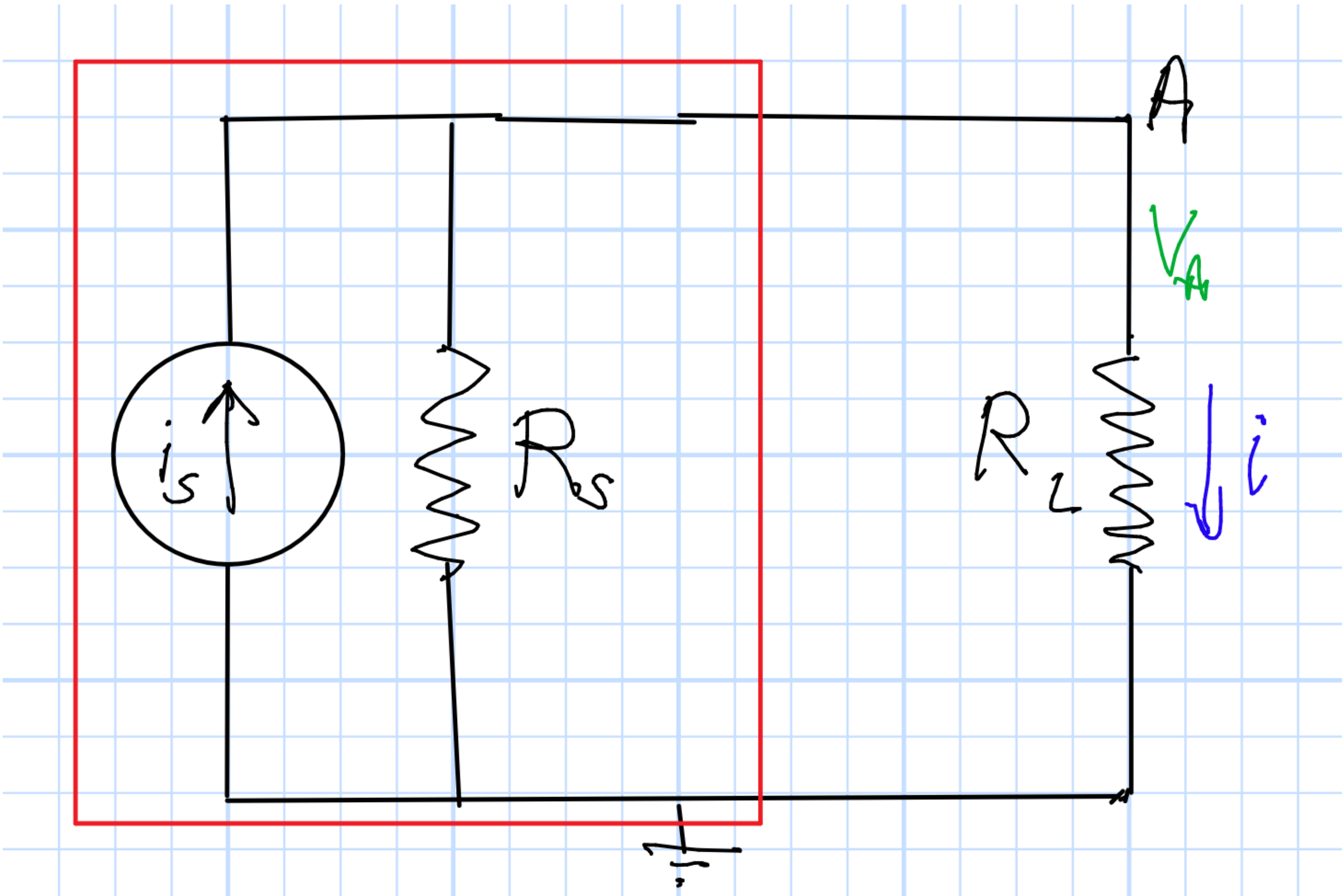


Why doesn't this work? How can we fix it?

Our models are too simple.

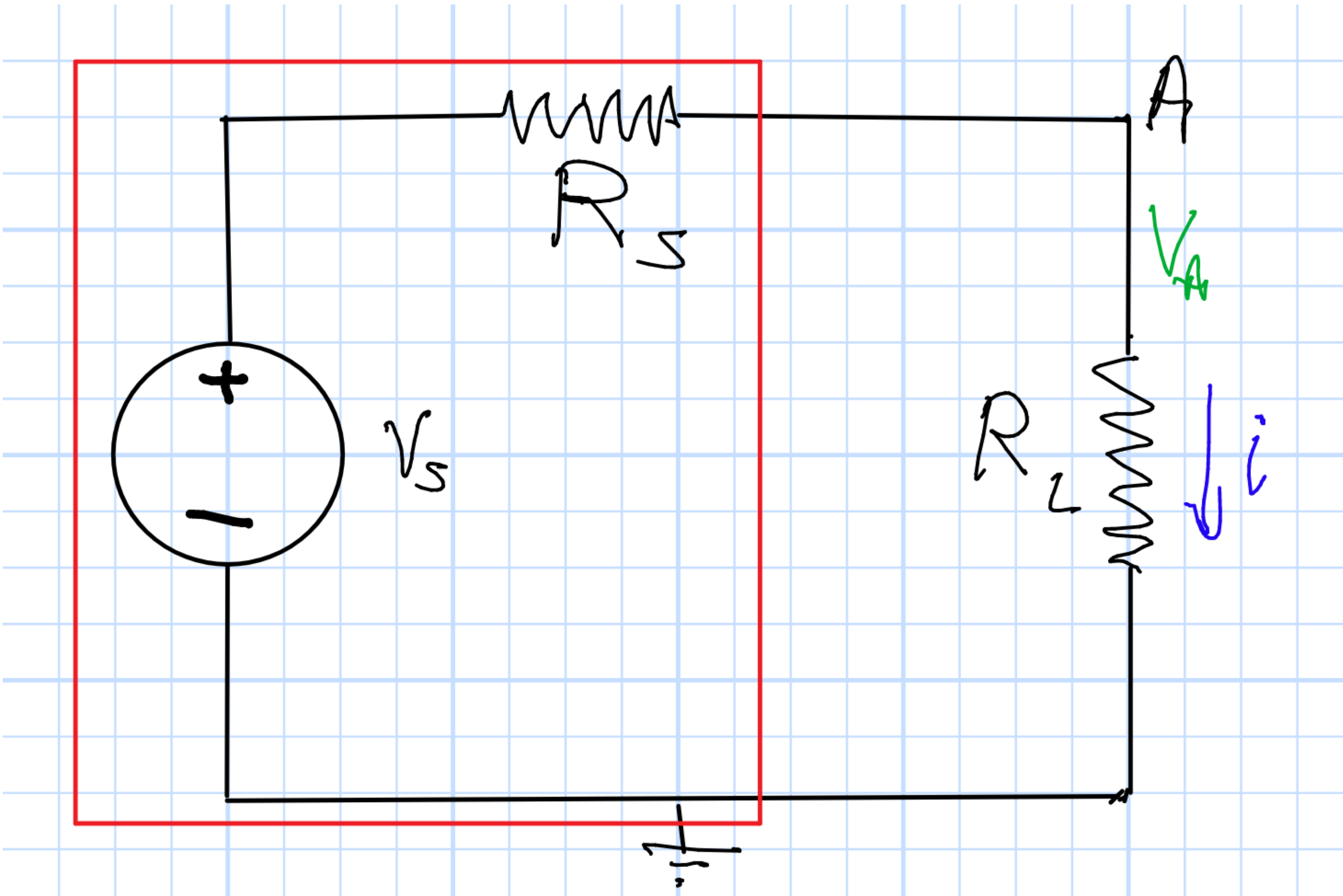
Better Current Source Model

Parallel Source Resistance



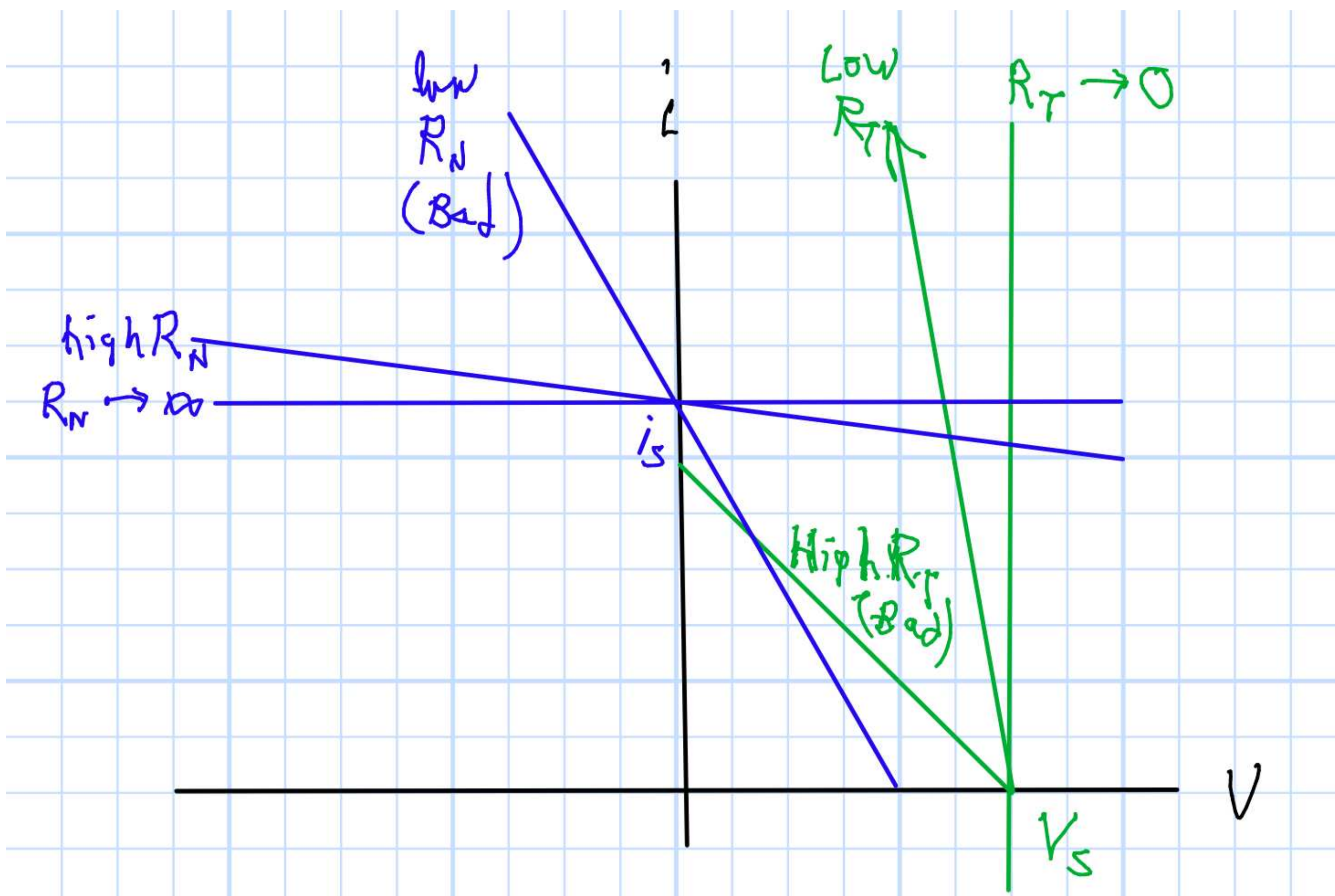
Better Voltage Source Model

Series Source Resistance



More Realistic Sources

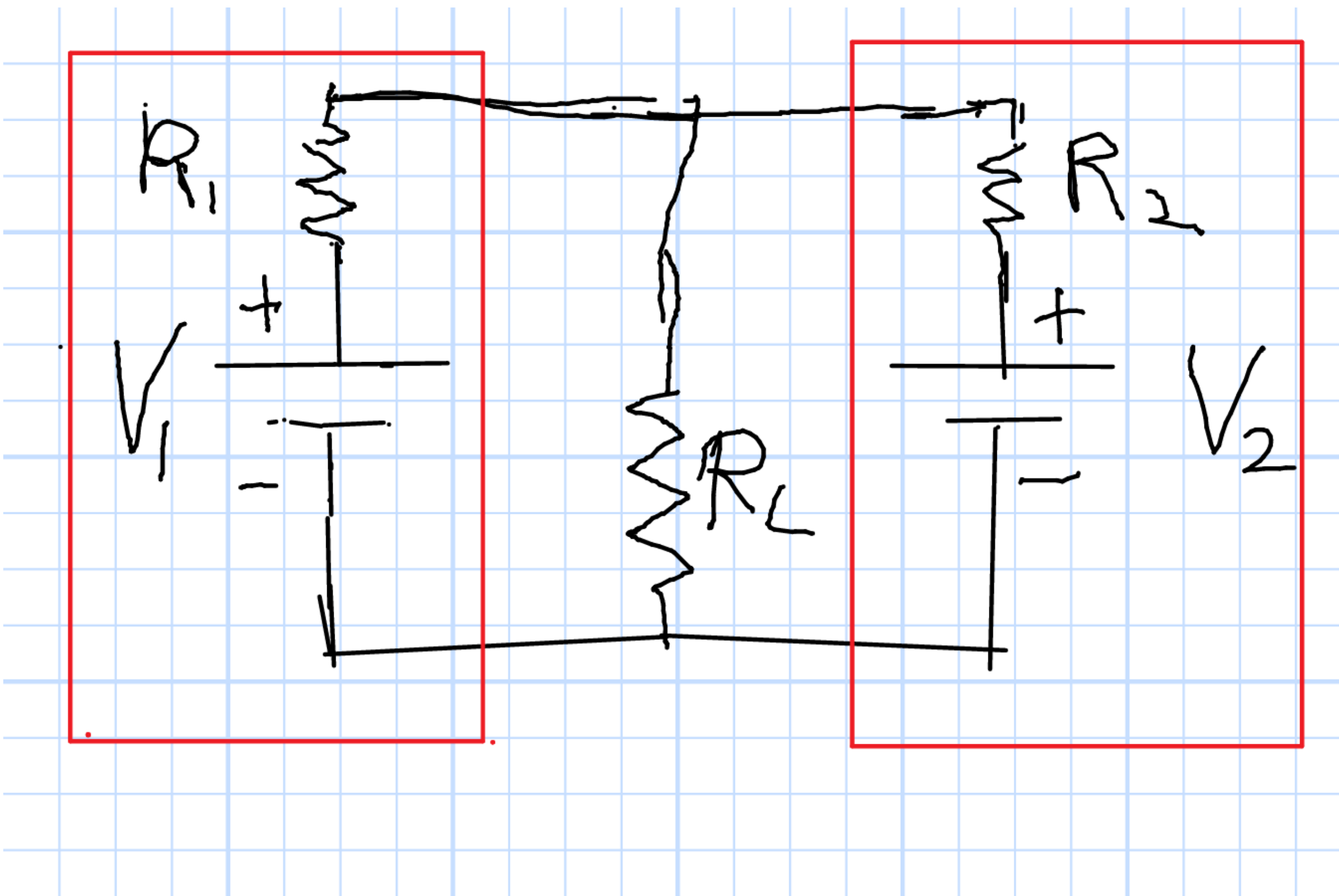
Terminal Characteristics of Current (blue) and Voltage (red) Sources



What is the characteristic curve for your power supply? Why?

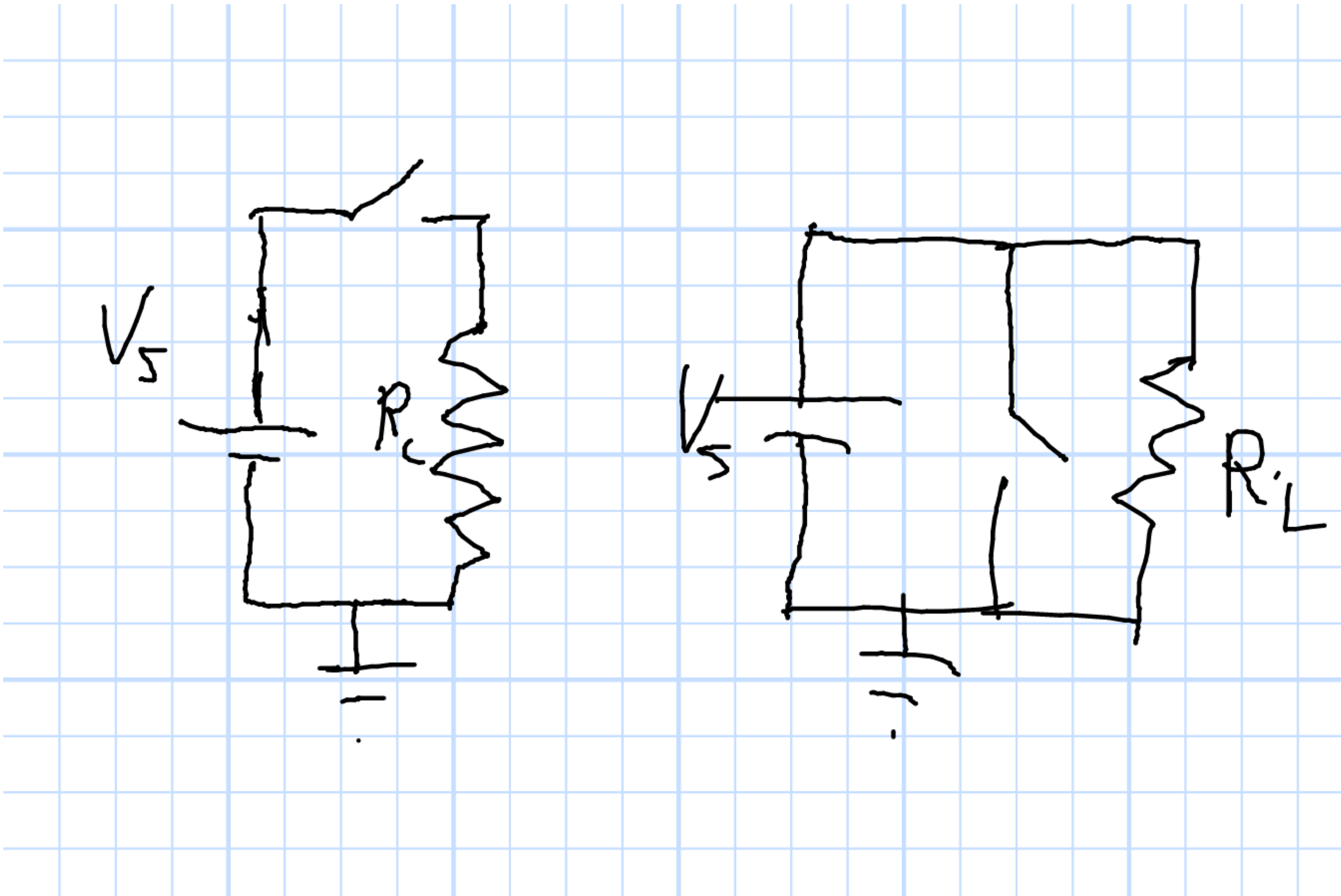
Now We Can Solve It

And Later We Will (But this is a Terrible Idea)



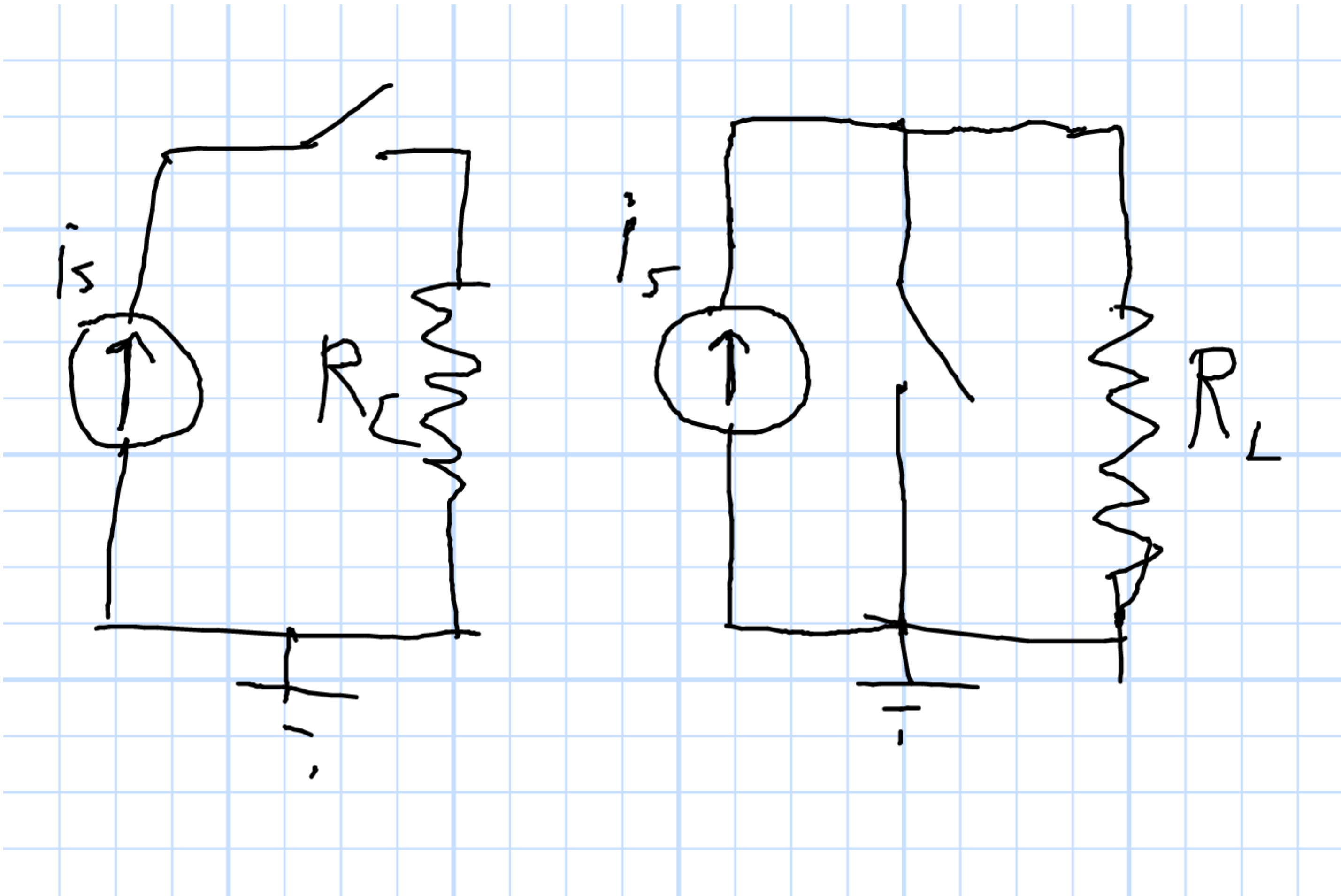
Aside: Switching Voltage

Which is Better? Why?



Aside: Switching Current

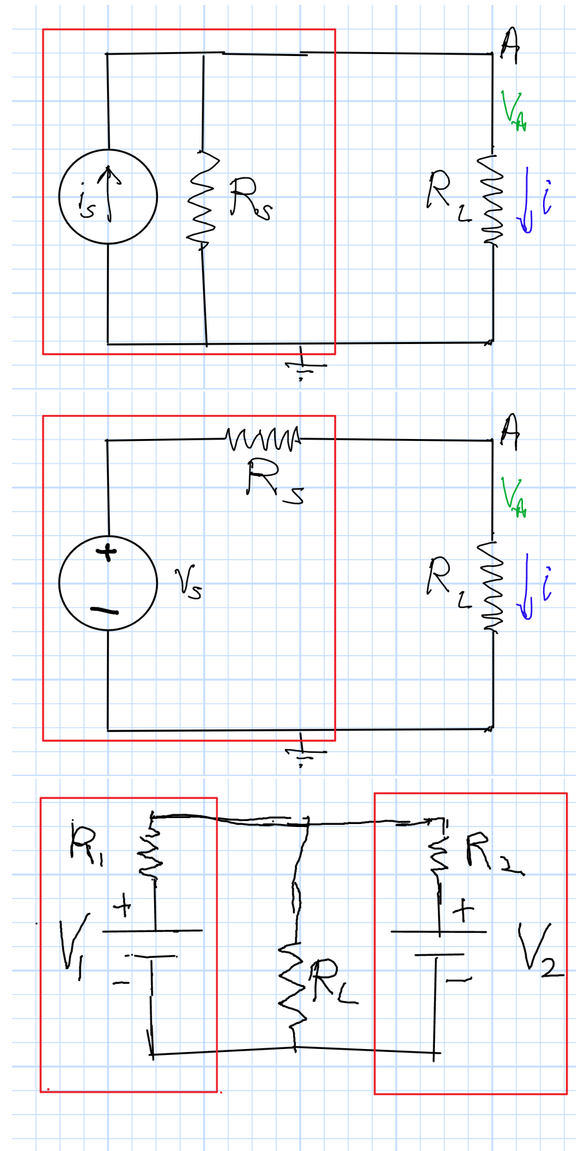
Which is Better? Why? Important for Laser Diodes



KVL, KCL: Simple Cases

- Simple KCL Examples
- Simple KVL Examples
- Series and Parallel Circuits
- Voltage and Current Dividers (Next Week)
- General Case (Next Week)

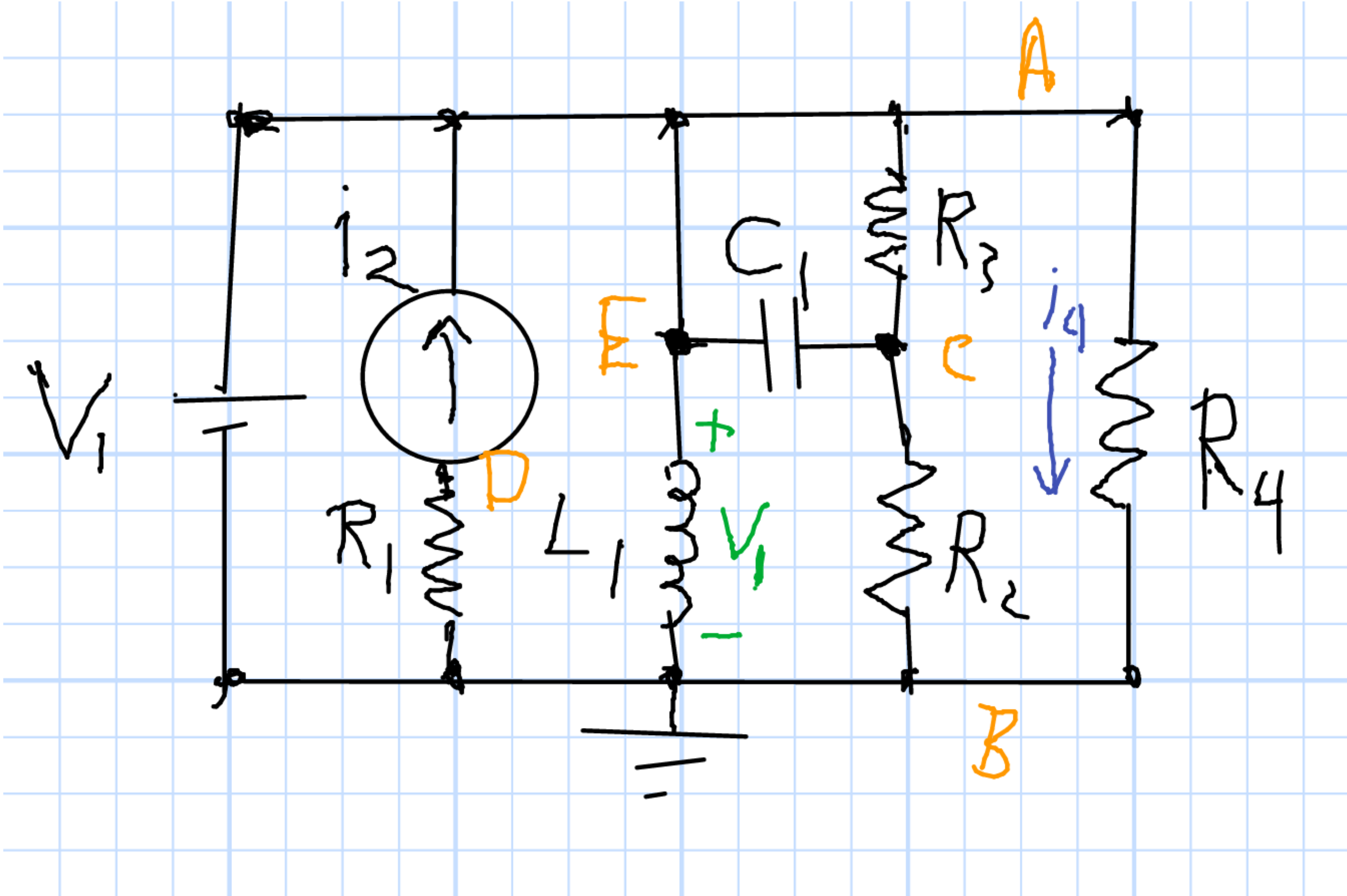
Can You Do These Now?



Nodes and Loops

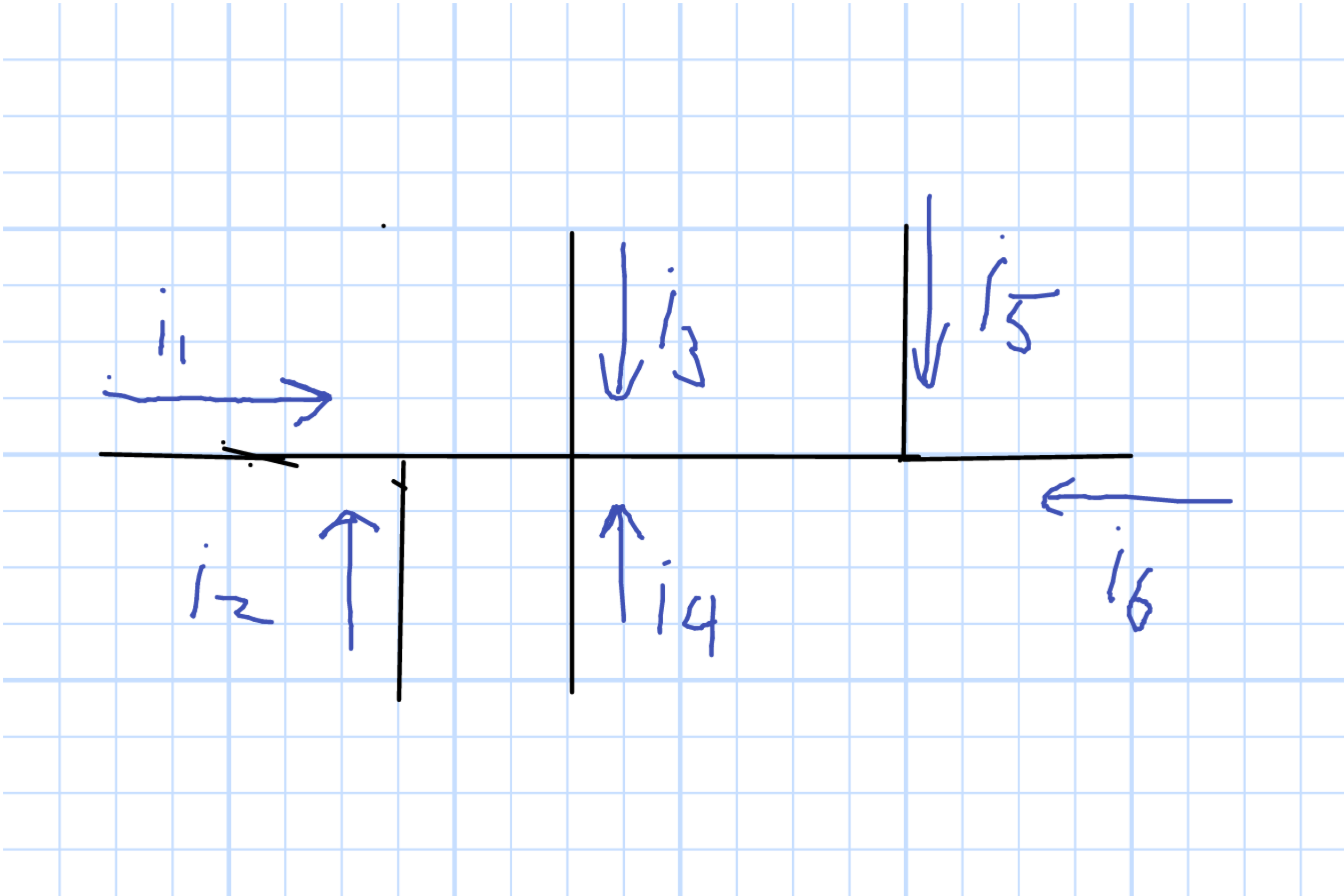
Where are the Nodes and Loops?

Where are the Essential Nodes? A B C D E



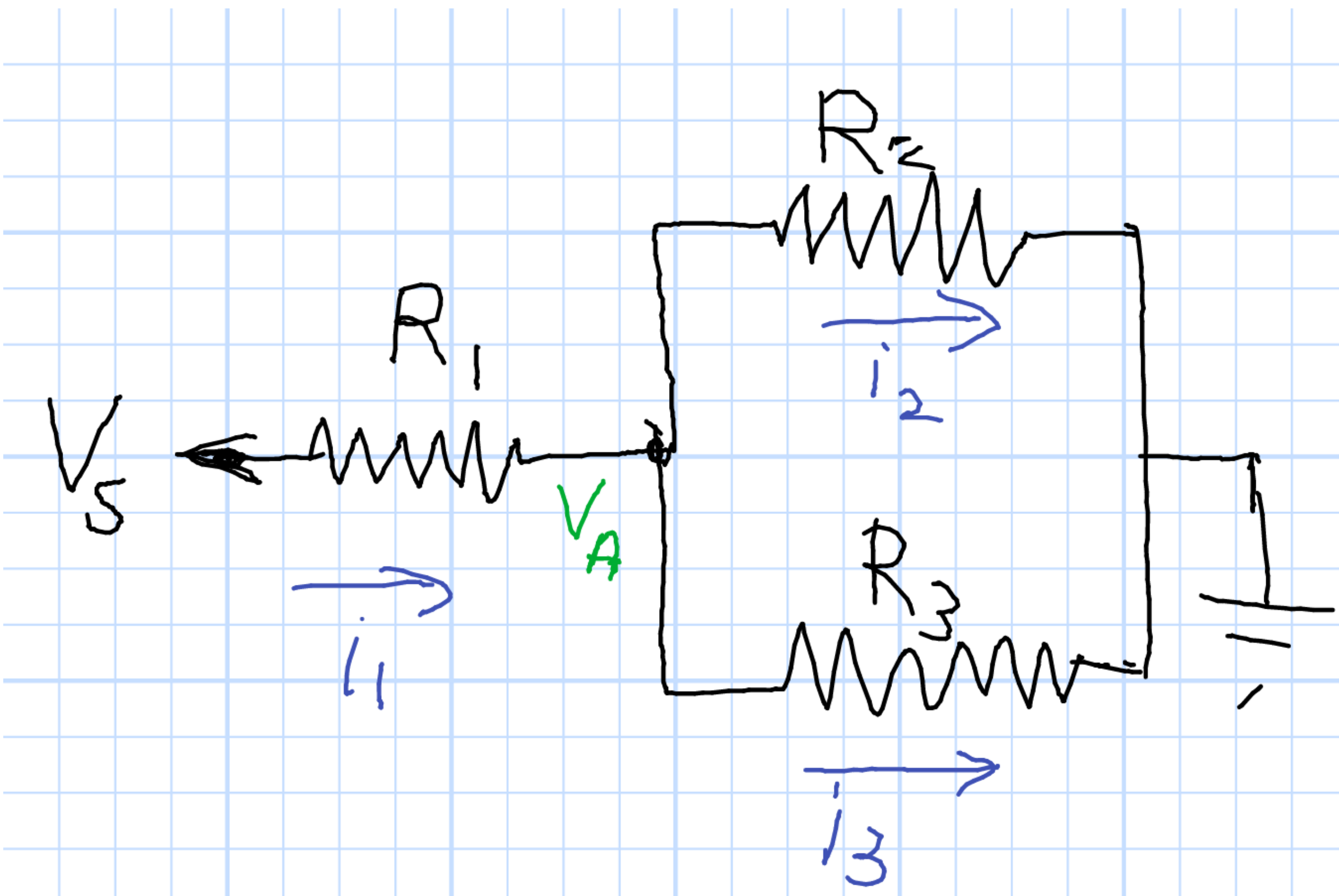
Remember Kirchoff's Current Law (KCL)

Conservation of Electrons $\sum i_n = 0$



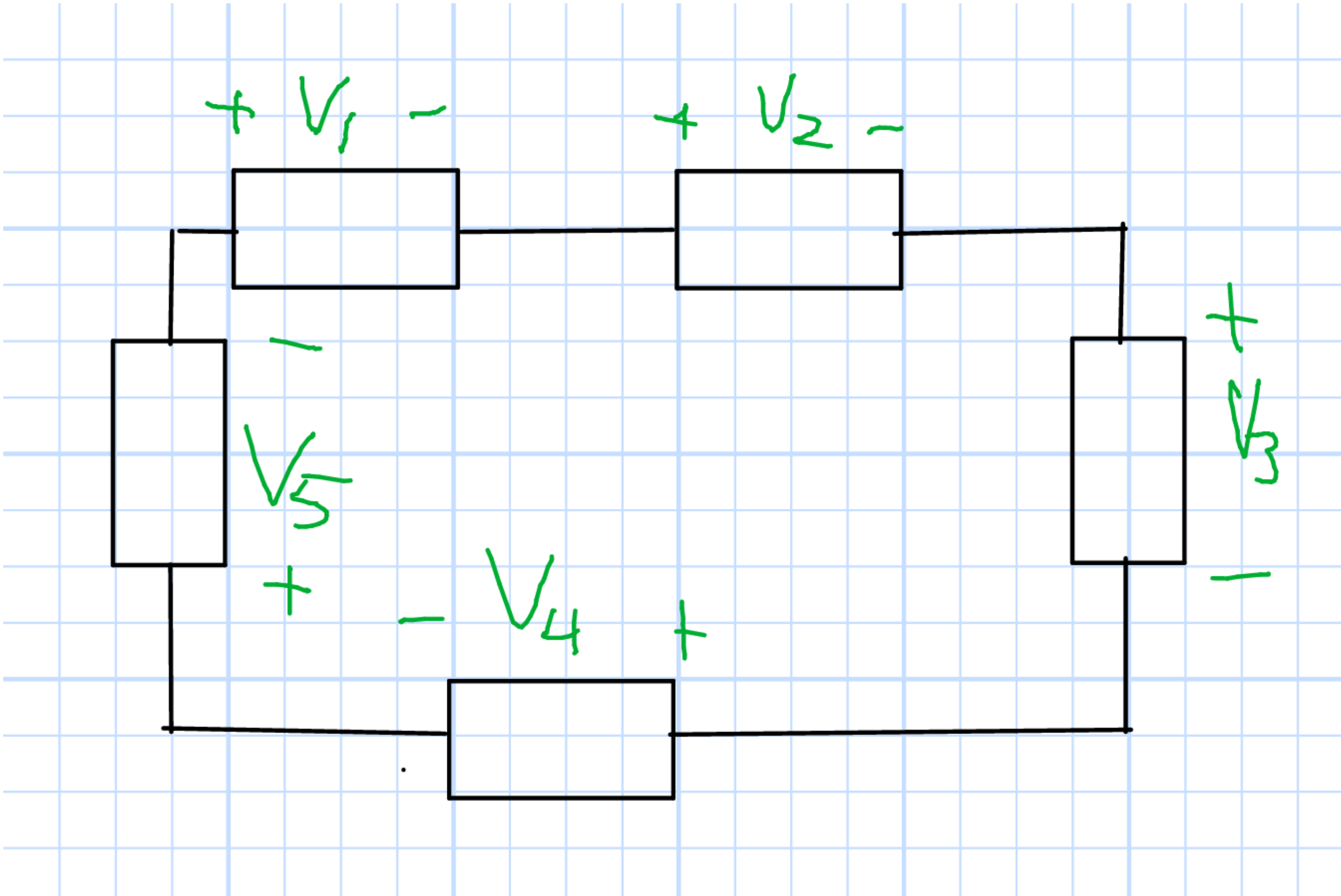
Kirchoff's Current Law

$i_1 = 1_2 + i_3$: Solve for V_A



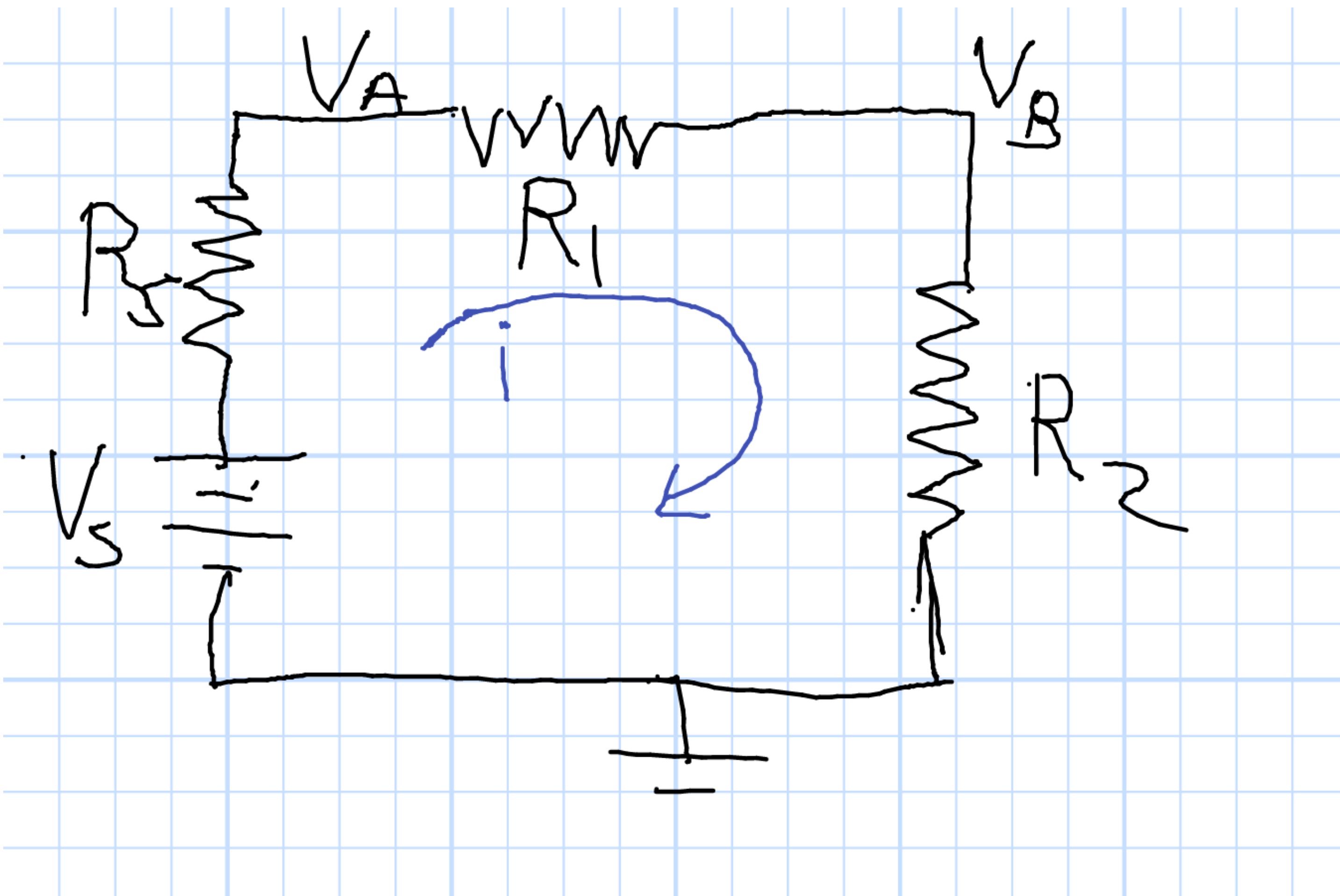
Remember Kirchoff's Voltage Law (KVL)

Conservation of Energy $\sum v_n = 0$



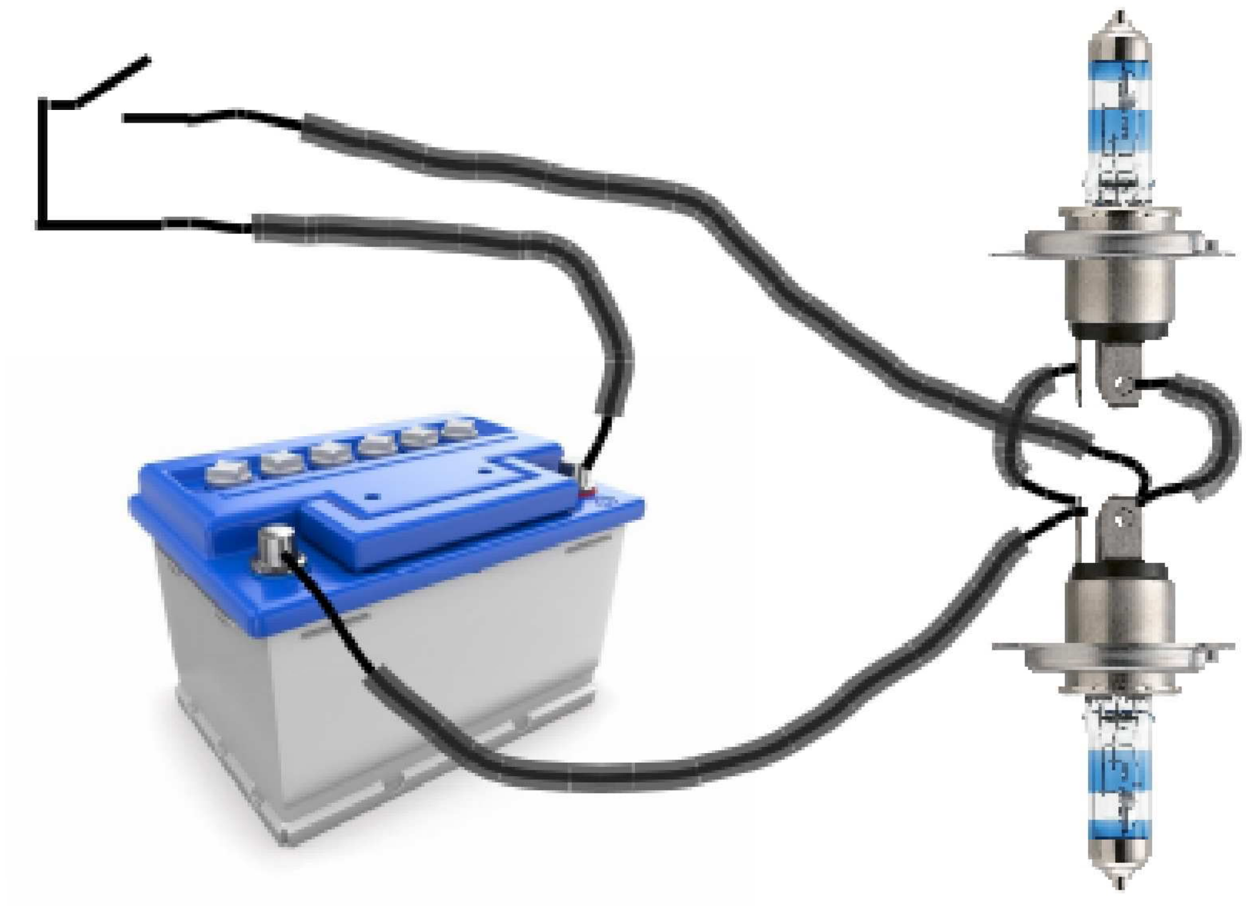
Kirchoff's Voltage Law

$v_s - iR_s - iR_1 - iR_2 = 0$: Solve for i

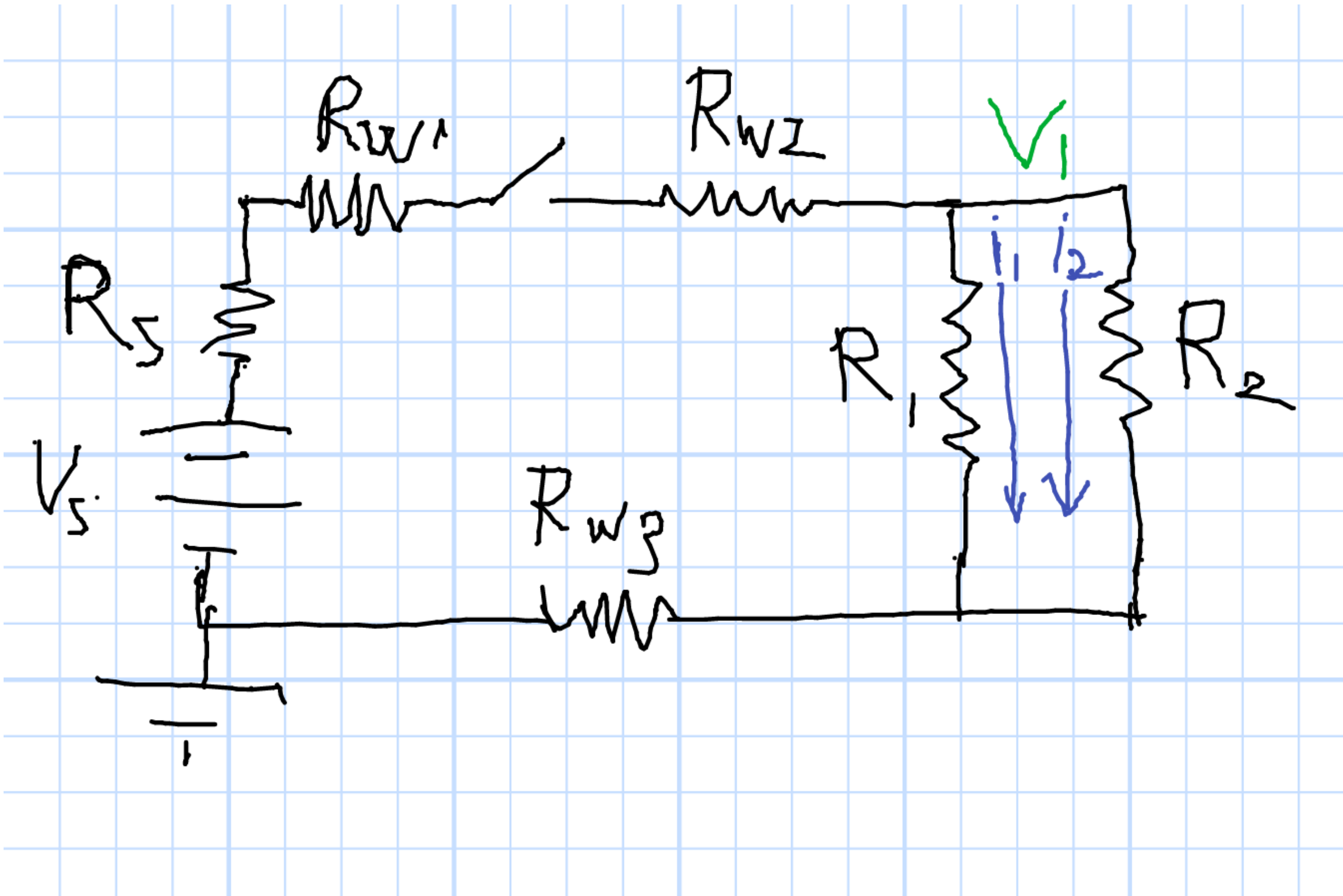


Use KCL and KVL

Draw the Circuit Using Symbols

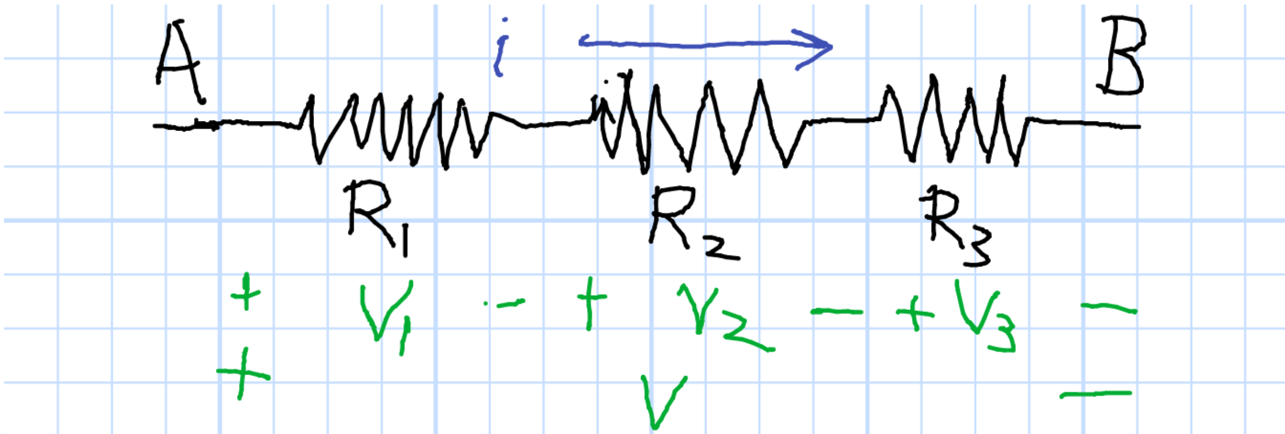


Car Headlights Circuit



Resistors in Series (1)

New Concept: Series and Parallel

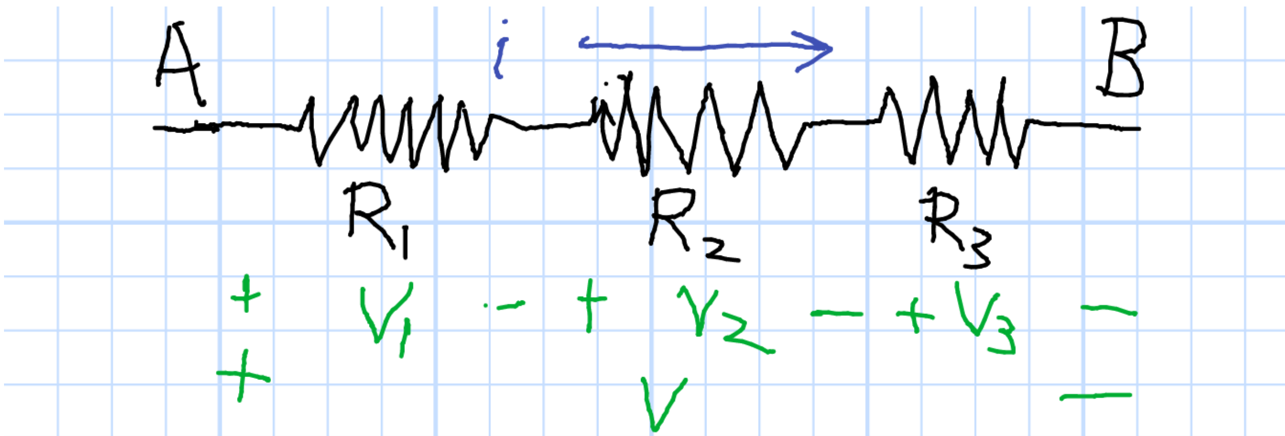


$$V = V_1 + V_2 + V_3 \qquad \text{KVL}$$

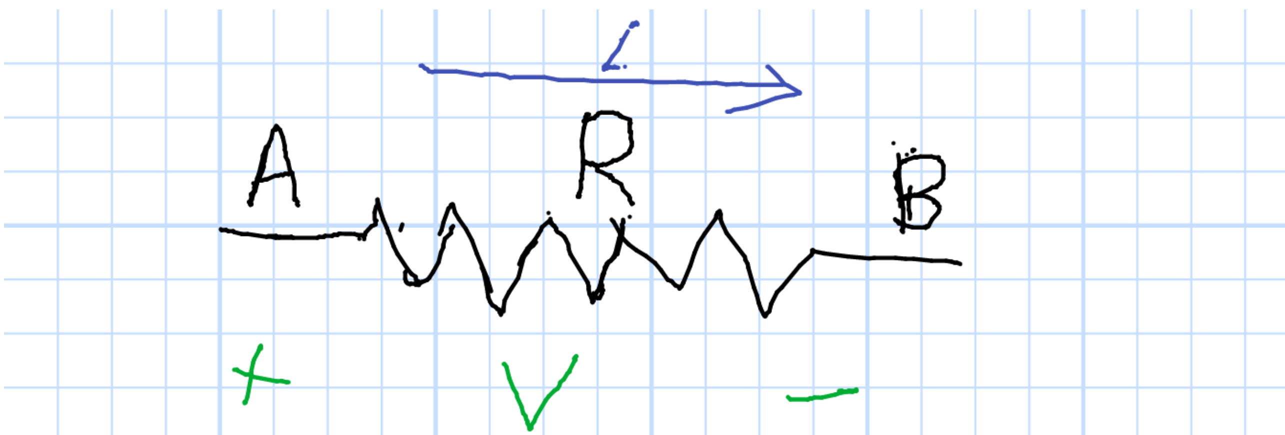
$$\frac{V}{i} = \frac{V_1}{i} + \frac{V_2}{i} + \frac{V_3}{i}$$

$$R = R_1 + R_2 + R_3$$

Resistors in Series (2)



$$R = R_1 + R_2 + R_3$$



Resistors in Series: Examples

Two 1kΩ Resistors in Series

$$R = R_1 + R_2$$

$$R = 2R_1$$

$$R = 2\text{k}\Omega$$

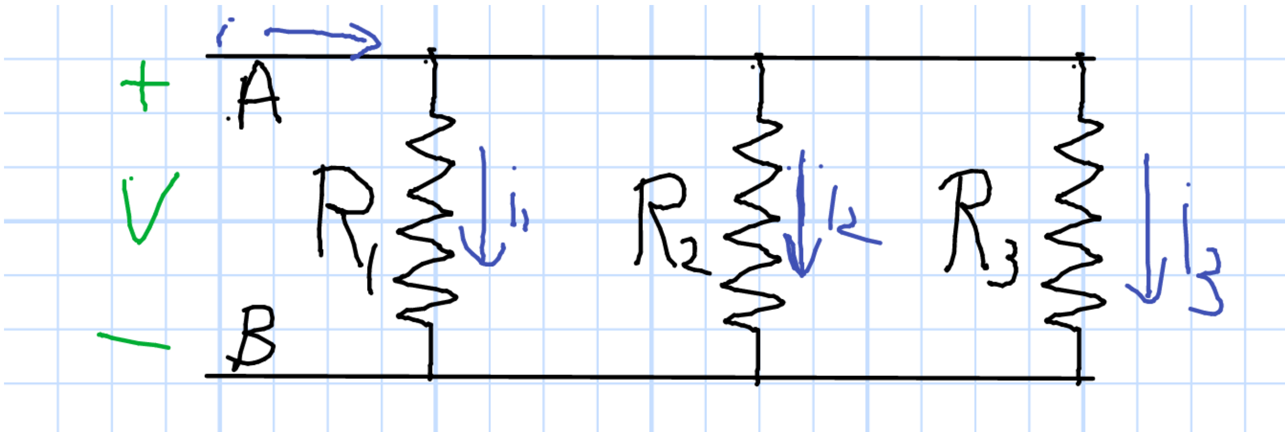
One Large Resistor and One Much Smaller

$$R = R_1 + R_2$$

$$R \approx Max\left(R_n\right)$$

For Example $R_2 = R_1/10$ $\qquad R = 1.1R_1 \qquad$ (10% error)

Resistors in Parallel (1)

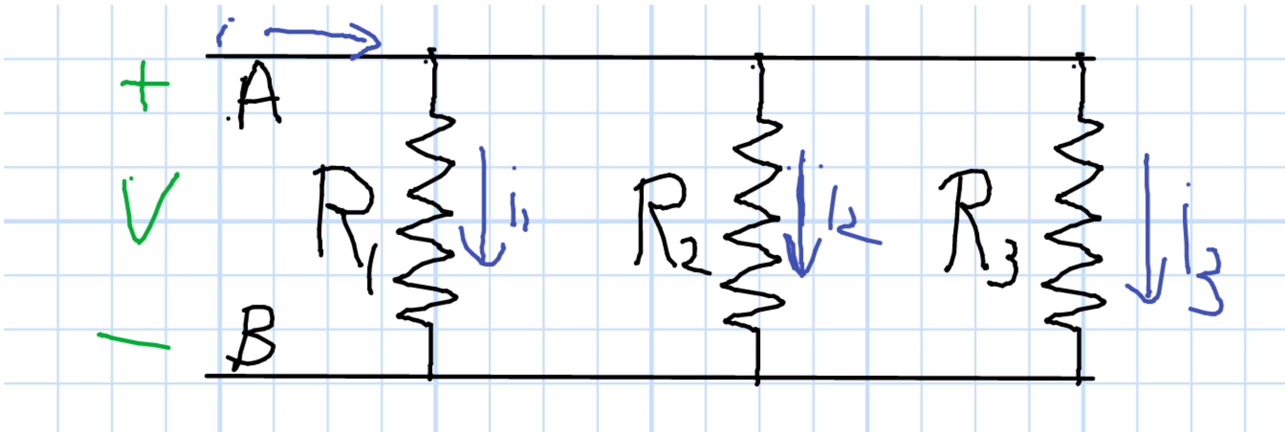


$$i = i_1 + i_2 + i_3 \qquad \text{KCL}$$

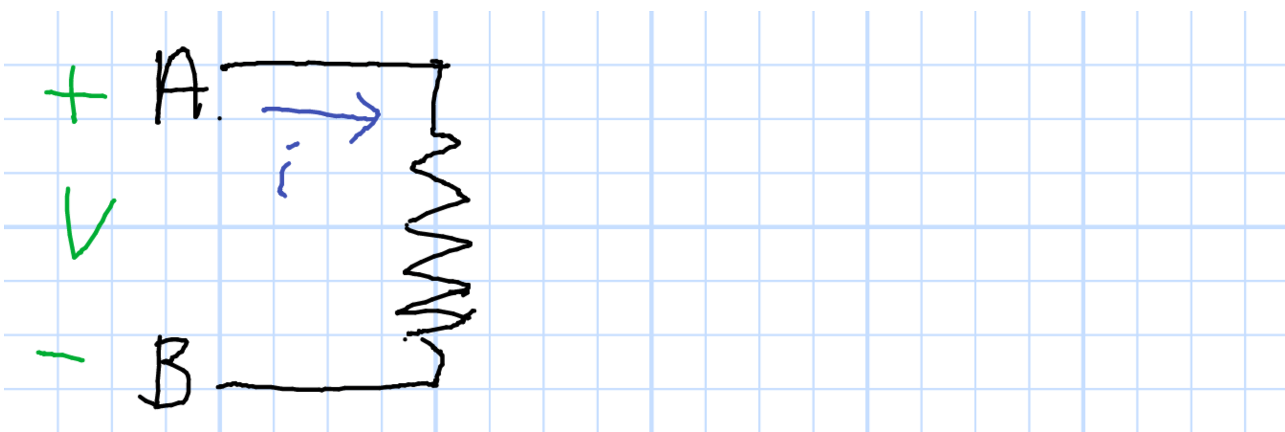
$$\frac{i}{V} = \frac{i_1}{V} + \frac{i_2}{V} + \frac{i_3}{V}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Resistors in Parallel (2)



$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$



Parallel–Resistor Equations

$$R = R_1 \parallel R_2$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

Conductances Add

$$G = G_1 + G_2$$

Resistors in Parallel: Example

Two 1kΩ Resistors in Parallel

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R = \frac{R_1}{2}$$

$$R = 500\Omega$$

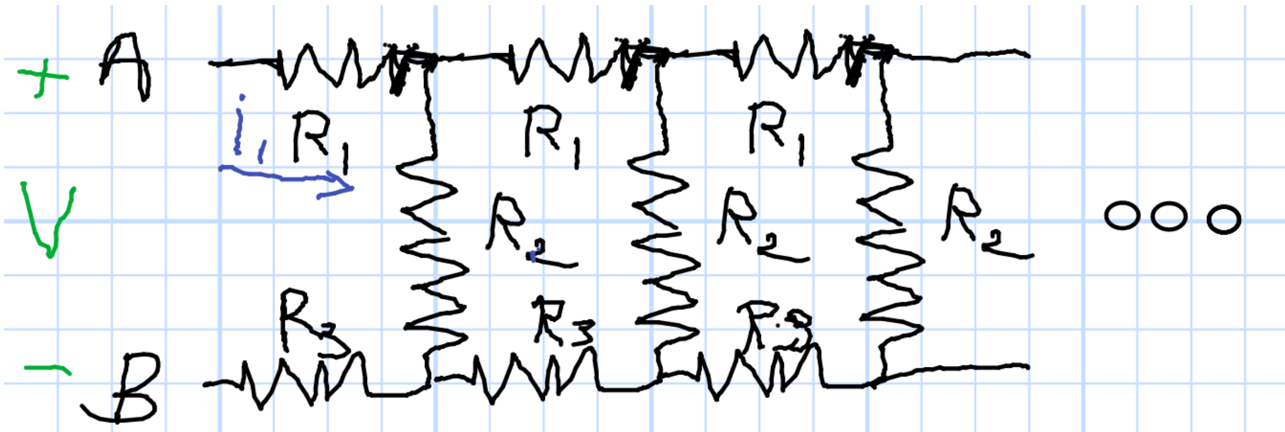
One Large Resistor and One Much Smaller

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R \approx Min\left(R_n\right)$$

For Example $R_2 = R_1/10$ $\qquad$ $R = 0.91R_1$ $\qquad$ (10% error)

Example: Ladder Network



Infinite Network: Equivalent Resistor? ($R_1 = R_2 = R_3 = 50\Omega$)
Assume the Answer is R
Add One More Link

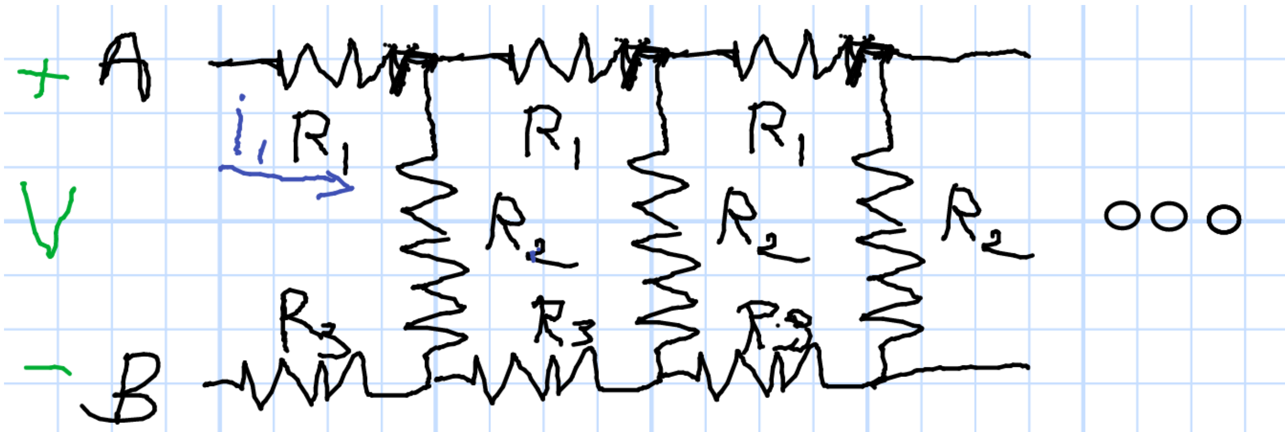
$$R = R_1 + (R_2 \parallel R) + R_3$$

$$R = R_1 + \frac{R_2 R}{R_2 + R} + R_3$$

$$R\left(R_2 + R\right) = R_1\left(R_2 + R\right) + R_2 R + R_3\left(R_2 + R\right)$$

$$R\left(R_2 + R - R_1 - R_2 - R_3\right) = R_1 R_2 + R_3 R_2$$

Ladder Solution



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$$R\left(R_2 + R - R_1 - R_2 - R_3\right) = R_1R_2 + R_3R_2$$

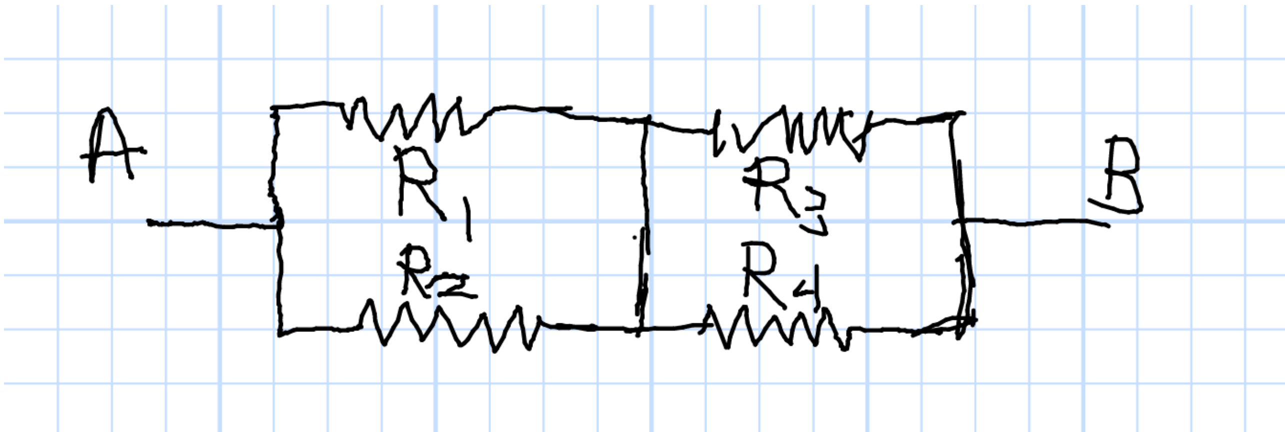
$$R^2 - R\left(R_1 + R_3\right) - R_1R_2 + R_3R_2 = 0$$

Possible Solutions

$$R = \frac{\left(R_1 + R_3\right) \pm \sqrt{\left(R_1 + R_3\right)^2 + 4R_2\left(R_1 + R_3\right)}}{2}$$

For All 50Ω Resistors, $R = 137\Omega$ (Failed Solution, $R = -37\Omega$)

Power Issues



What Resistors to Use?

$R = 1000$, All Resistors Equal

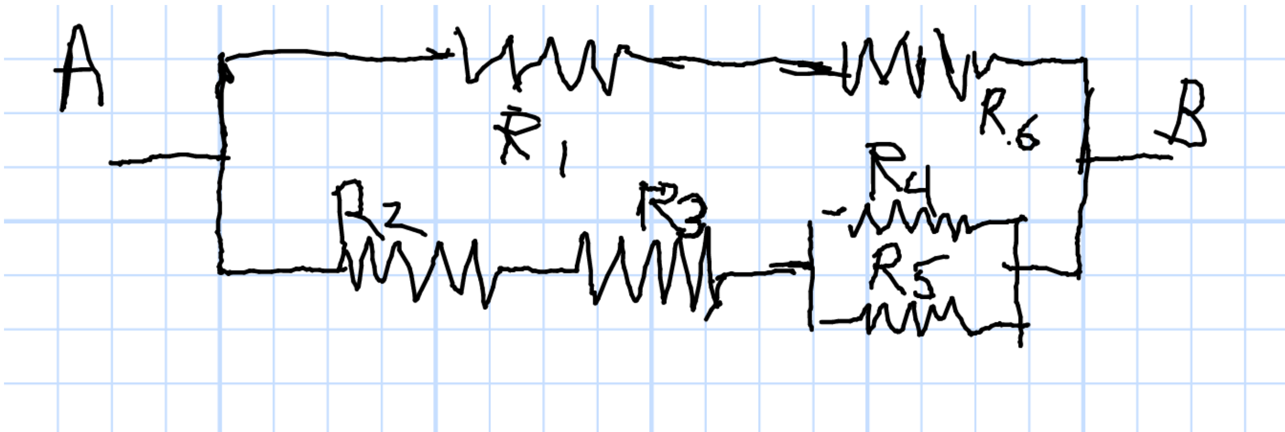
$$R_{1:4} = ?$$

What is the Power in Each Resistor as a Fraction of the Total?

$$\frac{P_n}{P_{total}} = ?$$

What if I leave out the vertical wire in the middle?

Equivalent Resistance (1)



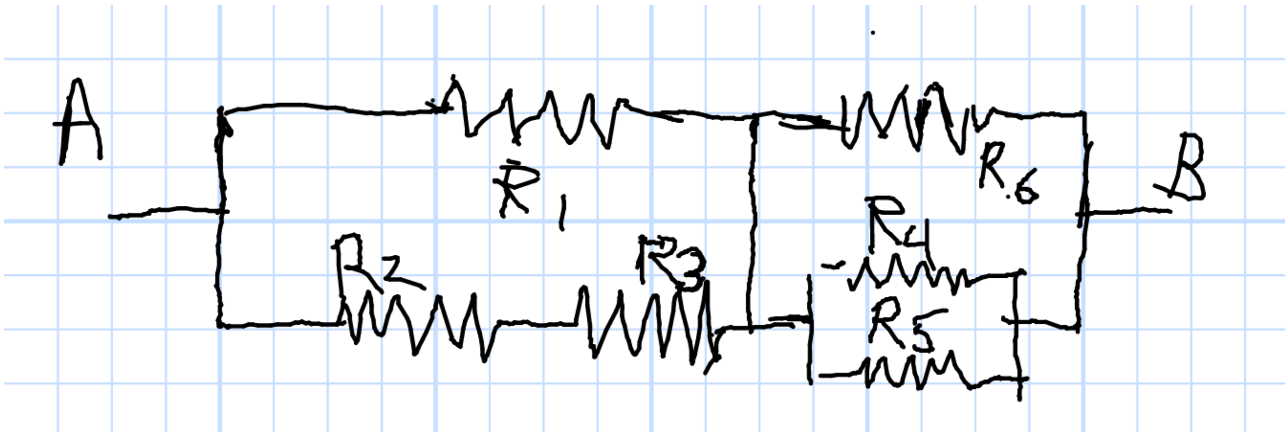
$$[R_1 + R_6] \parallel [R_2 + R_3 + (R_4 \parallel R_5)]$$

$$R_n = 50\Omega \qquad \text{All } n$$

$$R = [50 + 50] \parallel [50 + 50 + 25]$$

$$R = 55.6\Omega$$

Equivalent Resistance (2)



$$[R_1 \parallel (R_2 + R_3)] + [R_4 \parallel R_5 \parallel R_6]$$

$$R_n = 50\Omega \qquad \text{All } n$$

$$R = [50 \parallel (100)] + [50/3]$$

$$R = 50\Omega$$

Series and Parallel

Series

- Voltage Sources Add
- Current Sources Fail
- Resistors Add

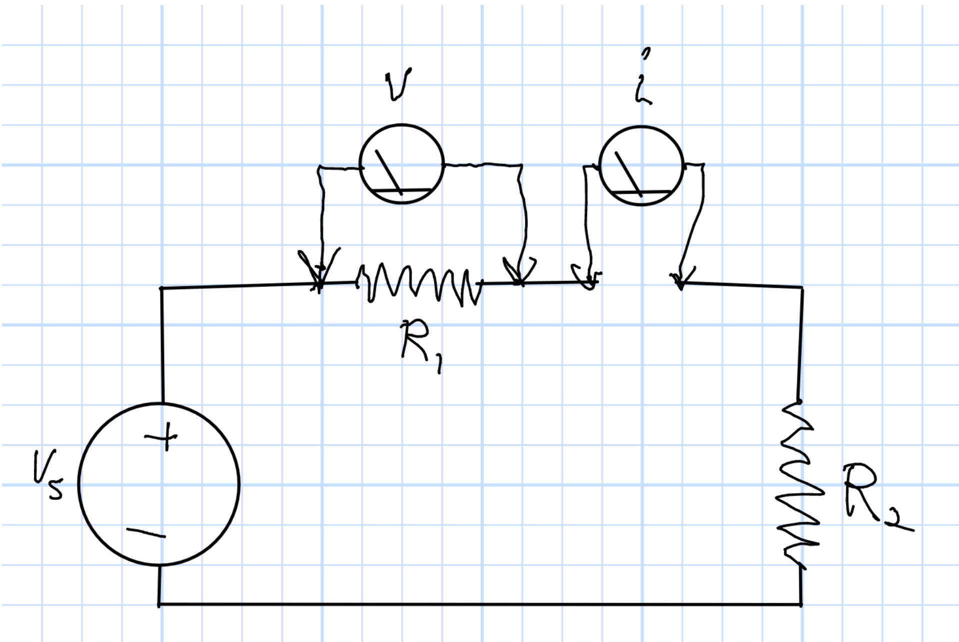
Parallel

- Voltage Sources Fail
- Current Sources Add
- Resistors Add Inverses

How can I prevent the failures?

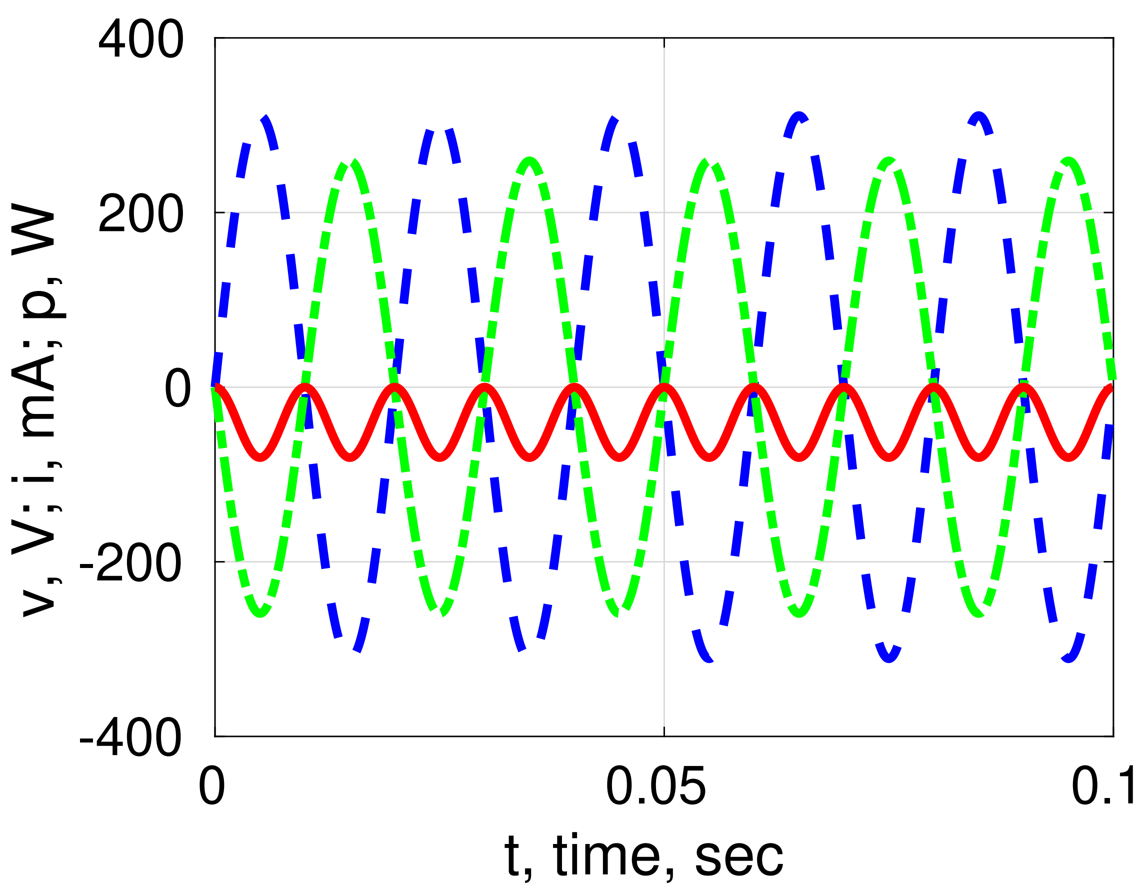
Measuring Voltage and Current

- Voltage Across Something
  - Meter Resistance High
- Current Through Something
  - Meter Resistance Low
  - Break Circuit for Current Measurement



AC Circuits: Sine Waves

$p = iv$, Voltage in Blue, Current in Green, Power in Red
Power at Twice the Operating Frequency



Another Cup of Coffee: RMS Voltage (or Current)

$$p(t) = i(t)\,v(t) \qquad\qquad p(t) = \frac{v^2(t)}{R}$$

$$v(t) = v_0 \cos(2\pi f t) \qquad v_0 = 311\text{Volts} \qquad f = 50\text{Hz}$$

$$v^2(t) = [v_0 \cos(2\pi f t)]^2 \qquad \frac{v^2(t)}{R} = \frac{v_0^2\left[\frac{1}{2} + \frac{1}{2}\cos(4\pi f t)\right]}{R}$$

$$\bar{P} = \frac{\overline{v_0^2}}{2R} = \frac{v_{RMS}^2}{R} = 1000 \text{ Watts} \qquad t = 42 \text{ s}$$

Comparable to the Previous Cup (1kW): See Next Slide

RMS Voltage Defined:

$$v_{RMS} = \frac{v_0}{\sqrt{2}} = \frac{v_{pp}}{2\sqrt{2}} = 220\text{V}$$

This problem was based on power in Europe or South America. How would it be different in North America?

The First Cup of Coffee

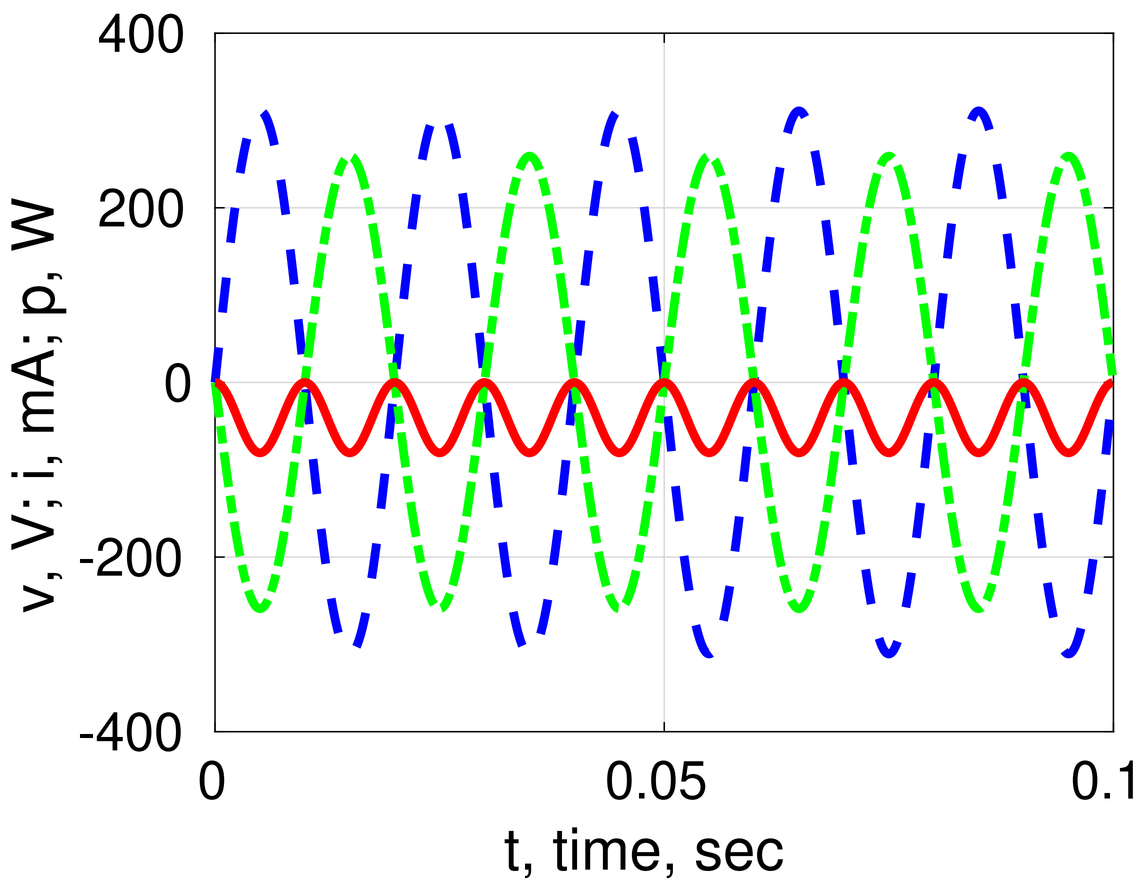
- Energy: Pt (Watts $\times$ Sec $=$ Joules)
- Heat a Cup of Water $T_0 = 20\text{C}$ to $T_1 = 60\text{C}$ (250mℓ)
- Energy Required 4.18J/K/mℓ
- 1kW Heater

$$Pt = 4.18\text{J/K/m}\ell \times 250\text{m}\ell \times (60C - 20C)$$

$$t = 42\text{s} \qquad R = 45\Omega$$

RMS Voltage

What Is the Peak–to–Peak Voltage, Zero–to–Peak, RMS



Voltage Measurements

- $v(t) = v_0 \sin 2\pi f t$ Convenient for Theory
- Peak–to–Peak $v_{pp} = 2v_0$ Easy on an Oscilloscope
- Root–Mean–Square (RMS) $v_{rms} = \frac{v_0}{\sqrt{2}}$ Multi–Meter
- Instantaneous power $P(t) = i(t)v(t)$
- Average power $P_{av} = i_{rms}v_{rms}$

Frequency Question

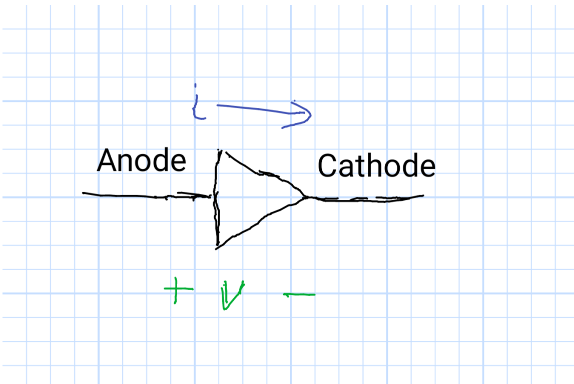
In the lab, you will build an ECG Instrument that is measures voltages.

Wire acts as an antenna. You will see signals from the power wires.

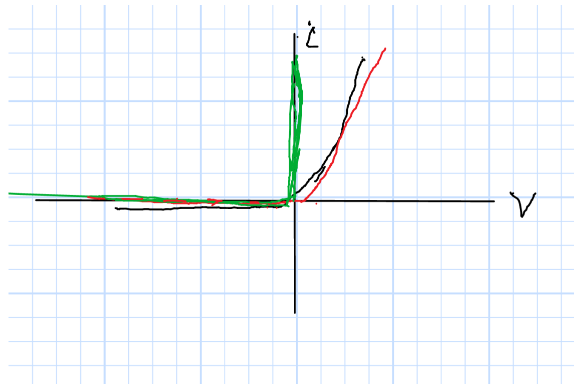
What will be the frequency of those signals . . .

- In the US?
- In Europe?
- In Your Home Country?

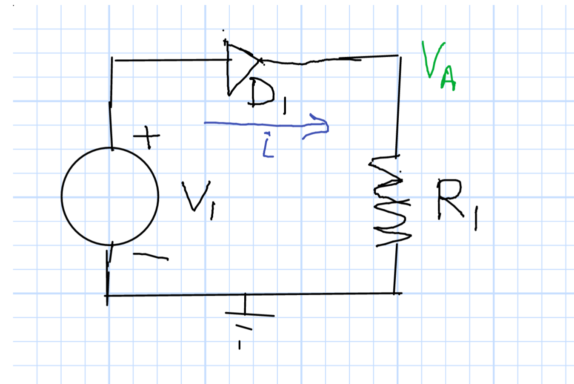
Diodes, Briefly



Diode Symbol



I–V Curves



Circuit

- Diodes are Pretty Good Conductors One Way
- Diodes are Pretty Good Insulators the Other Way
- Light Emitting Diodes Emit Light When they Conduct
- They are Nonlinear
- There are Several Piecewise–Linear Approximations
- The Details Can Get Messy