Circuits and Signals: Biomedical Applications
Week 8

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Oct 2023

Week 8 Agenda: Circuits and Sine/Cosine Waves

- General Concepts of Sinusoidal Analysis
- C, L, Reactance and Impedance
- Eli the Ice Man
- Series and Parallel
- Voltage and Current Dividers
- One More Op–Amp Circuit
- Thévenin Equivalent
- Norton Equivalent

Solving Problems (1)

Real Quantity	Positive Freq.		Negative Freq.	Phasor
$A\cos\omega t$	$\frac{A}{2}e^{j\omega t}$	$+$	$\frac{A}{2}e^{-j\omega t}$	$A\angle 0$
$A\sin\omega t$	$\frac{-jA}{2}e^{j\omega t}$	$+$	$\frac{jA}{2}e^{-j\omega t}$	$A\angle 90$
$A\cos(\omega t+\phi)$	$\frac{Ae^{j\phi}}{2}e^{j\omega t}$	$+$	$\frac{Ae^{-j\phi}}{2}e^{-j\omega t}$	$A\angle\phi$
$A\cos\phi\cos\omega t - \ldots$ $A\sin\phi\sin\omega t =$				
$a\cos\omega t + b\sin\omega t$	$\frac{a+jb}{2}e^{j\omega t}$	$+$	$\frac{a-jb}{2}e^{-j\omega t}$	$a+jb$

Problem Solution

Unknown $= x + jy$

Pos. Freq. $+$ C.C.

$$2\mathrm{Re}\left[\frac{x+jy}{2}e^{j\omega t}\right]$$

$$\mathrm{Re}\left[(x+jy)\,e^{j\omega t}\right]$$

Solving Problems (2)

Known Sources: $i\cos(\omega t+\phi_s)=\frac{ie^{j\phi_s}}{2}e^{j\omega t}+\frac{ie^{-j\phi_s}}{2}e^{-j\omega t}$

Resistors

$v=iR$ $\qquad$ $\mathbf{V}=\mathbf{I}R$

Capacitors

$i=C\frac{dv}{dt}$ $\qquad$ $\mathbf{I}=j\omega C\mathbf{V}$ $\qquad$ $\mathbf{V}=\frac{1}{j\omega C}\mathbf{I}$

Inductors

$v=L\frac{di}{dt}$ $\qquad$ $\mathbf{V}=j\omega L\mathbf{I}$

Unknowns: $v\cos(\omega t+\phi_u)=\frac{ve^{j\phi_u}}{2}e^{j\omega t}+\frac{ve^{-j\phi_u}}{2}e^{-j\omega t}$

Easy? . . .

Easy!

Capacitor (1)

- Capacitor Equation

$$i = C\frac{dv}{dt}$$

- Sinusoidal Voltage

$$v = \mathbf{V}e^{j\omega t}$$

- Current

$$i = C\frac{dv}{dt} = Cj\omega\mathbf{V}e^{j\omega t}$$

$$i = \mathbf{I}e^{j\omega t} = Cj\omega\mathbf{V}e^{j\omega t}$$

$$\mathbf{I} = j\omega C\mathbf{V}$$

- Impedance (New Concept)

$$Z = \frac{\mathbf{V}}{\mathbf{I}} = \frac{1}{j\omega C}$$

- Example $C = 1\mu\text{F}$

$$Z = \frac{1}{j\omega C} \qquad \omega = 2\pi f$$

$$V = 10\text{V at } f = 50\text{Hz}$$

- Current

$$I = \frac{V}{Z} \qquad i = Ie^{j\omega t}$$

```
>> C=1e-6;V=10;f=50;
>> Z=1/(1j*2*pi*f*C)
Z =
   0.0000e+00 - 3.1831e+03i
>> I=V/Z
I =
   0.0000 + 0.0031i
```

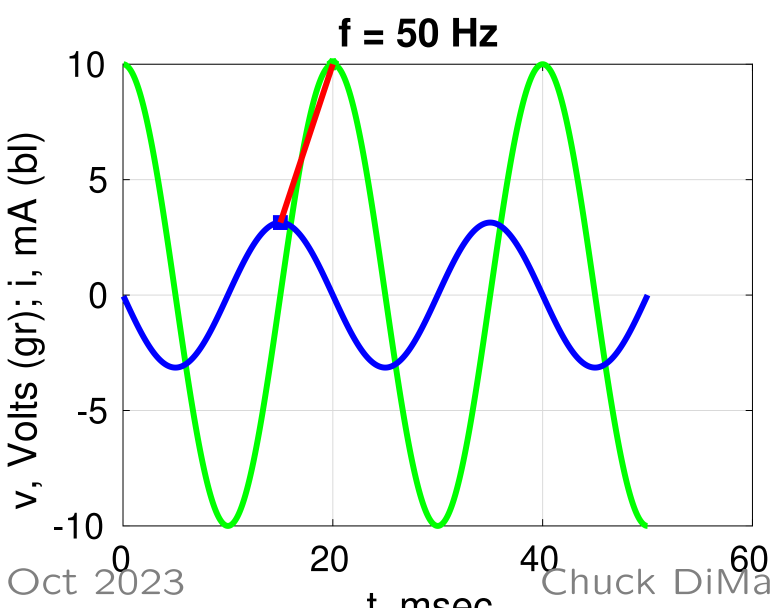
$$I = 3.1\text{mA}\angle 90^\circ$$

Capacitor (2); Current Leads

$C = 1\mu$F $\quad V = 10$V at f

f	Z	$\|I\|$
50Hz	$-j3200\Omega$	3.1mA
500Hz	$-j320\Omega$	31mA
5kHz	$-j32\Omega$	310mA
50kHz	$-j3.2\Omega$	3.1A

Current Peak is Earlier




```
t=[0.001:0.001:1]*0.05;
C=1e-6;V=10;f=50;omega=2*pi*f;
Z=1/(1j*omega*C);I=V/Z;
vc=real(V*exp(1j*omega*t));
ic=real(I*exp(1j*omega*t));
fv=find(vc==max(vc),1);
fi=find(ic==max(ic),1);
figure;plot(t*1e3,vc,'g',...
  t*1e3,ic*1e3,'b',...
 t(fv)*1e3,vc(fv),'g*',...
 t(fi)*1e3,ic(fi)*1e3,'bs',...
 [t(fv),t(fi)]*1e3,...
   [vc(fv),ic(fi)*1e3],'r');
grid on;xlabel('t, msec');
ylabel('v, Volts (gr); i, mA (bl)')
title('f = 50 Hz');
```

Inductor: Current Lags

- Inductor Equation

$$v = L\frac{di}{dt}$$

- Sinusoidal Current

$$i = \mathbf{I}e^{j\omega t}$$

- Voltage

$$v = L\frac{di}{dt} = Lj\omega\mathbf{I}e^{j\omega t}$$

$$v = \mathbf{V}e^{j\omega t} = Lj\omega\mathbf{I}e^{j\omega t}$$

$$\mathbf{V} = j\omega L\mathbf{I}$$

- Impedance

$$Z = \frac{\mathbf{V}}{\mathbf{I}} = j\omega L$$

- Example $L = 5\text{H}$

$$Z = j\omega L$$

- Current

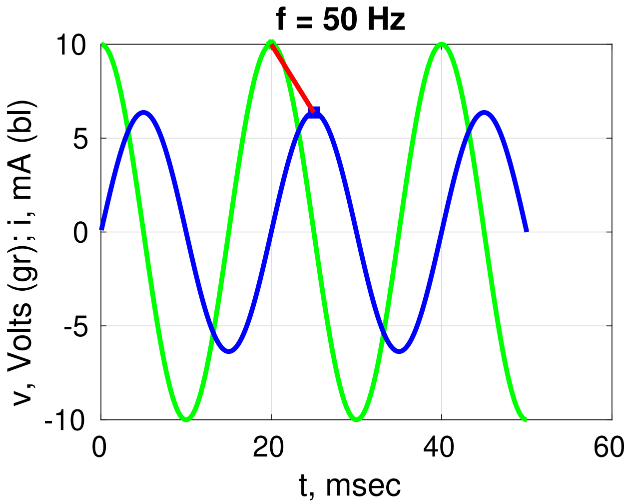
$$I = \frac{V}{Z} \qquad i = Ie^{j\omega t}$$

```
>> L=5;V=10;f=50;omega=2*pi*f;
>> Z=1j*omega*L
Z =
   0.0000e+00 + 1.5708e+03i
>> I=V/Z
I =
   0.0000 - 0.0064i
```

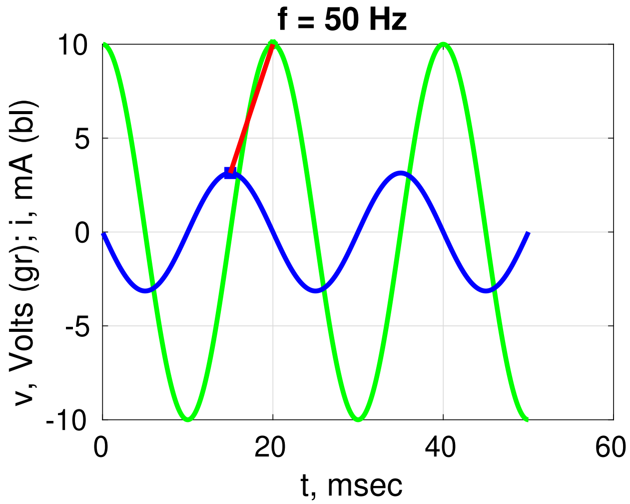
$$Z = 1600\Omega\angle 90^\circ$$

$$I = 6.4\text{mA}\angle{-90^\circ}$$

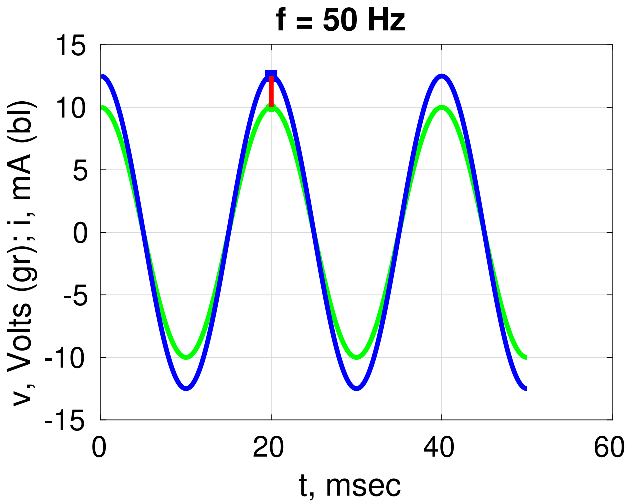
R, L, and C



$L = 500$mH (Peak v before i)



$C = 1\mu$F (Peak v after i)



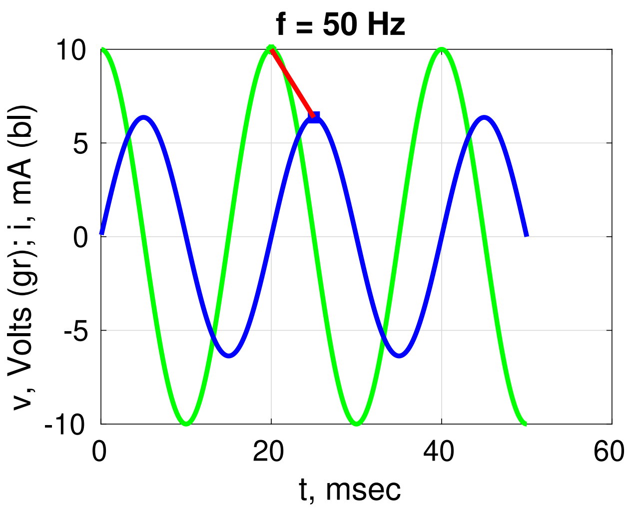
500Ω Resistor

$$Z_C = 3200\Omega\angle{-90^\circ}$$

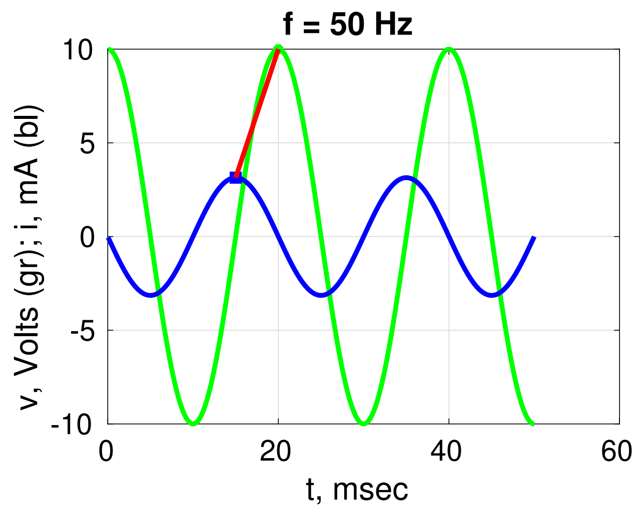
$$Z_L = 1600\Omega\angle{+90^\circ}$$

$$Z_R = 500\Omega\angle 0^\circ$$

Eli the Ice Man



500mH Inductor



1μF Capacitor

- E for Electromotive Force (Voltage)
- Inductor (ELI)
  E (Voltage) Leads I (Current):
  Peak of Voltage (Green) Before Current (Blue)
- Capacitor (ICE)
  I (Current) Leads E (Voltage):
  Peak of Current (Blue) Before Voltage (Green)

Resistance, Reactance, Impedance

- Resistance, R

$$Z_R = R \qquad \mathbf{V} = \mathbf{I}R$$

- Reactance, X New Concept

$$Z = jX \qquad \mathbf{V} = \mathbf{I}jX$$

- Examples

$$Z_C = jX_C = \frac{1}{j\omega C} = \frac{-j}{\omega C} \qquad Z_L = jX_L = j\omega L$$

- Impedance

$$Z = R + jX \qquad \mathbf{V} = \mathbf{I}Z$$

AC Circuits & Steady–State Sinusoids

- Voltage Sources, Current Sources, Resistors

$$v = iR$$

- Voltage Sources, Current Sources, Impedances

$$\mathbf{V} = \mathbf{IZ} = \mathbf{I}R + j\mathbf{I}X$$

- Everything We Learned Before Works
  - Series and Parallel Circuits
  - Voltage and Current Dividers
  - Node and Mesh Analysis
  - Even Op–Amps
- Just Remember the Numbers are Complex

$$i = \text{Re}\left(\mathbf{I}e^{j\omega t}\right) \qquad v = \text{Re}\left(\mathbf{V}e^{j\omega t}\right)$$

Series and Parallel

- Resistors in Series

$$R = R_1 + R_2$$

- Resistors in Parallel

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

- Inductors in Series

$$j\omega L = j\omega L_1 + j\omega L_2$$

- Inductors in Parallel

$$\frac{1}{j\omega L} = \frac{1}{j\omega L_1} + \frac{1}{j\omega L_2}$$

- Capacitors in Series

$$\frac{1}{j\omega C} = \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}$$

- Capacitors in Parallel

$$\frac{1}{1/j\omega C} = \frac{1}{1/j\omega C_1} + \frac{1}{1/j\omega C_2}$$

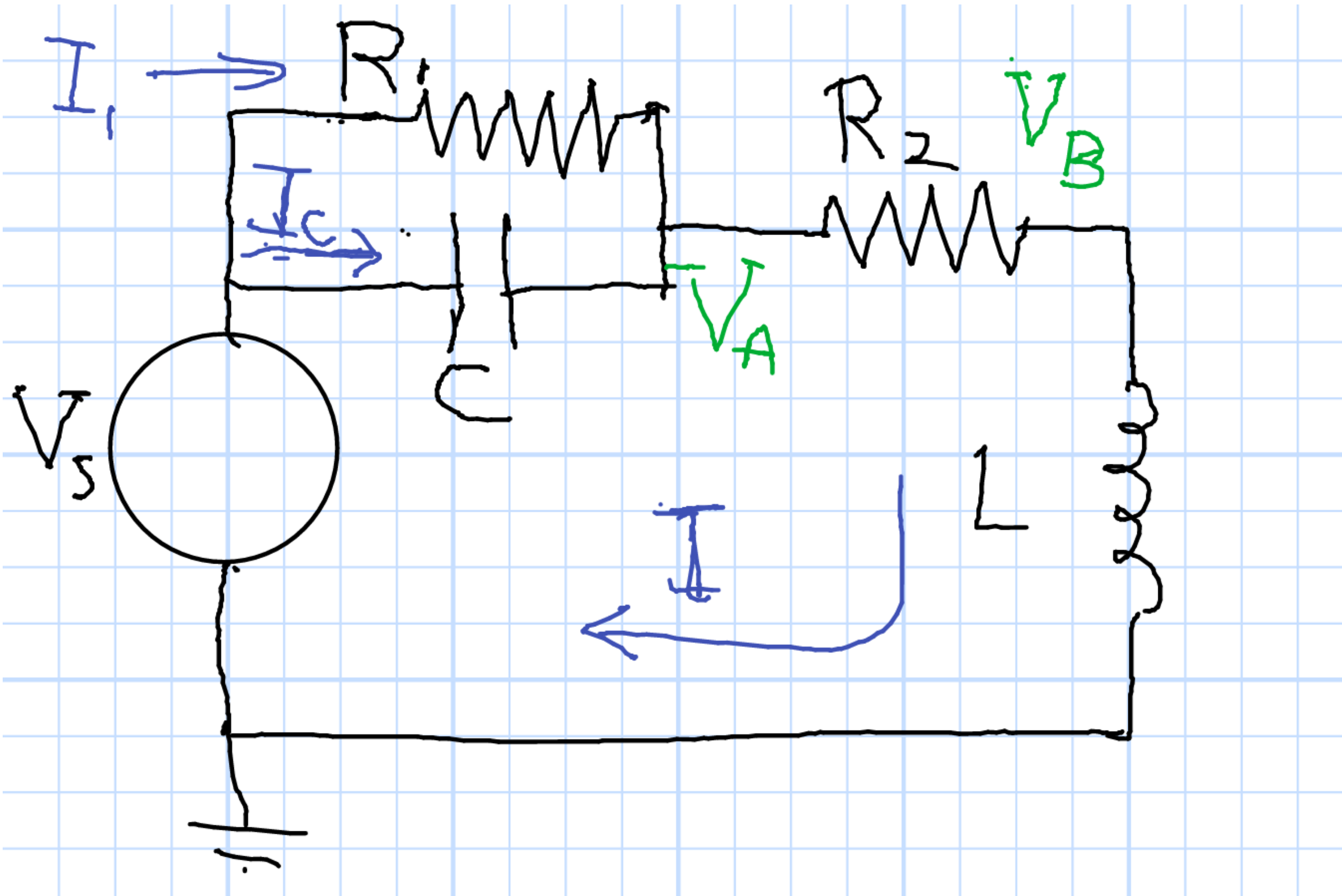
- Impedances in Series

$$Z = Z_1 + Z_2$$

- Impedances in Parallel

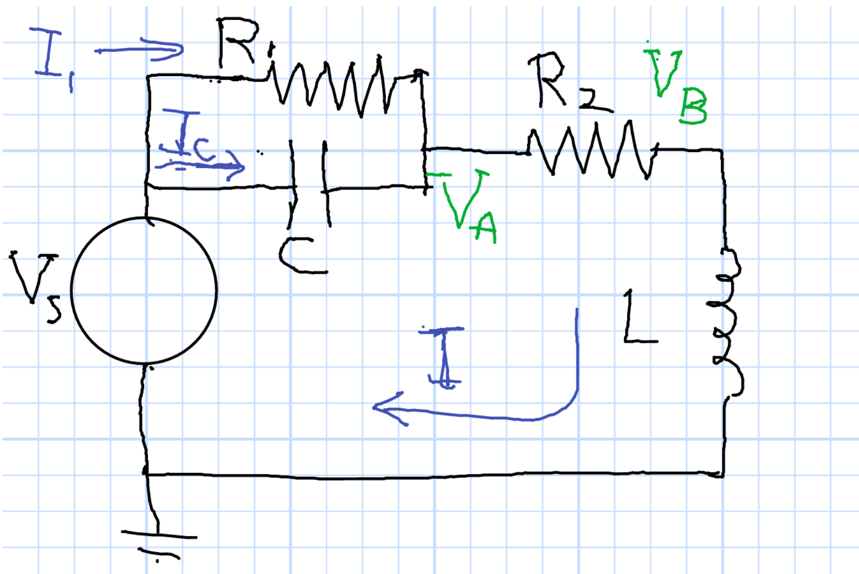
$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

Voltage Divider



$R_1 = 1\text{k}\Omega$, $R_2 = 2\text{k}\Omega$, $C = 470\text{pF}$, $L = 3\text{mH}$, $V_s = 20\text{V}$, $f = 100\text{kHz}$

Overall Impedance



$$Z = \left(R_1 \parallel \frac{1}{j\omega C}\right) + R_2 + j\omega L$$

```
V=20;omega=2*pi*1e5;
ZC=1/(1j*omega*C);
ZL=1j*omega*L;
Z1C=1/(1/R1+1/ZC);
>> Z=Z1C+R2+ZL
Z =
   2.9198e+03 + 1.6133e+03i
>> abs(Z)
ans =
   3.3359e+03
>> angle(Z)*180/pi
ans =
   28.9229
```

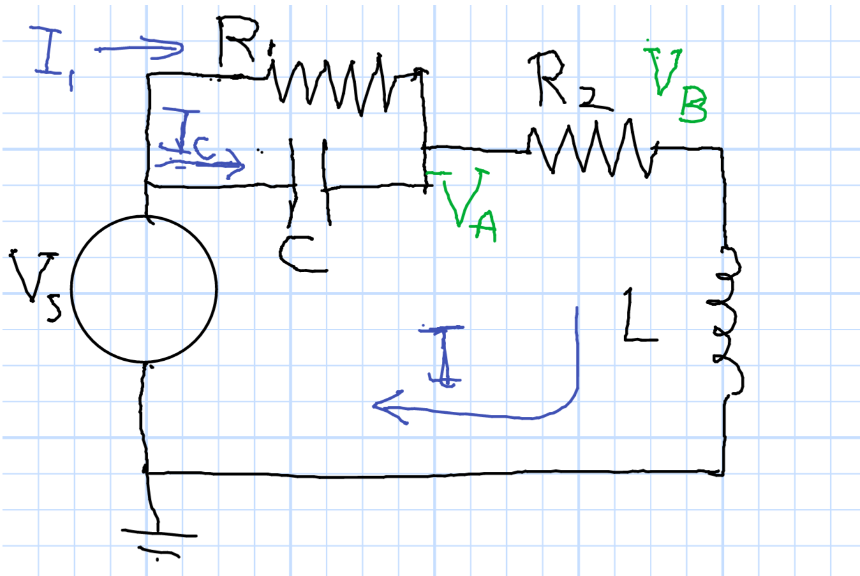
$R_1 = 1\text{k}\Omega$ $\qquad R_2 = 2\text{k}\Omega$

$C = 470\text{pF}$ $\qquad L = 3\text{mH}$

$V_s = 20\text{V}$ at $f = 100\text{kHz}$

$Z = 3.3\text{k}\Omega\angle 29^\circ$

Current



$R_1 = 1\text{k}\Omega$ $\quad R_2 = 2\text{k}\Omega$

$C = 470\text{pF}$ $\quad L = 3\text{mH}$

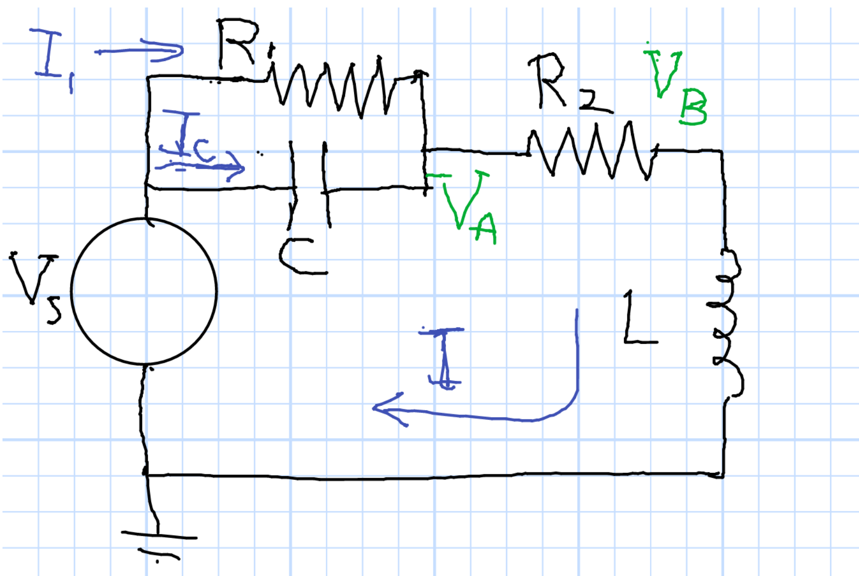
$V_s = 20\text{V}$ at $f = 100\text{kHz}$

$$I = \frac{V}{Z} = \frac{20\text{V}}{Z}$$

```
>> I=V/Z
I =
   0.0052 - 0.0029i
>> abs(I)
ans =
    0.0060
>> angle(I)*180/pi
ans =
  -28.9229
```

$$I = 6\text{mA}\angle -29^\circ$$

Current Divider



$R_1 = 1\text{k}\Omega$ $\quad R_2 = 2\text{k}\Omega$

$C = 470\text{pF}$ $\quad L = 3\text{mH}$

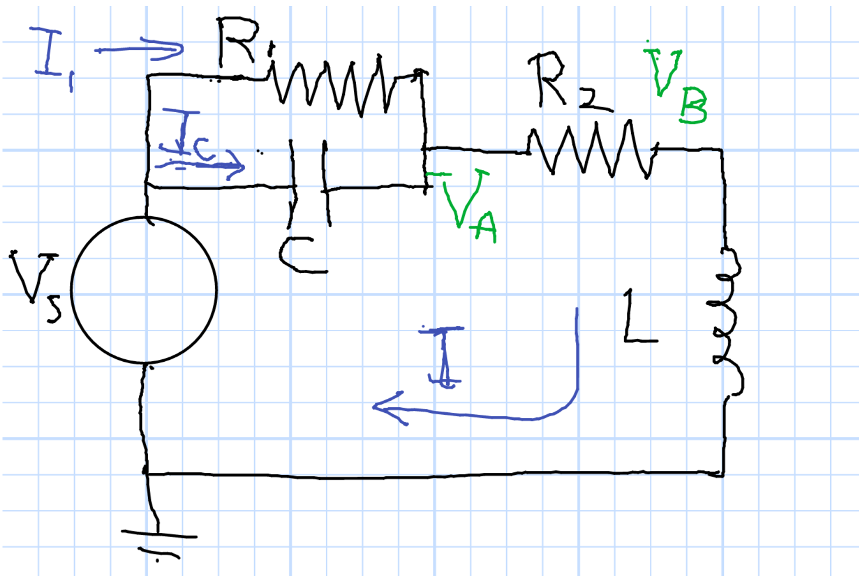
$V_s = 20\text{V}$ at $f = 100\text{kHz}$

$$I_C = I\frac{R_1}{R_1 + Z_C}$$

$$I_1 = I\frac{Z_C}{R_1 + Z_C}$$

```
IC=I*R1/(R1+ZC)
IC =
   0.0012 + 0.0012i
I1=I*ZC/(R1+ZC)
I1 =
   0.0040 - 0.0041i
Icheck=IC+I1, I
Icheck =
   0.0052 - 0.0029i
I =
   0.0052 - 0.0029i
```

Voltage Divider



$R_1 = 1\text{k}\Omega$ $\quad R_2 = 2\text{k}\Omega$

$C = 470\text{pF}$ $\quad L = 3\text{mH}$

$V_s = 20\text{V at } f = 100\text{kHz}$

$$V_A = V\frac{R_2 + j\omega L}{Z}$$

```
>> VA=V*(R2+ZL)/Z
VA =
  15.9609 + 4.0924i
>> abs(VA),angle(VA)*180/pi
ans =
   16.4772
ans =
   14.3809
```

$$V_A = 16.4\text{V}\angle 14^\circ$$

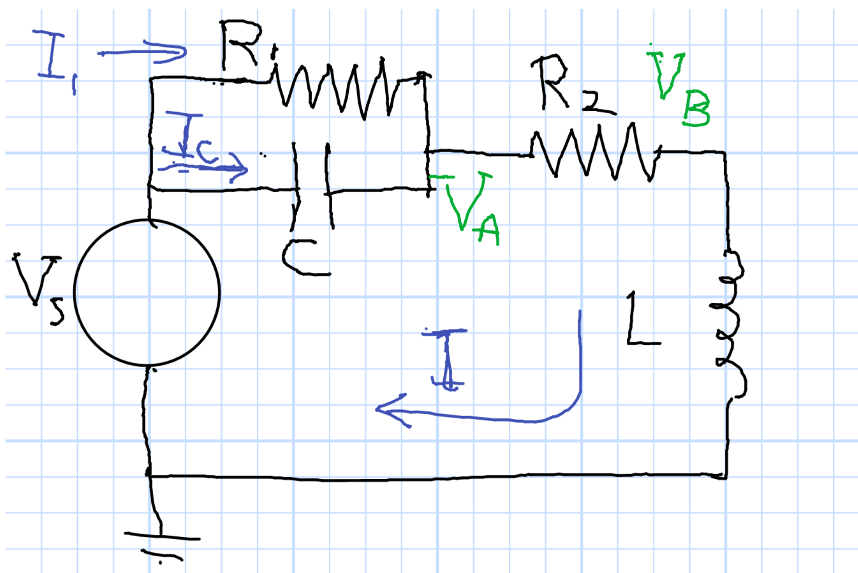
$$V_B = V\frac{j\omega L}{Z}$$

$$= 11.3\text{V}\angle 61^\circ$$

Questions

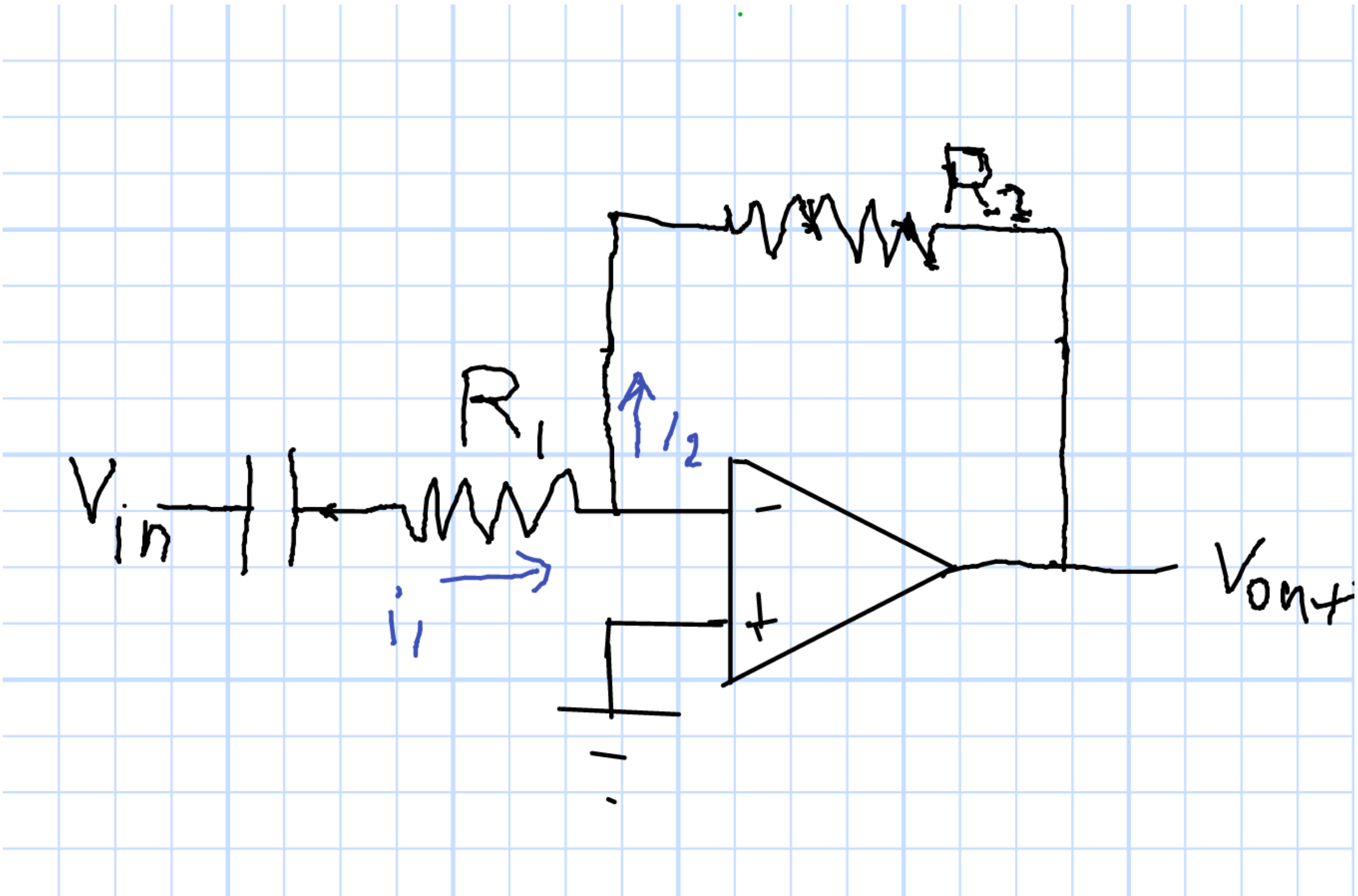
How much does all of this change in the current circuit if the input to the oscilloscope is 1MΩ?

How about if it is 50Ω?

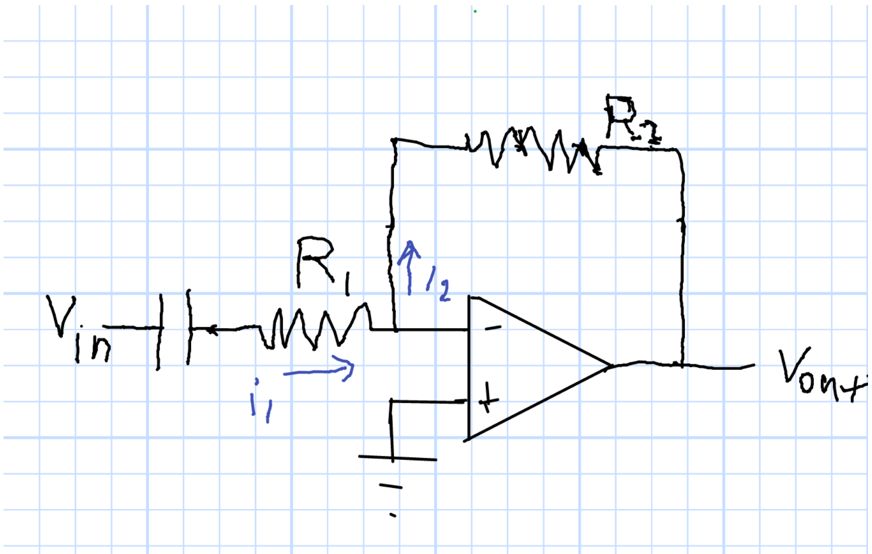


$R_1 = 1$kΩ, $R_2 = 2$kΩ $Z_C = -j3.4$kΩ, $Z_L = j1.9$kΩ,
$V = 20$V, $f = 100$kHz

AC–Coupled Op–Amp Circuit



AC–Coupled Amplifier Solution



$$A_v = -\frac{R_2}{R_1 + \frac{1}{j\omega C}}$$

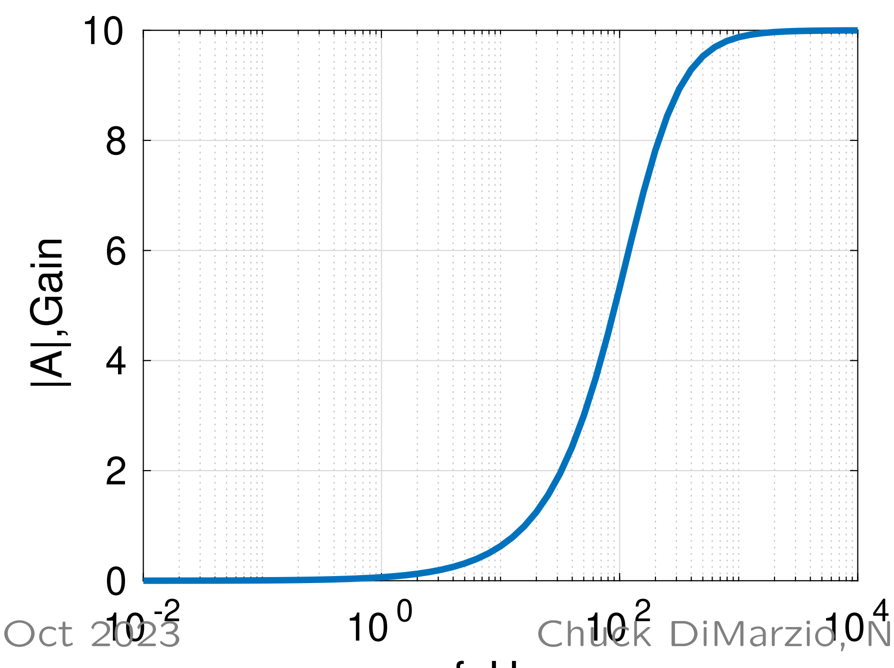
$R_1 = 1\text{k}\Omega$ $\quad$ $R_2 = 10\text{k}\Omega$

$C = 1\mu\text{F}$ $\quad$ $X_C = \frac{1}{j2\pi f C}$

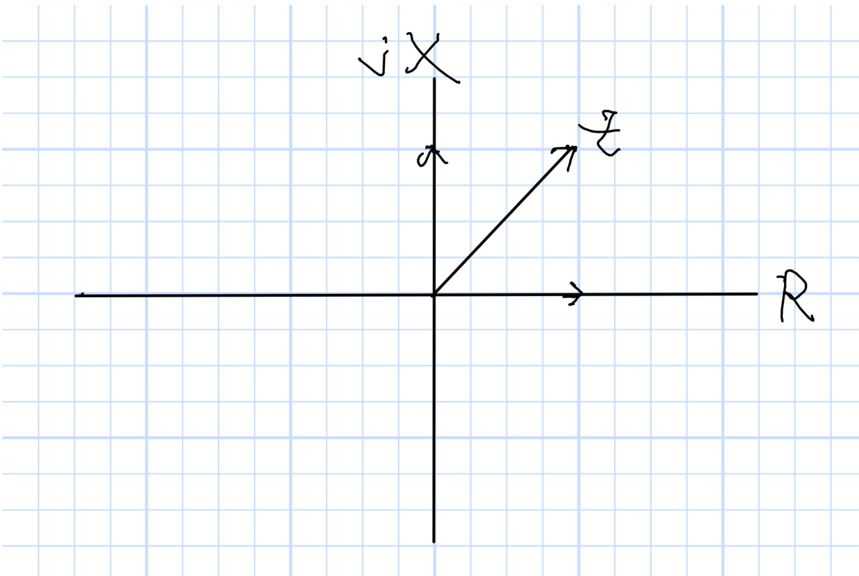
$$X_C = -160\text{k}\Omega \text{ at } 1\text{Hz}$$

$$X_C = -160\Omega \text{ at } 1\text{kHz}$$

```
>> f=10.^[-2:0.1:4];
>> R1=1000;R2=10e3;C=1e-6;
>> A=-R2./(R1+1./(1j*2*pi*f*C));
>> figure;semilogx(f,abs(A));
grid on;
>> xlabel('f, Hz');
ylabel('|A|,Gain');
```



Cutoff



- Series Components

$$Z = R + jX$$

$$|X| = R \quad \rightarrow \quad |Z| = \sqrt{2}R$$

- Series Components

$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{jX}$$

$$|X| = R \quad \rightarrow \quad |Z| = \sqrt{\frac{1}{2}}R$$

- Voltage Gain Changes by $\sqrt{2}$
- Power Gain Changes by 2