Circuits and Signals: Biomedical Applications
Week 9

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EECE–2150
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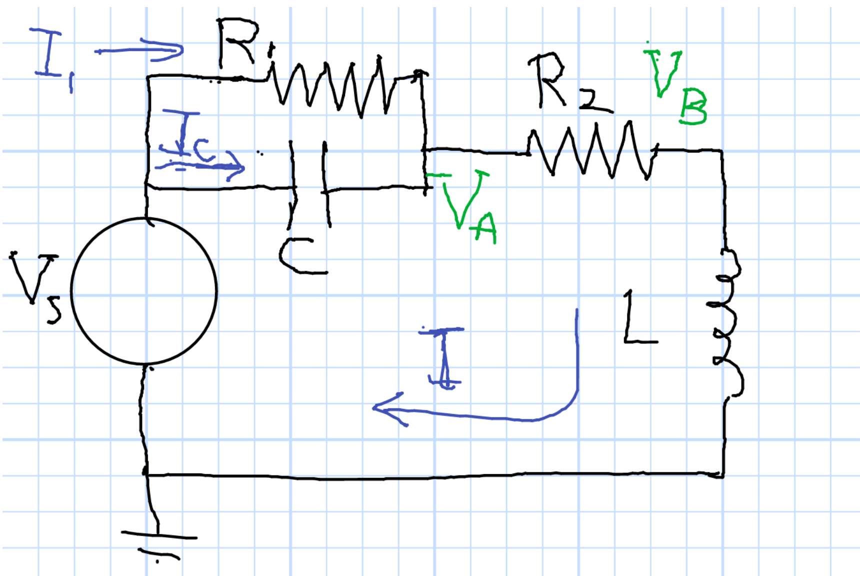
Week 9 Agenda: Circuits and Sine/Cosine Waves

- Power, RMS, Peak–to–Peak, Oscilloscopes and Meters
- Resistive and Reactive Power, Power Factor
- Thévenin Equivalent
- Norton Equivalent
- Fourier Series

Power



$R_1 = 1\text{k}\Omega$ $\qquad R_2 = 2\text{k}\Omega$

$C = 470\text{pF}$ $\qquad L = 3\text{mH}$

$V_s = 20\text{V}$ at $f = 100\text{kHz}$

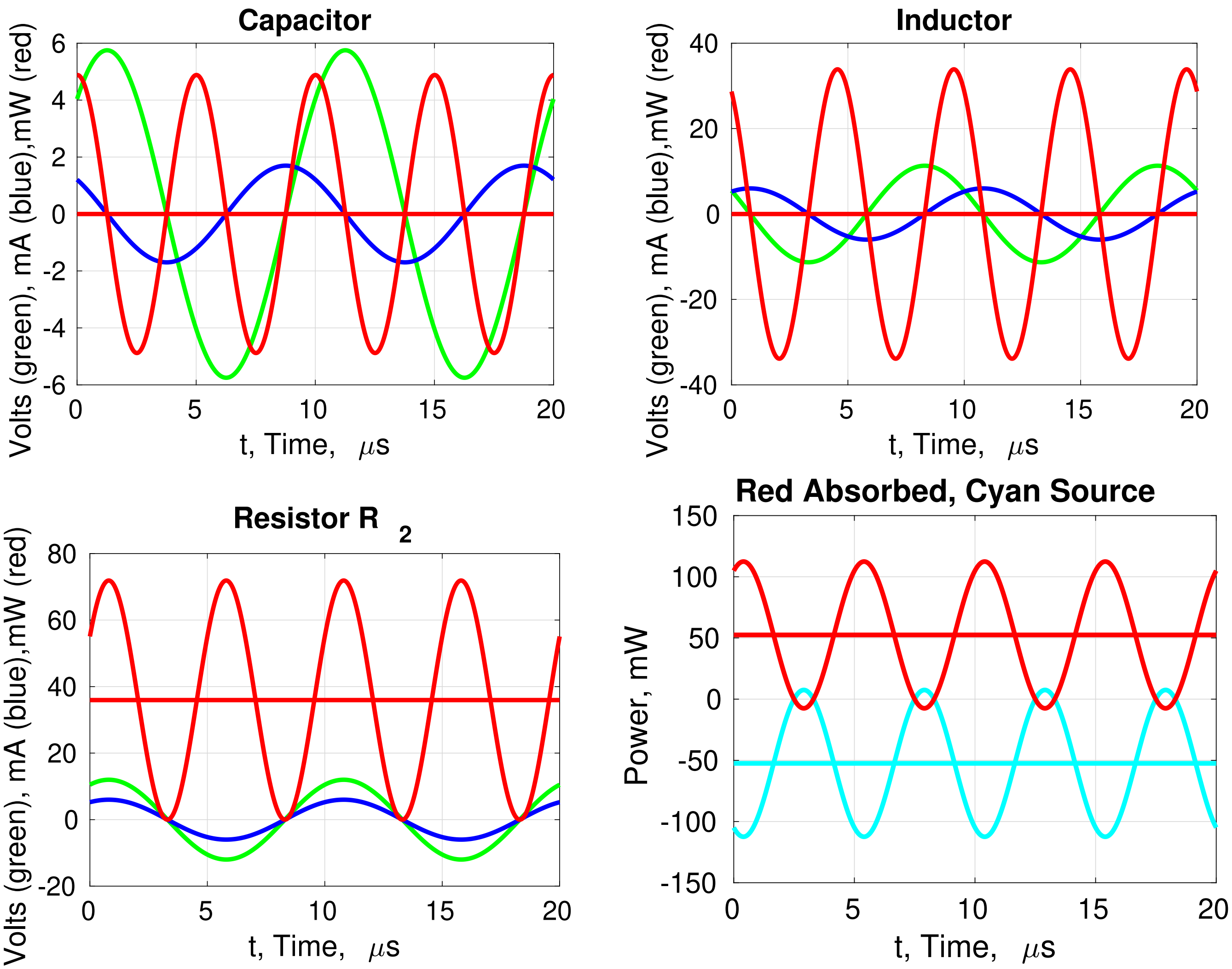
Capacitor: $V_C = V_S - V_A$

$v_c = \text{Re}\left(V_C e^{j\omega t}\right) \qquad i_c = \text{Re}\left(I_C e^{j\omega t}\right)$

$$p_c = v_c i_c$$

```
>> t=[0:0.001:1]*2/f;
>> vc=real(VC*exp(1j*omega*t));
ic=real(IC*exp(1j*omega*t));
pc=vc.*ic;
figure;plot(t*1e6,vc,'g',...
   t*1e6,ic*1e3,'b',...
   t*1e6,pc*1e3,'r',...
   t([1,end])*1e6,...
    real(VC*conj(IC))/2*[1,1]*1e3,..
    'r');
grid on;
```

Power Summary



Note That Source Absorbs Power at Times.

Resistive & Reactive Power (1)

- Power Equations

$$v = \text{Re}\left(Ve^{j\omega t}\right) \quad i = \text{Re}\left(Ie^{j\omega t}\right) \qquad p = vi$$

- Complex Notation (Remember I^* is Conjugate of I.

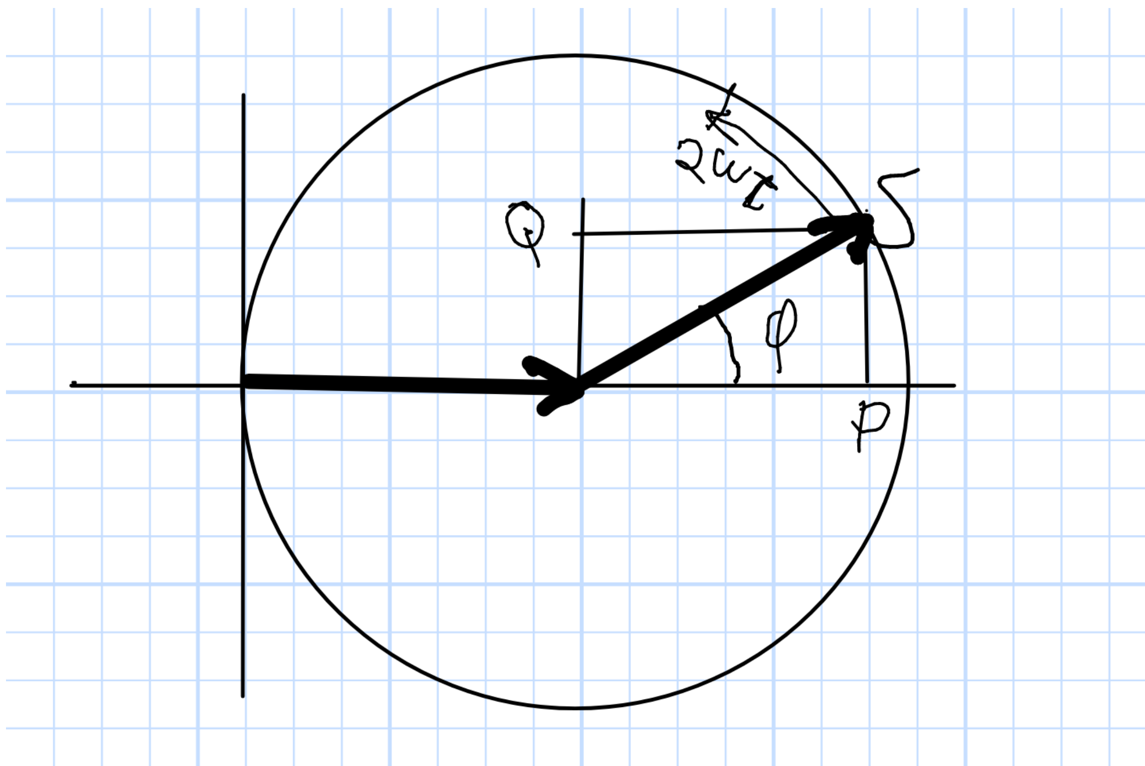
$$v = \frac{V}{2}e^{j\omega t} + \frac{V^*}{2}e^{-j\omega t} \qquad i = \frac{I}{2}e^{j\omega t} + \frac{I^*}{2}e^{-j\omega t}$$

$$p = \frac{VI^*}{4} + \frac{V^*I}{4} + \frac{VI}{4}e^{j2\omega t} + \frac{V^*I^*}{4}e^{-j2\omega t}$$

$$p = \text{Re}\left(\frac{VI^*}{2}\right) + \frac{\text{Re}\left(VIe^{j2\omega t}\right)}{2}$$

Resistive & Reactive Power (2)

$$p = \mathrm{Re}\left(\frac{VI^*}{2}\right) + \frac{\mathrm{Re}\left(VIe^{j2\omega t}\right)}{2}$$



Average Power

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$$p = \text{Re}\left(\frac{VI^*}{2}\right) + \frac{\text{Re}\left(VIe^{j2\omega t}\right)}{2}$$

$$\phi = \text{Angle}\left(\frac{VI^*}{2}\right)$$

- Time Average (over whole number of cycles)

$$\overline{p} = \text{Re}\left(\frac{VI^*}{2}\right) = \frac{|VI|}{2}\cos\phi$$

- RMS Voltage and Current

$$\overline{p} = \frac{|V|}{\sqrt{2}}\frac{|I|}{\sqrt{2}}\cos\phi = V_{RMS}I_{RMS}\cos\phi$$

Complex Power

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$$p = \text{Re}\left(\frac{VI^*}{2}\right) + \frac{\text{Re}\left(VIe^{j2\omega t}\right)}{2} \qquad \bar{p} = \text{Re}\left(\frac{VI^*}{2}\right)$$

- Complex Power

$$\mathbf{S} = \frac{\mathbf{VI}^*}{2}$$

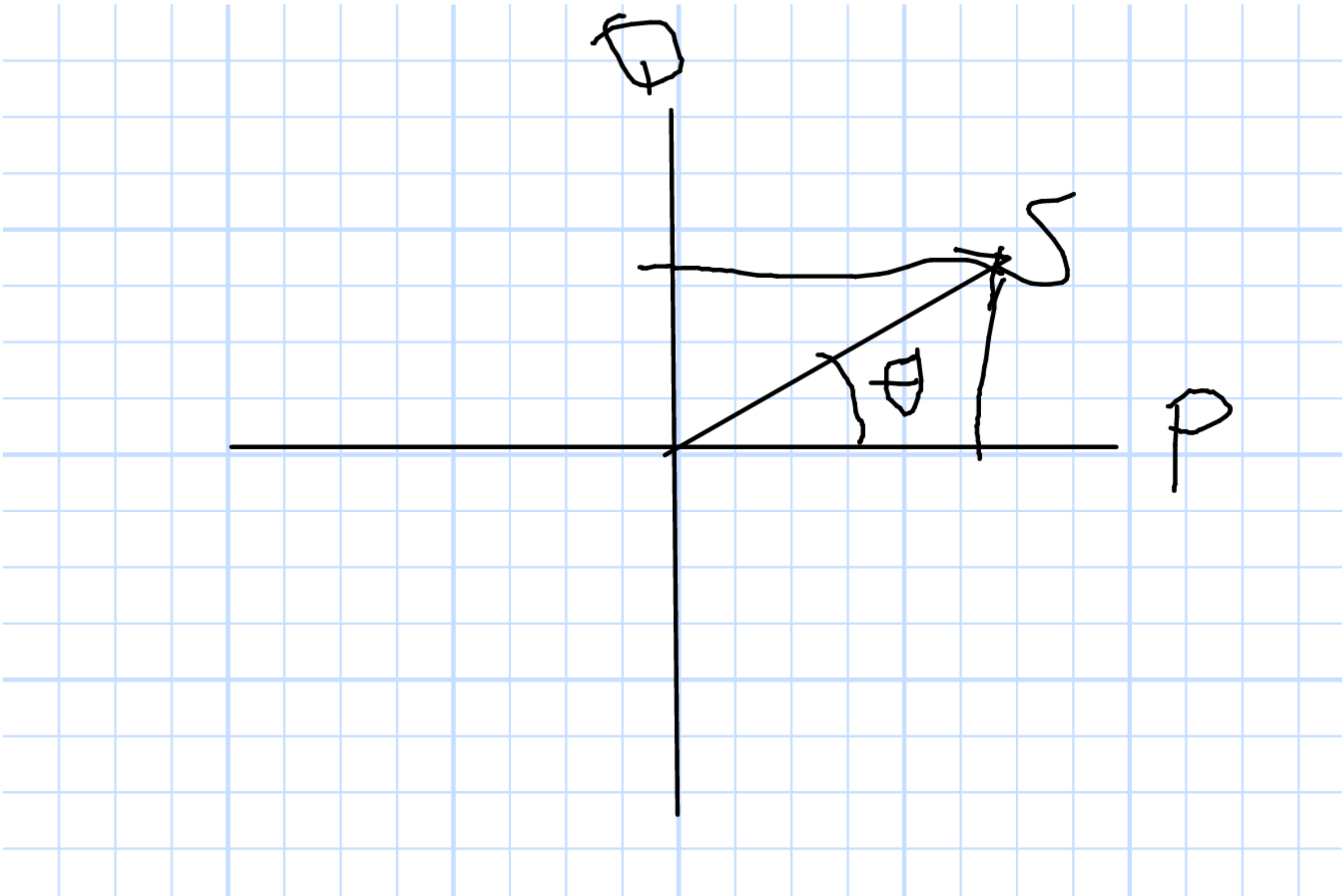
- Resistive Power

$$P = \text{Re}\,\mathbf{S}$$

- Reactive Power

$$Q = \text{Im}\,\mathbf{S}$$

Power Factor



$\mathbf{S} = P + jQ$. Power Factor: $\cos\theta$

P Watts, S Volt-Amps = VA, Q, Volt-Amps-Reactive = VAR

Generators

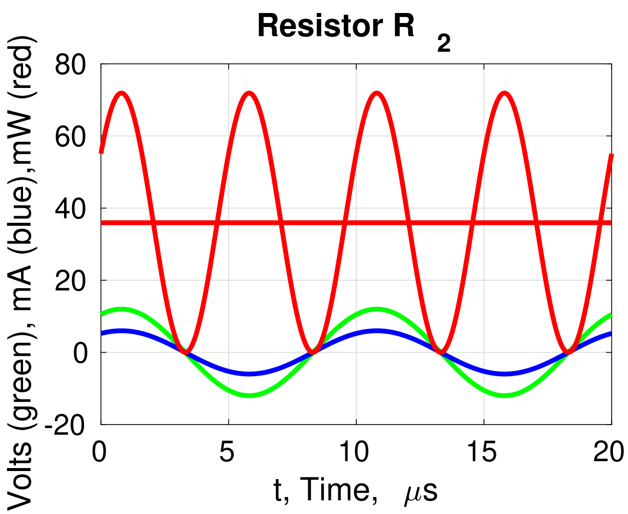
150 kVA (exmod.uk)

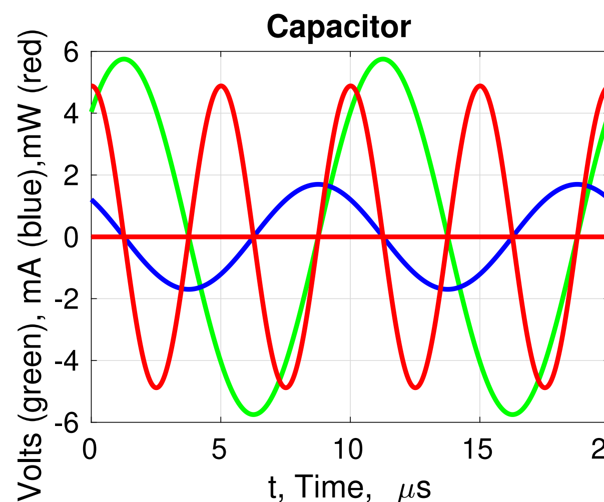
600 MVA (nuclearstreet.com)

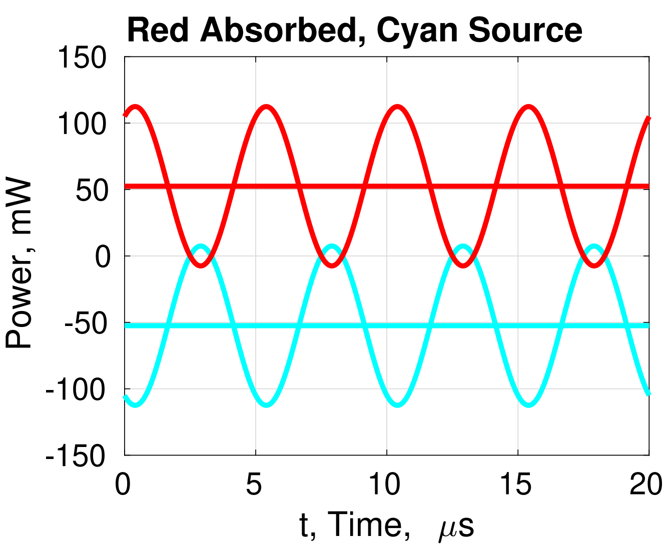
1 kVA indiamart.com

1177MVA (transformer-technology.com)

Power Summary







- Resistor; $\phi = 0$

$$\overline{p} = v_{rms} i_{rms} = \frac{v_{rms}^2}{R}$$

- Cap/Inductor; $\phi = \pm 90^\circ$

$$\overline{p} = 0$$

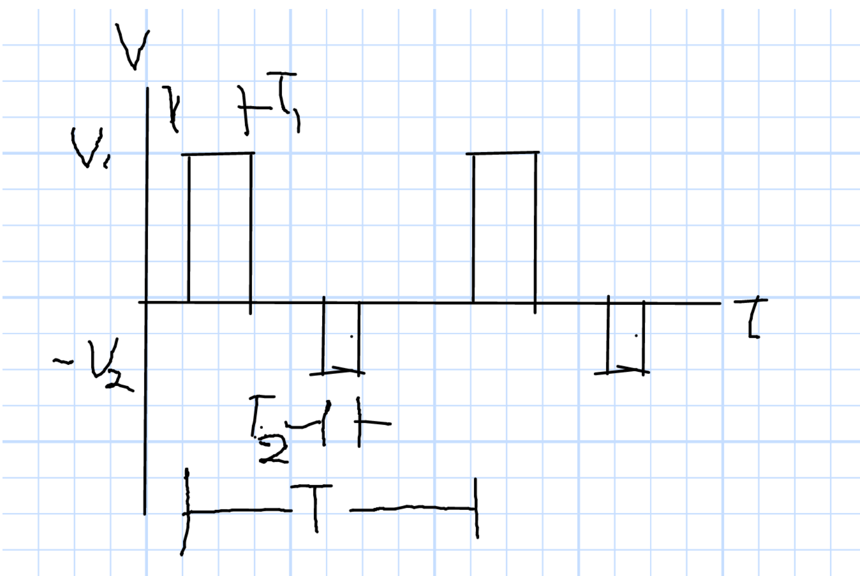
- General Sinusiods

$$\overline{p} = v_{rms} i_{rms} \cos\phi$$

RMS More Generally

$$p = \frac{v^2}{R} \qquad \bar{p} = \frac{v_{rms}^2}{R} \qquad \bar{p} = \frac{w(t:t+T)}{T}$$

$$\bar{p} = \frac{1}{T}\int_t^{t+T} \frac{v^2}{R} dt$$



$$\bar{p} = \frac{1}{RT}\left(v_1^2 t_1 - v_2^2 t_2\right)$$

$$\bar{p} = \frac{v_{rms}^2}{R}$$

$$v_{rms}^2 = \frac{v_1^2 t_1 - v_2^2 t_2}{T}$$

Voltage Readings

- Equations

$$v = \mathrm{Re}\left(Ve^{j\omega t}\right) = 20\mathrm{V}\cos\omega t$$

$$v_{max} = |V| \qquad v_{min} = -|V|$$

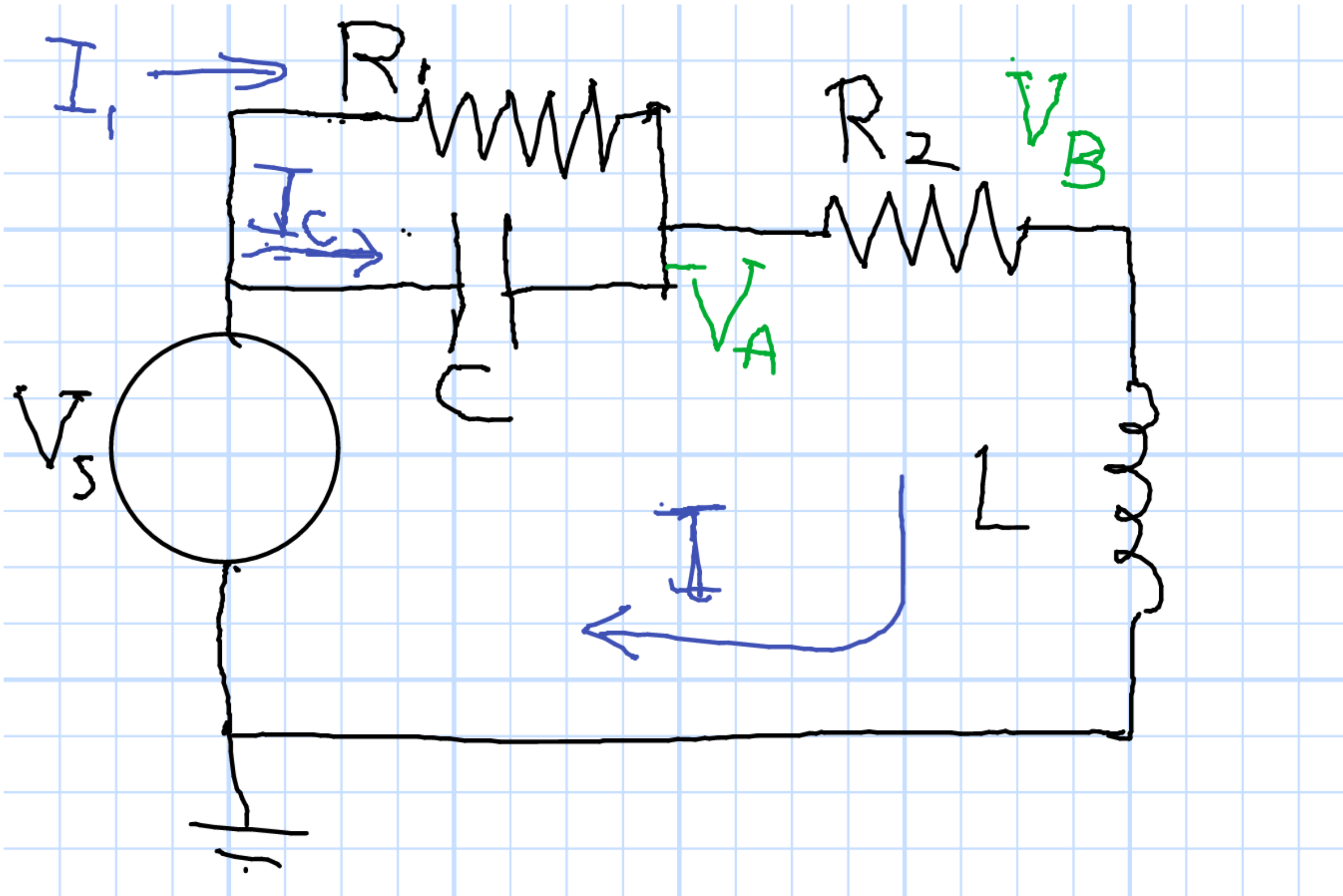
- Volt Meter Measures RMS

$$v_{rms} = \frac{|V|}{\sqrt{2}} = 14\mathrm{V}$$

- Oscilloscope Measures Peak–to–Peak

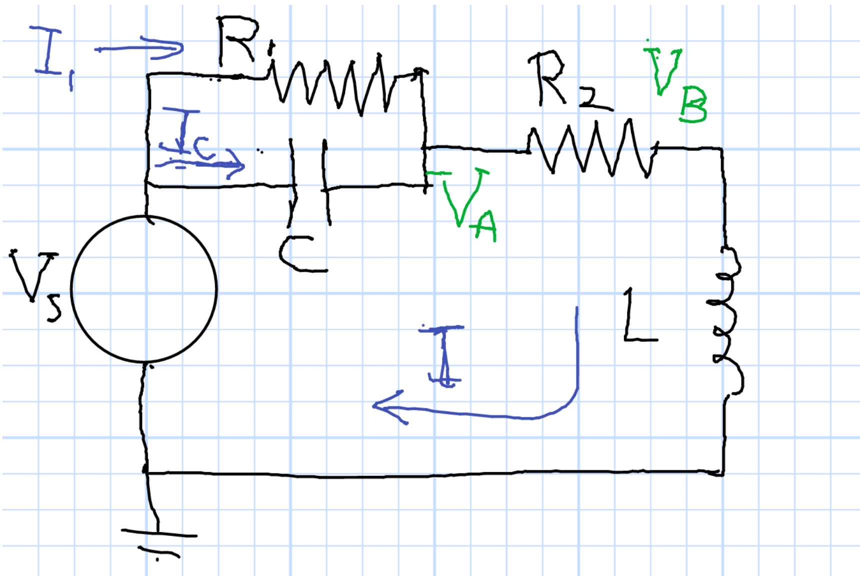
$$v_{pp} = 2|V| = 40\mathrm{V}$$

Thévenin and Norton Again



$R_1 = 1\text{k}\Omega$, $R_2 = 2\text{k}\Omega$, $C = 470\text{pF}$, $L = 3\text{mH}$, $V_s = 20\text{V}$, $f = 100\text{kHz}$

Open–Circuit Voltage



$R_1 = 1\text{k}\Omega$, $R_2 = 2\text{k}\Omega$, $C = 470\text{pF}$, $L = 3\text{mH}$, $V_s = 20\text{V}$, $f = 100\text{kHz}$

See Slides 8

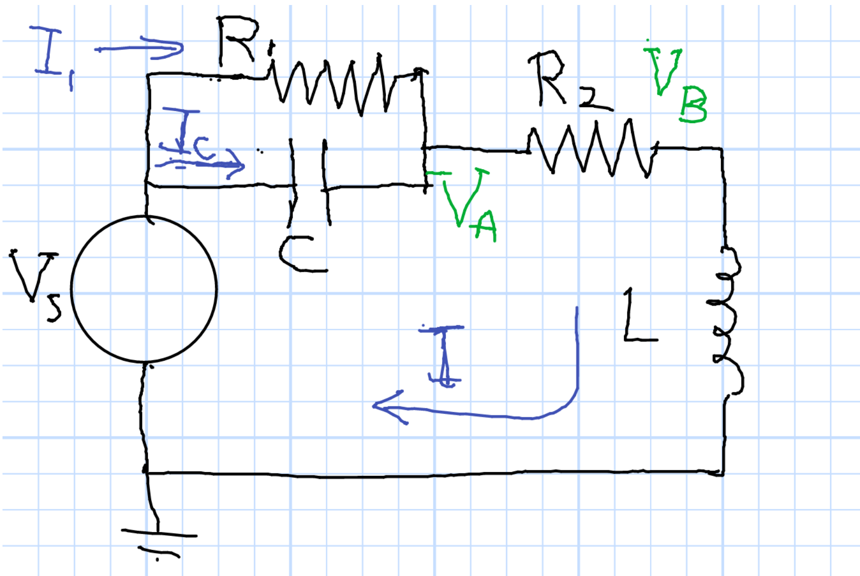
$$Z = \left(R_1 \parallel \frac{1}{j\omega C}\right) + R_2 + j\omega L$$

$$= 3.3\text{k}\Omega\angle 29^\circ$$

$$V_{OC} = V_B = V_S\frac{j\omega L}{Z} =$$

$$11.3\text{Volts}\angle 61^\circ$$

Impedance



$R_1 = 1\text{k}\Omega$, $R_2 = 2\text{k}\Omega$, $C = 470\text{pF}$, $L = 3\text{mH}$, $V_s = 20\text{V}$, $f = 100\text{kHz}$

- Zero the Source(s)
- Short the Voltage Source

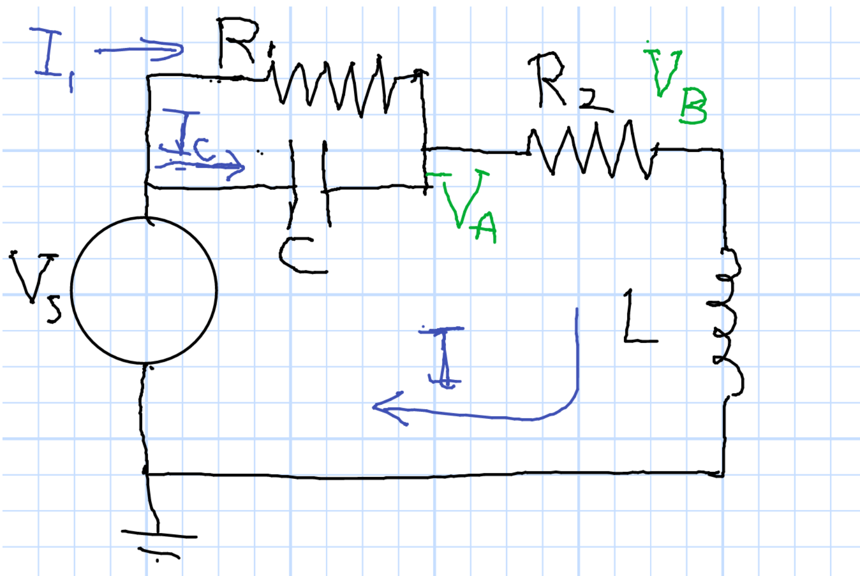
$$V_s = 0$$

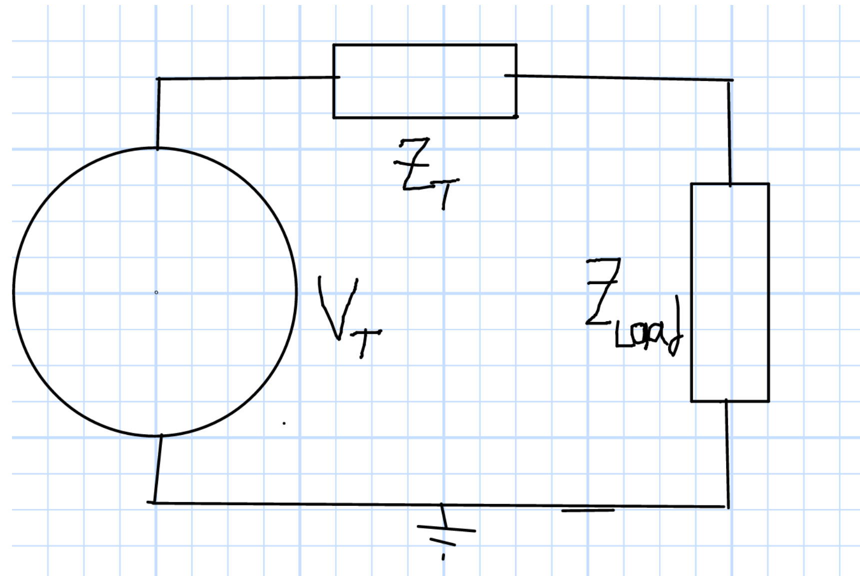
- Open a Current Source
  None Here

$$Z = \left[\left(R_1 \parallel \frac{1}{j\omega C}\right) + R_2\right] \parallel 1 j\omega L$$

$$= 1.66\text{k}\Omega\angle 55.8^\circ$$

Thévenin Equivalent





$R_1 = 1\text{k}\Omega$, $R_2 = 2\text{k}\Omega$, $C = 470\text{pF}$, $L = 3\text{mH}$, $V_s = 20\text{V}$, $f = 100\text{kHz}$

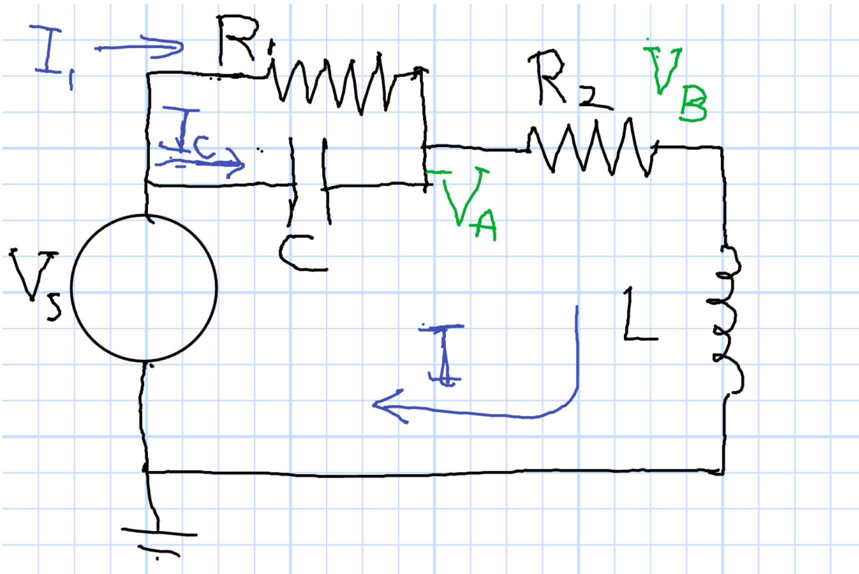
$$V_T = V_{OC} = V_B = V_S \frac{j\omega L}{Z} =$$

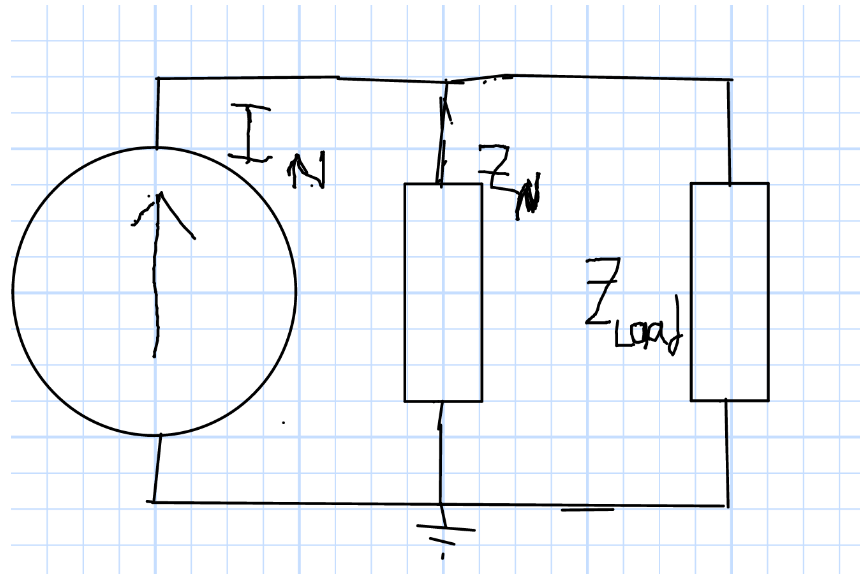
$$11.3\text{Volts}\angle 61^\circ$$

$$Z_T = \left[\left(R_1 \parallel \frac{1}{j\omega C}\right) + R_2\right] \parallel 1j\omega L$$

$$= 1.66\text{k}\Omega\angle 55.8^\circ$$

Norton Equivalent





$R_1 = 1\text{k}\Omega$, $R_2 = 2\text{k}\Omega$, $C = 470\text{pF}$, $L = 3\text{mH}$, $V_s = 20\text{V}$, $f = 100\text{kHz}$

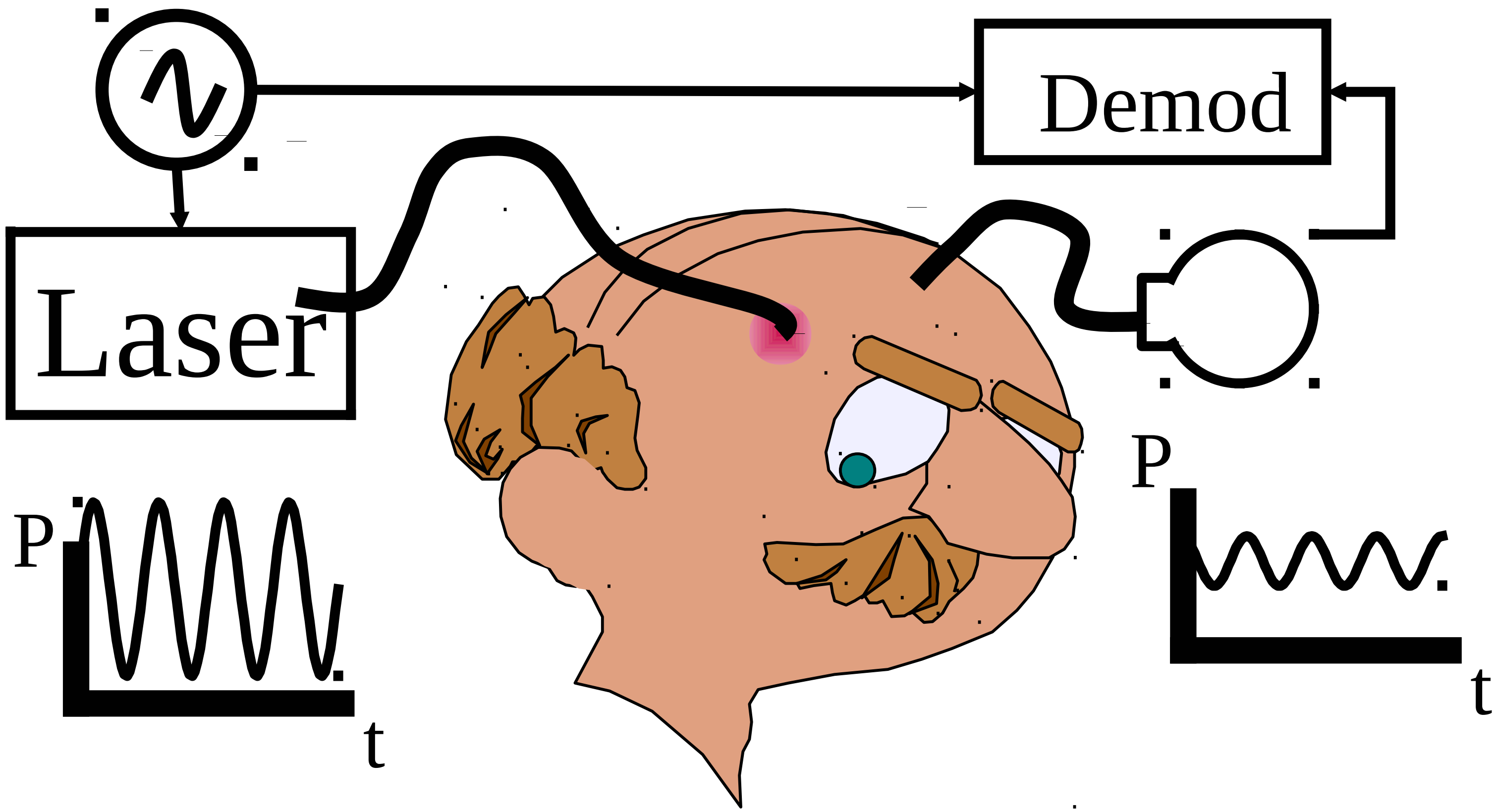
$$V_T = 11.3\text{Volts}\angle 61^\circ$$

$$I_N = \frac{V_T}{Z_T} = 6.8\text{mA}\angle 5.2^\circ$$

$$Z_N = Z_T = 1.66\text{k}\Omega\angle 55.8^\circ$$

Frequency–Domain Analysis

Diffuse Optical Tomography: DC, Pulse, Sine



Fourier Series

- Sinusoids From Last Week

$$v(t) = \mathrm{Re}\left(\mathbf{V}e^{j\omega t}\right) = \frac{\mathbf{V}}{2}e^{j\omega t} + \frac{\mathbf{V}^*}{2}e^{-j\omega t}$$

- General Periodic Function

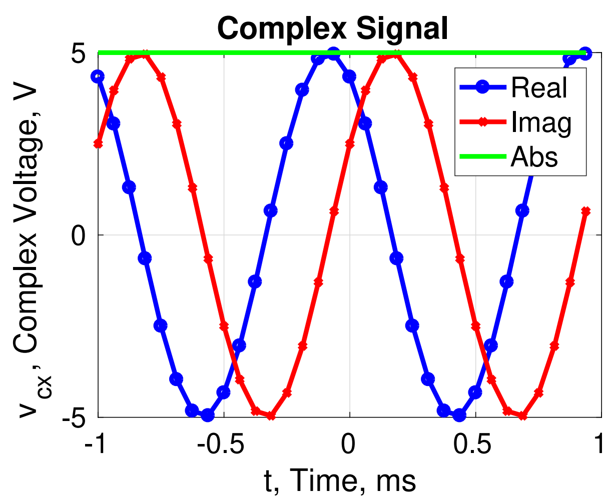
$$v(t) = \sum_{n=-\infty}^{\infty}\left(\frac{\mathbf{V}_n}{2}e^{jn\omega t}\right)$$

- Constraint for Real Functions of Time

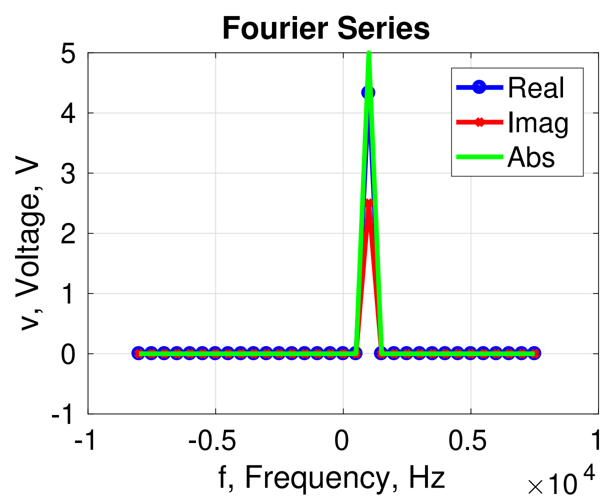
$$\mathbf{V}_{-n} = \mathbf{V}_{+n}^*$$

- Given a Periodic $v(t)$, We Can Find $\mathbf{V}_n$ for All n.

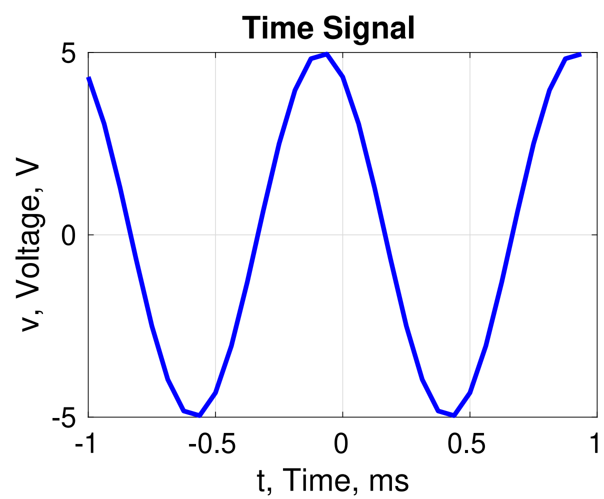
Fourier Series of Complex and Real Signals



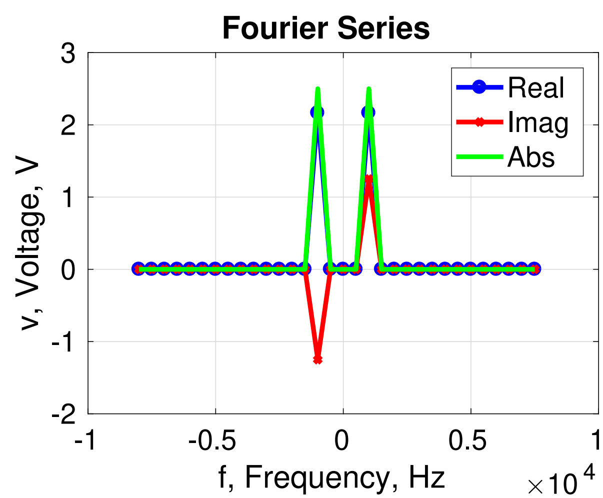
$$V e^{j30\pi/180} e^{j\omega t}$$



$$V(-f) = 0$$

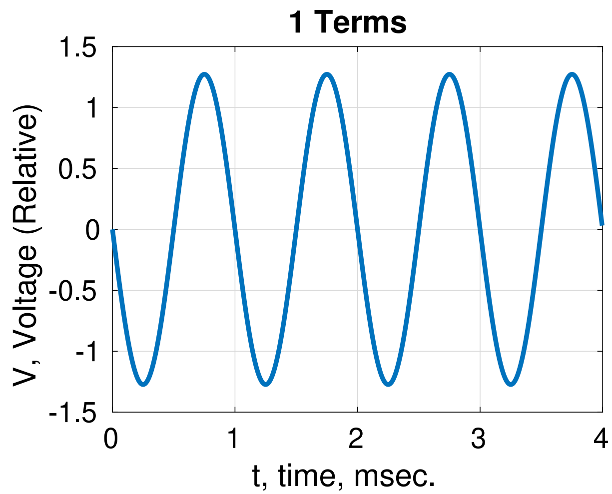


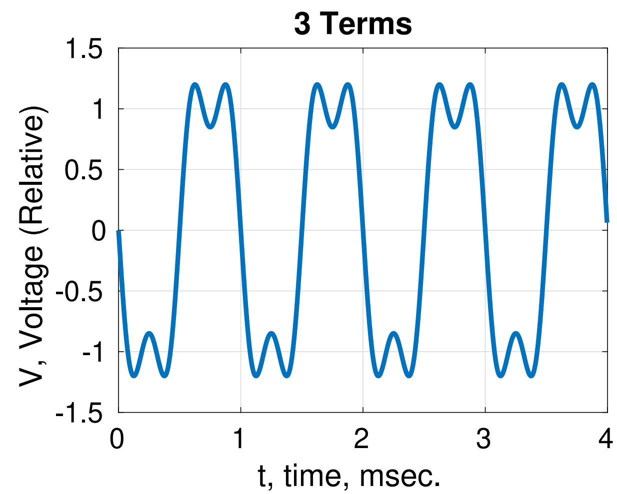
$$\cos(\omega t + 30^\circ)$$



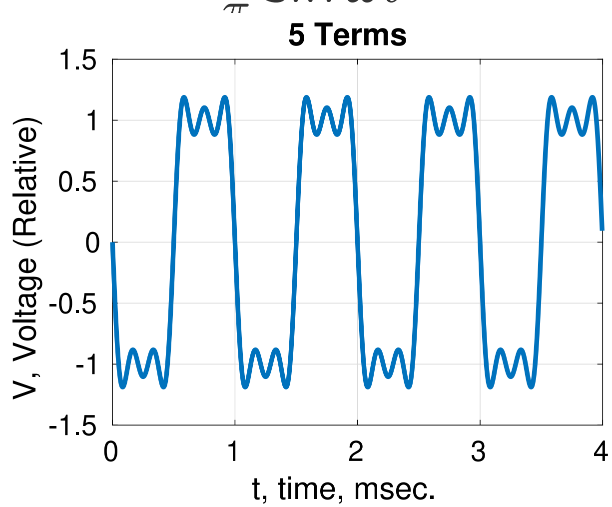
$$V(-f) = V^*(f)$$

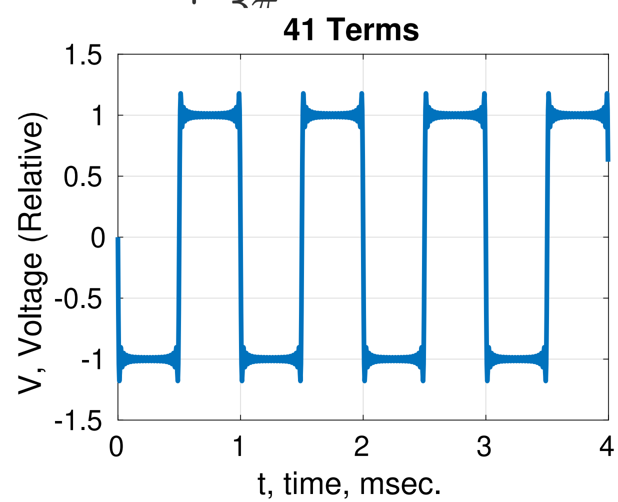
Square Wave





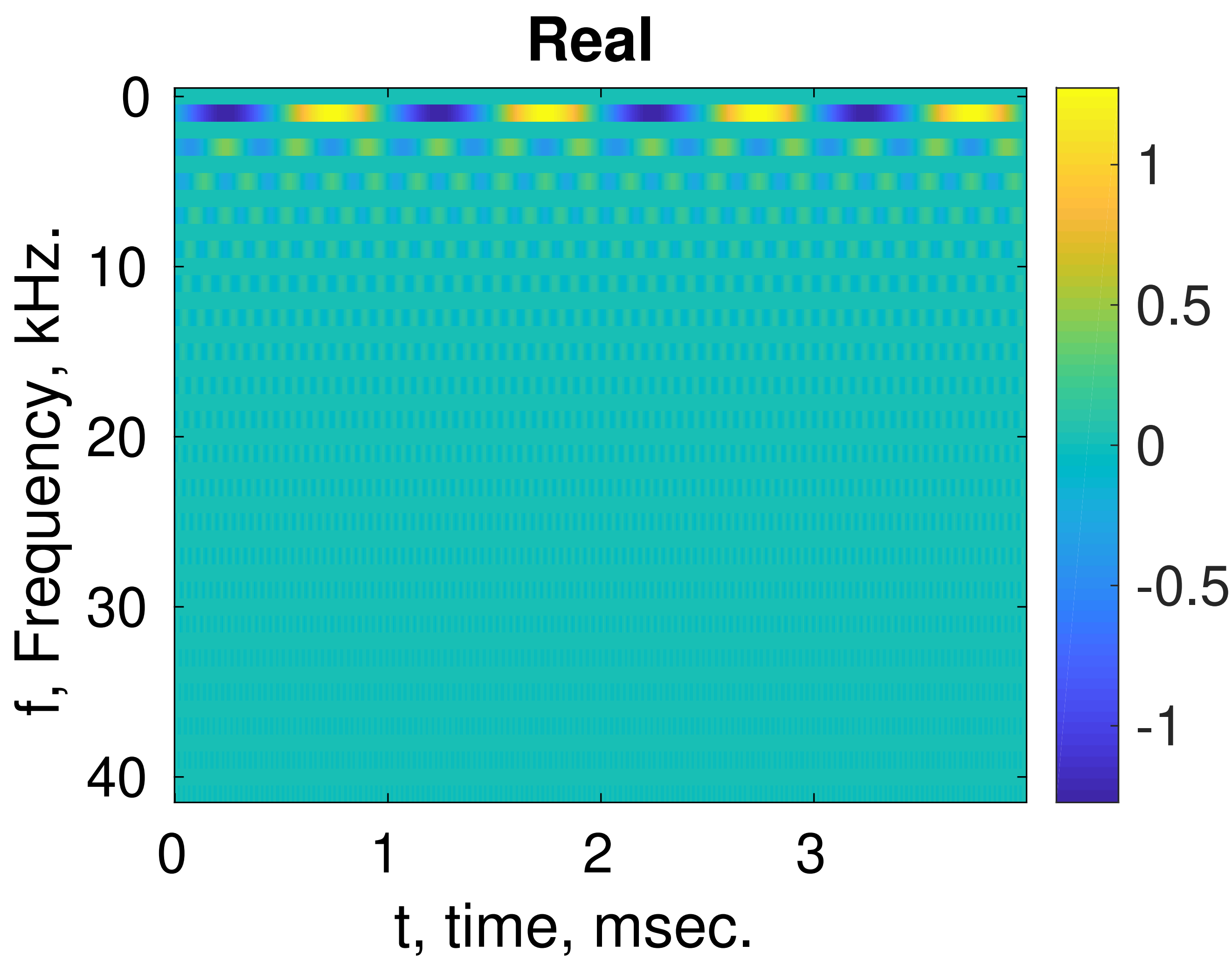
$$\frac{4}{\pi}\sin\omega t \qquad +\frac{4}{3\pi}\sin 3\omega t$$





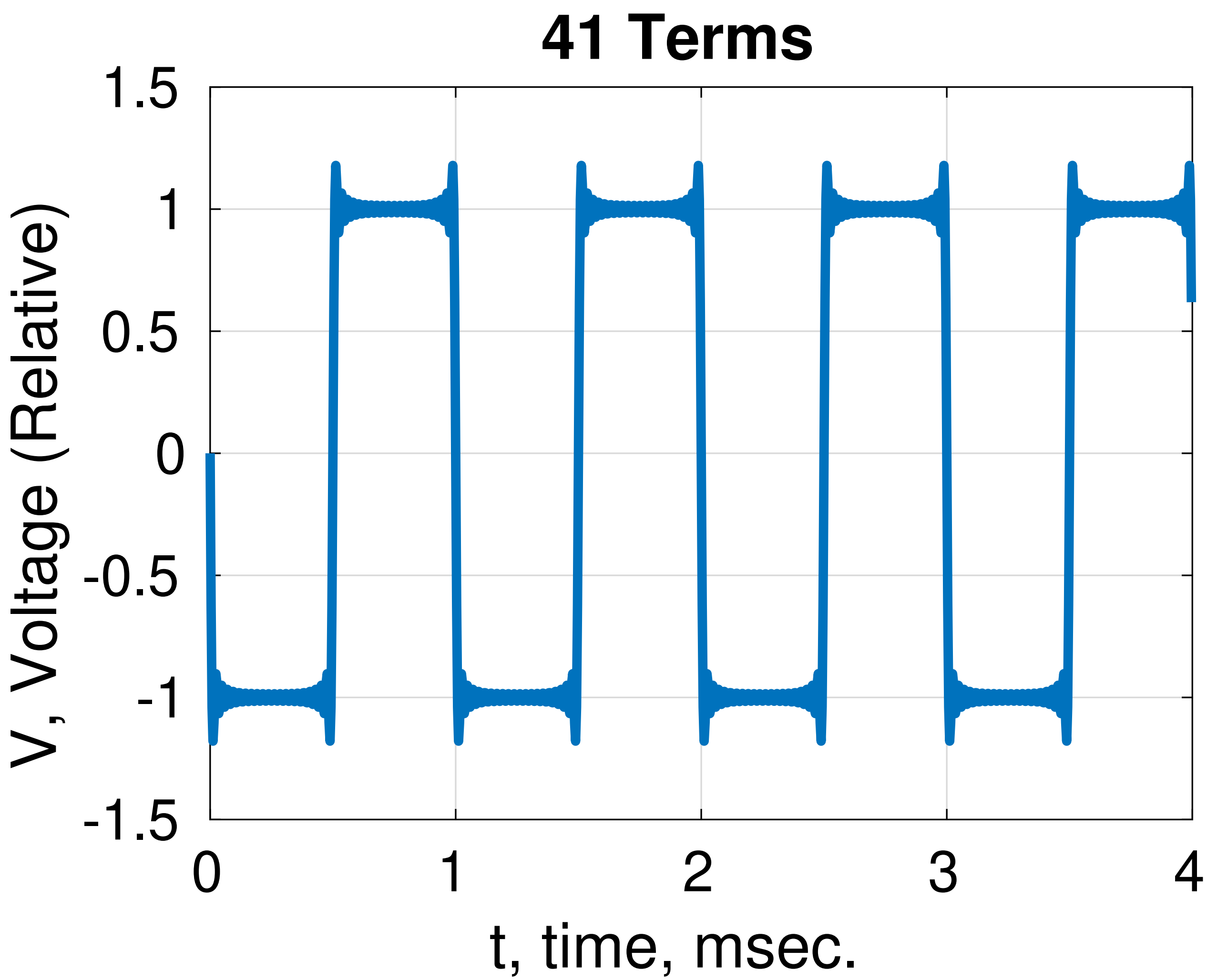
$$+\frac{4}{5\pi}\sin 5\omega t \qquad +\cdots+\frac{4}{41\pi}\sin 41\omega t$$

Harmonics Added



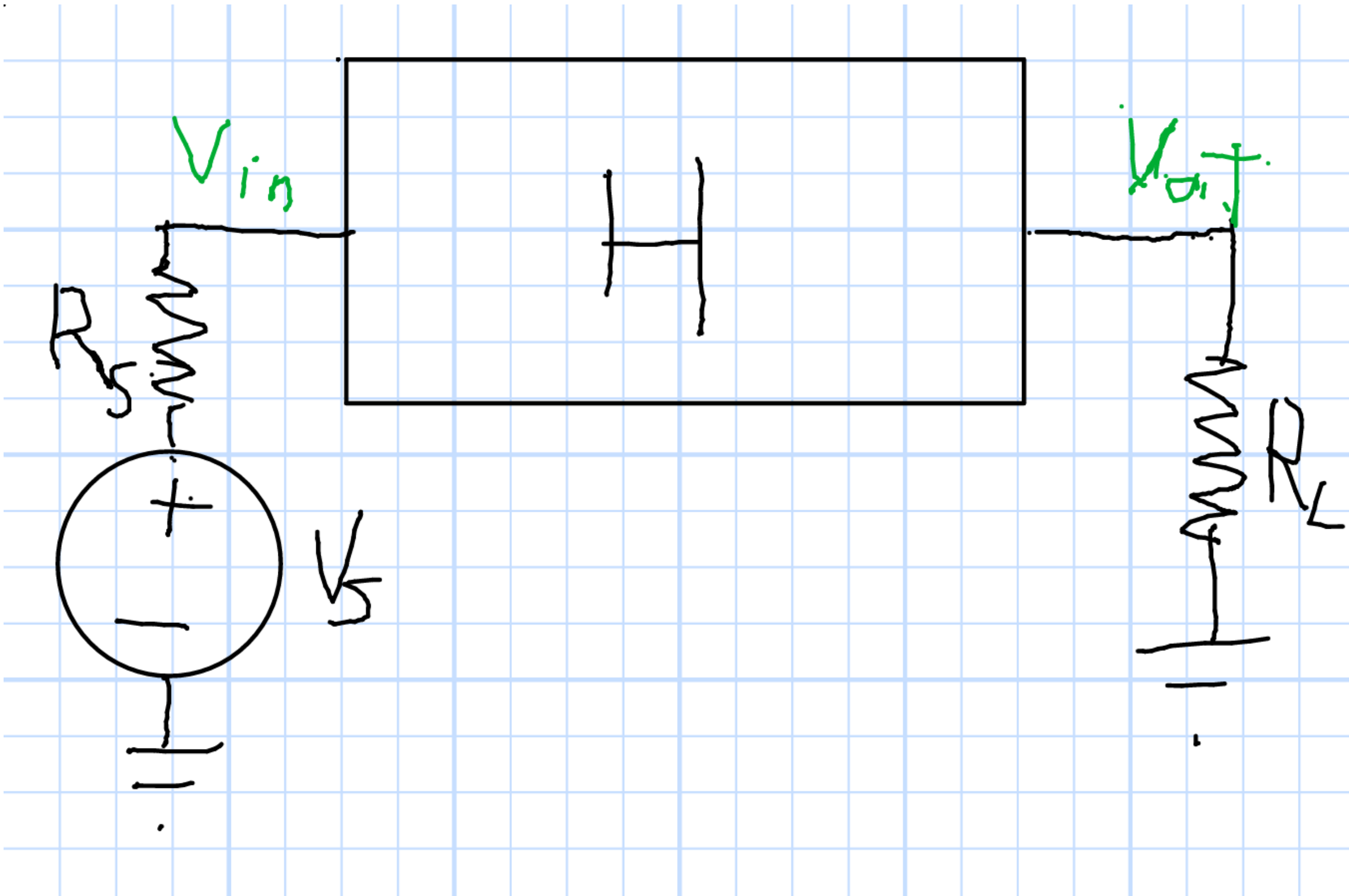
No Even Harmonics, Higher Harmonics Contribute Little

Square Wave Approximation



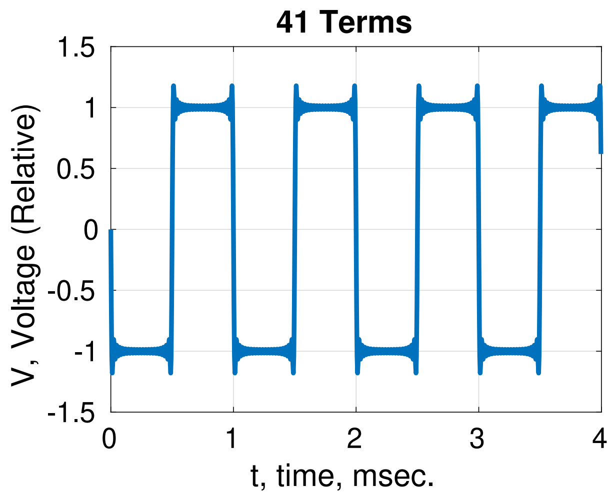
More Harmonics Would Reduce the Ringing (Gibbs Phenomenon)

Transfer Function Concept

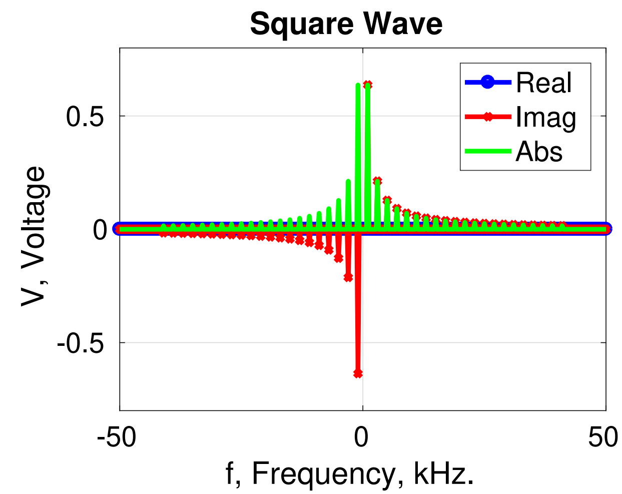


$\mathbf{V}_{out}(\omega) = H(\omega)\,\mathbf{V}_{in}(\omega)$ and No Coupling

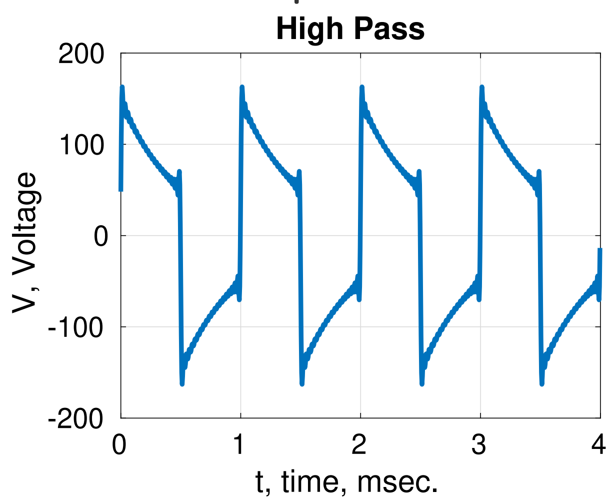
High–Pass Filter/Amplifier



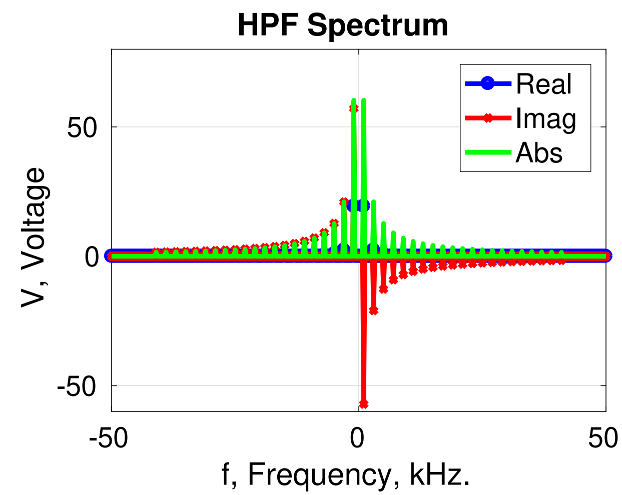
Input



Fourier Series

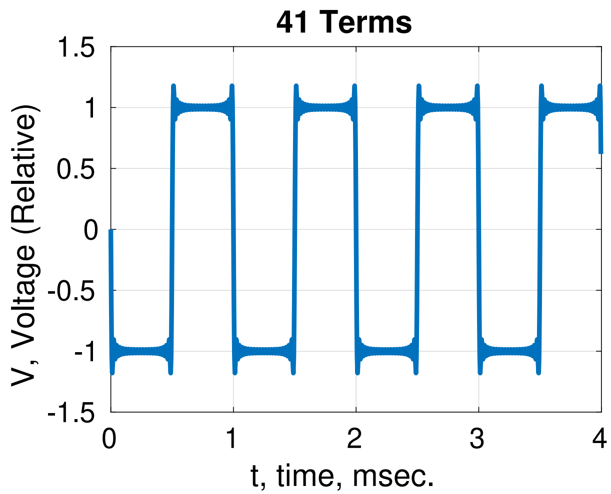


HPF Output

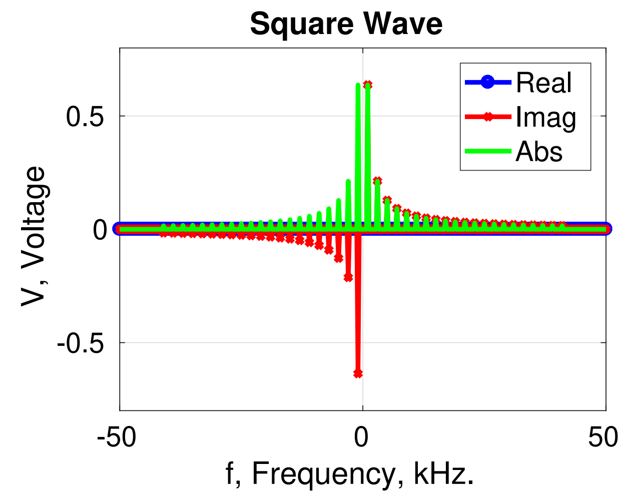


Fourier Series

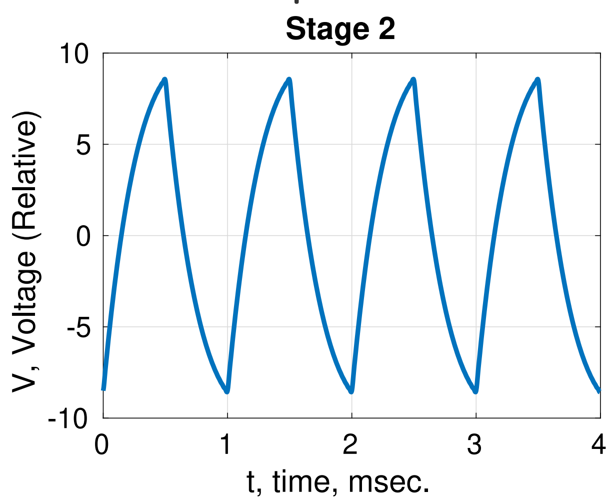
High–Pass and Low Pass



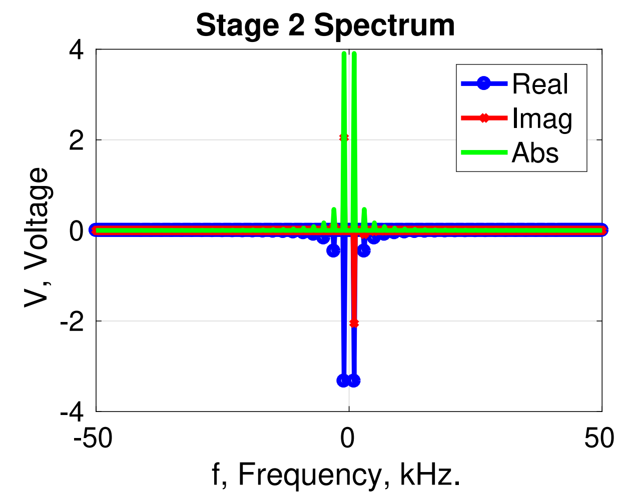
Input



Fourier Series

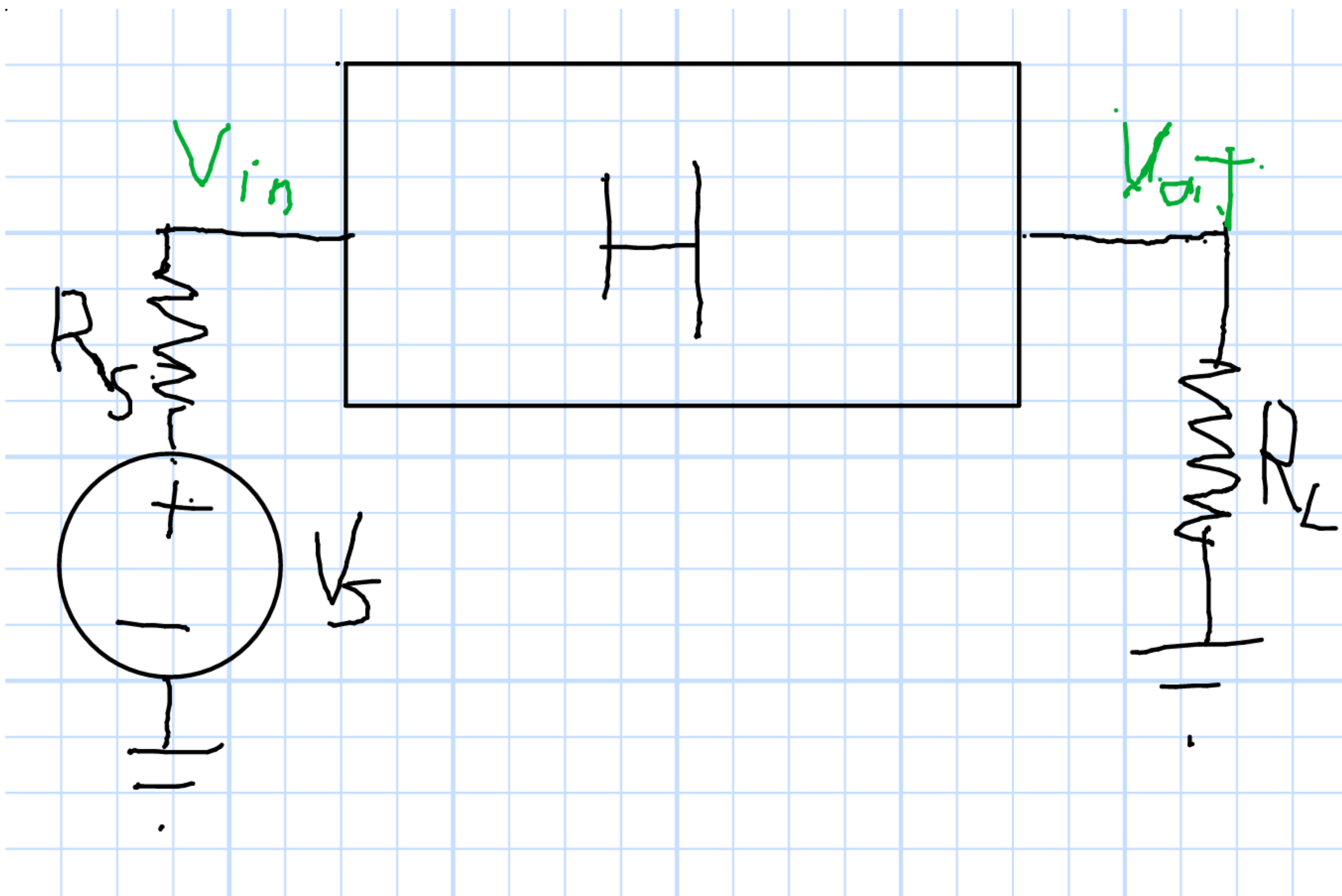


2–Filter Output

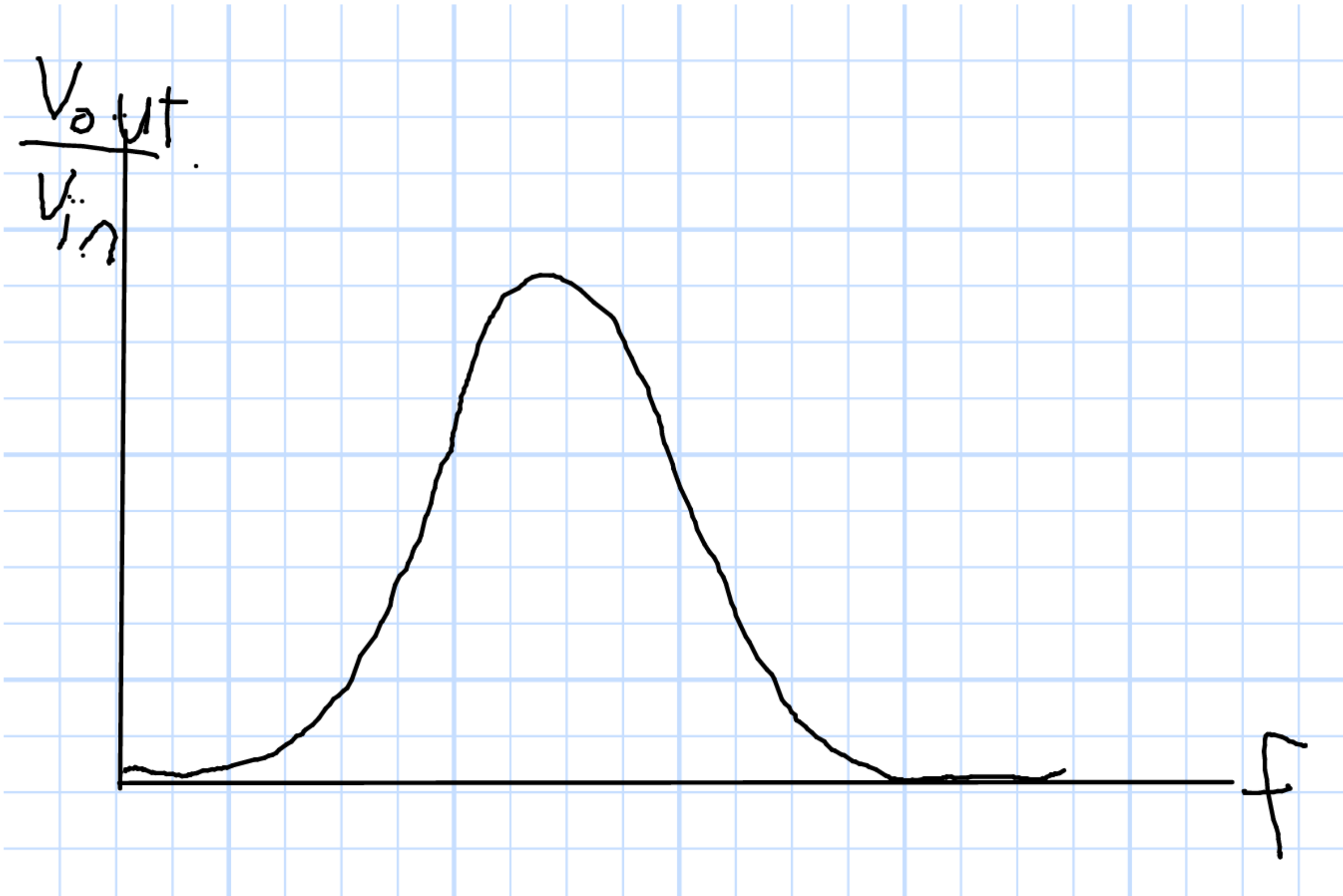


Fourier Series

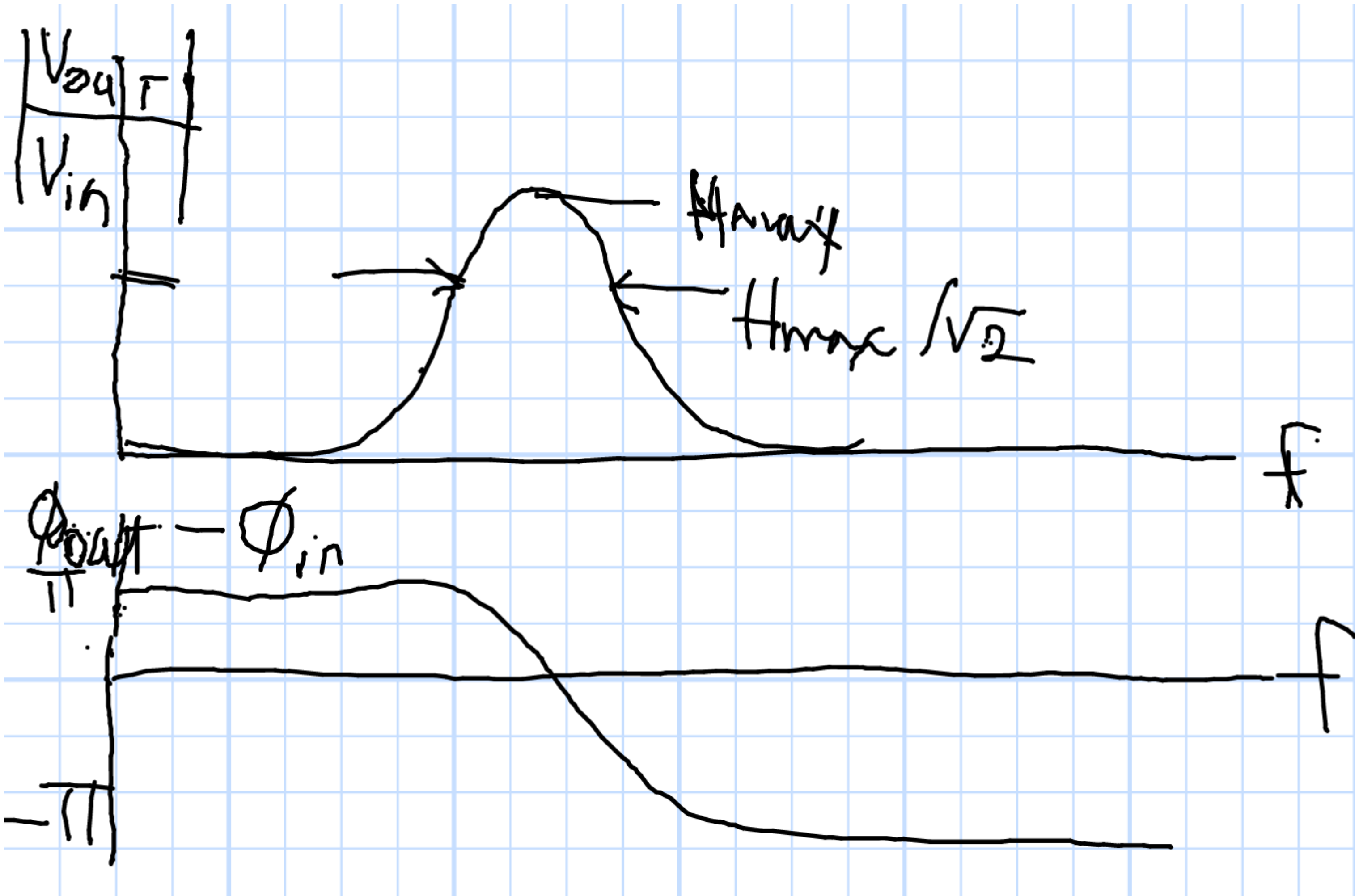
Transfer Function Concept



Transfer Function Concept

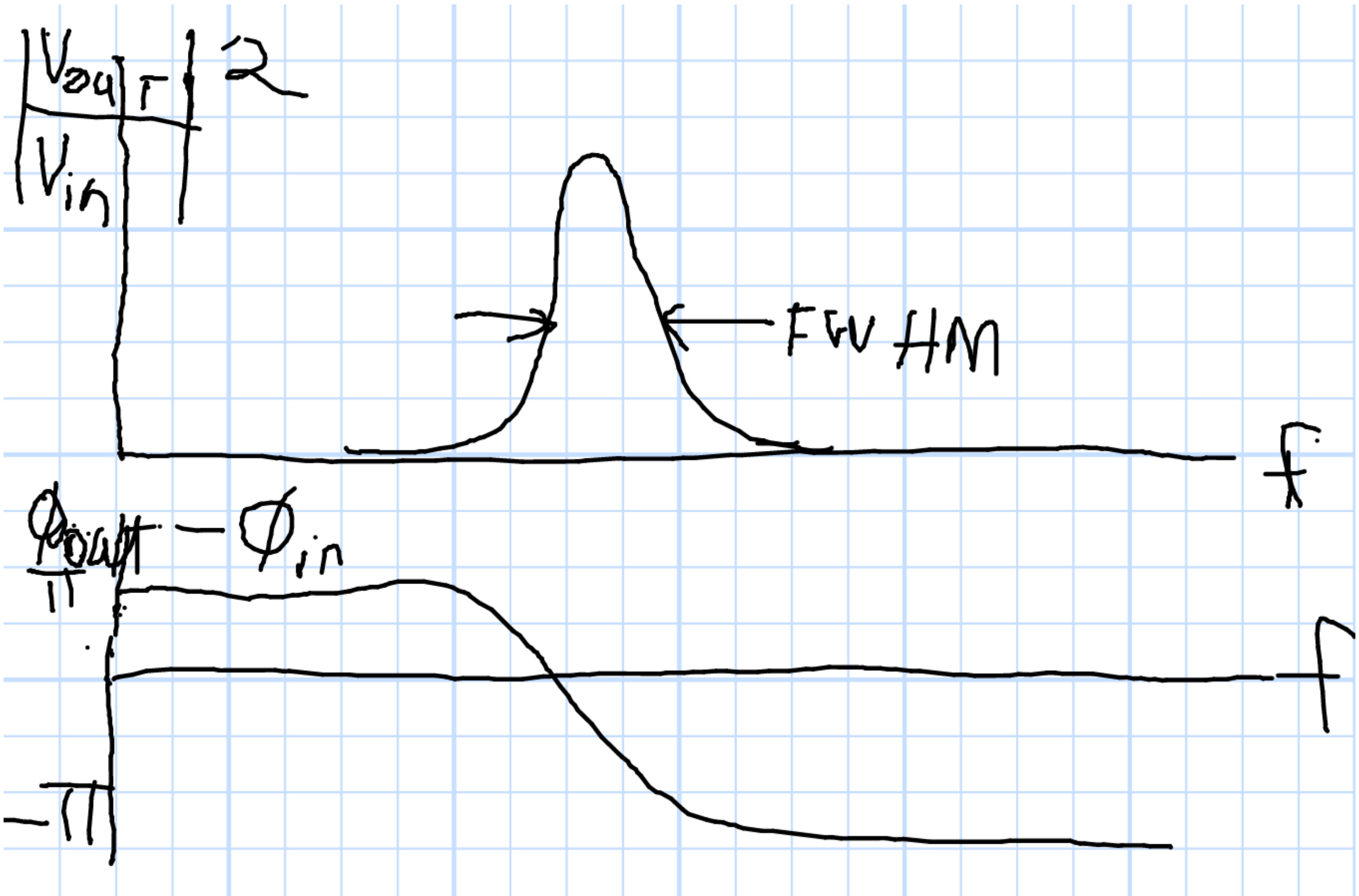


But It's Complex



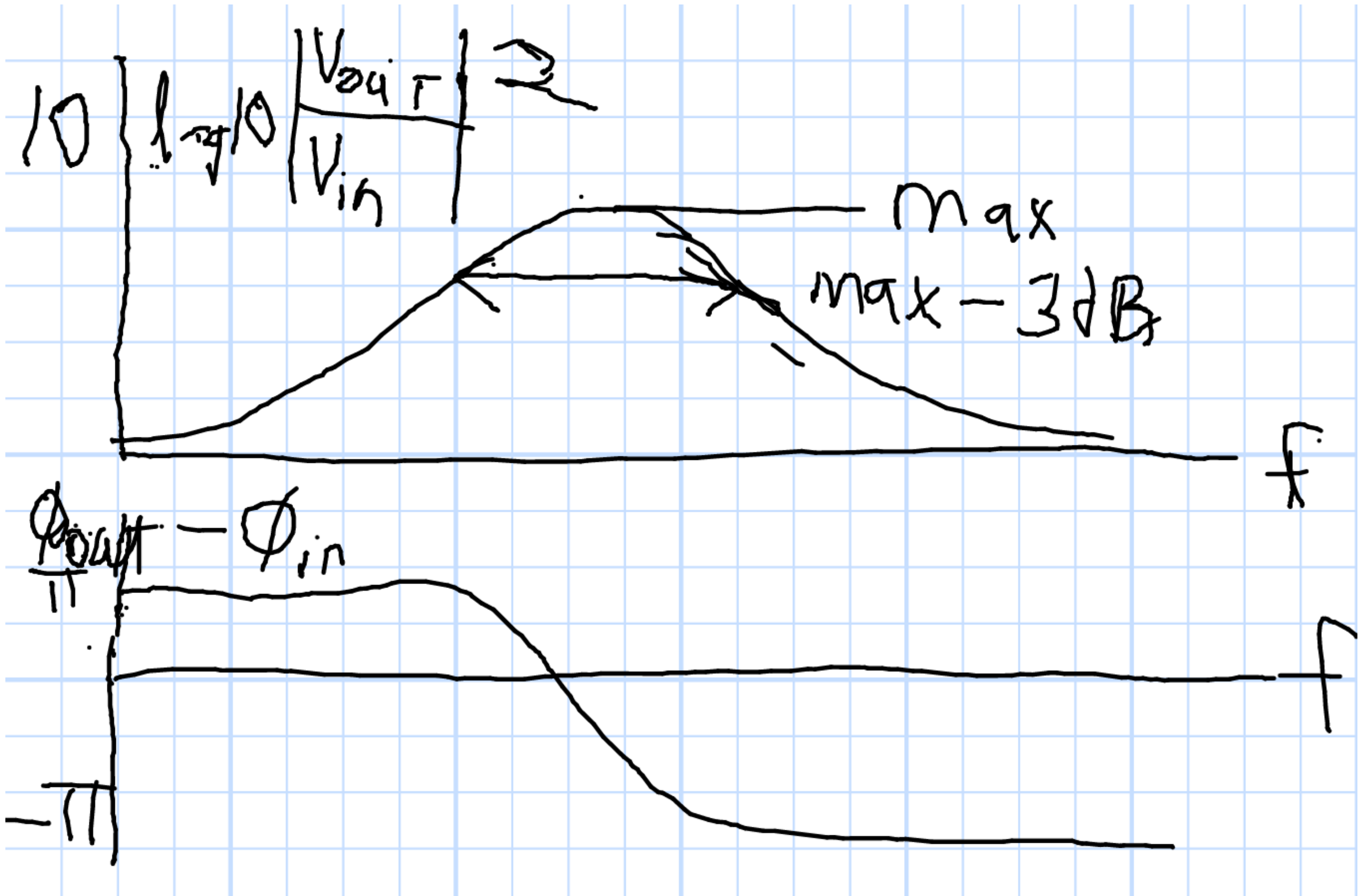
Plot Amplitude and Phase

And It's Nice to Talk About Power



Square the Amplitude.

And Sometimes We Want a Log Scale



Use Decibels

DeciBels, dB

- Always Refers to a Ratio

$$\frac{|V_{out}|}{|V_{in}|} \qquad \text{Or} \qquad \frac{P_{out}}{P_{in}} =? \left|\frac{V_{out}}{V_{in}}\right|^2$$

- Log Base 10

$$\log_{10}\frac{P_{out}}{P_{in}}$$

- But That's too Coarse; Multiply by 10

$$R_{dB} = 10\log_{10}\frac{P_{out}}{P_{in}} = 20\log_{10}\frac{|V_{out}|}{|V_{in}|} \qquad \frac{|V_{out}|}{|V_{in}|} = 10^{R_{dB}/20}$$

- lower case d, Capital B (Named for Alexander Graham Bell)

Some Ratios in dB

dB	Power Ratio	Voltage Ratio
-40	0.0001	0.01
-30	0.001	$\sqrt{0.001} \approx 0.032$
-20	0.01	0.1
-10	0.1	$\sqrt{0.1} \approx 0.32$
-6	0.25	0.5
-3	0.5	$\sqrt{0.5} \approx 0.71$
0	1	1
3	2	$\sqrt{2} \approx 1.4$
6	4	2
10	10	$\sqrt{10} \approx 3.2$
20	100	10
30	1000	$\sqrt{1000} \approx 32$
40	10000	100

The cool thing is that they add.

Some Special References

- Electronics and Optics dBm

$$10\log_{10}\frac{P}{1\text{mW}}$$

- Sometimes Used for Voltage, Assuming 50 Ohms

$$10\log_{10}\frac{|V|^2}{50\Omega\times 1\text{mW}}$$

- Example: $1\text{V}_{RMS}\rightarrow 13\text{dBm}$

- Radar Meteorology dBZ (One 1mm raindrop per m^3)

- Acoustics dBu and more

Comment on Digitization

- Eight–Bit Digitizer
  - Smallest Step = 1, Max Error = 0.5
  - Largest Value $2^8 - 1 = 255$
  - Ratio

$$20 \log_{10} \frac{255}{1} = 48\text{dB}$$

- In General, for n–bit Digitizer

$$20 \log_{10} \frac{2^n - 1}{1} \approx 20 \log_{10} 2^n = 20n \log_{10} 2$$

Dynamic Range in dB $\approx 6n$