Electrical Engineering
Week 14

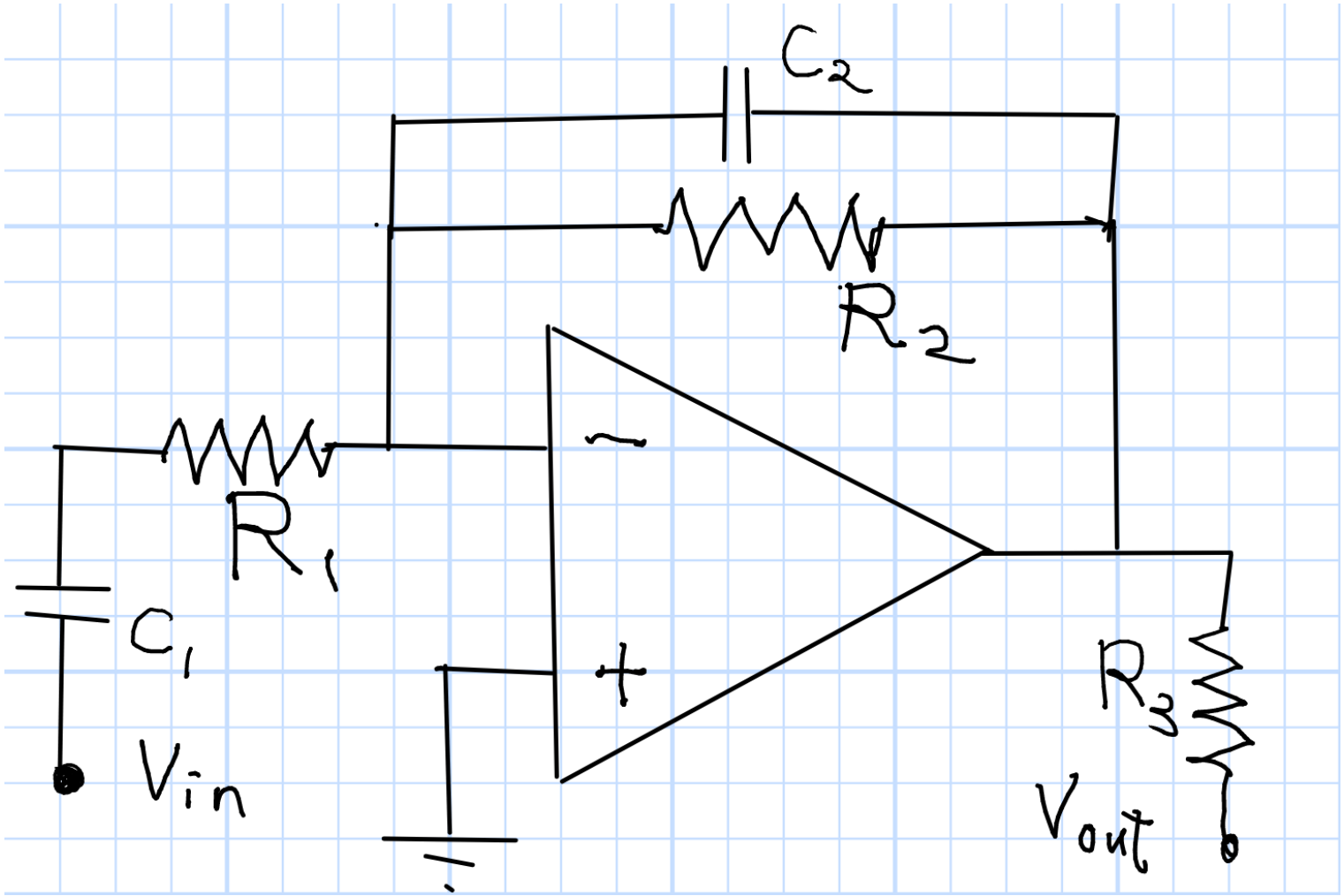
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EECE–2210
Northeastern University

Dec 2021

Week 14 Agenda: Title

- Transfer Function
- Review for Exam

Bandpass Filter

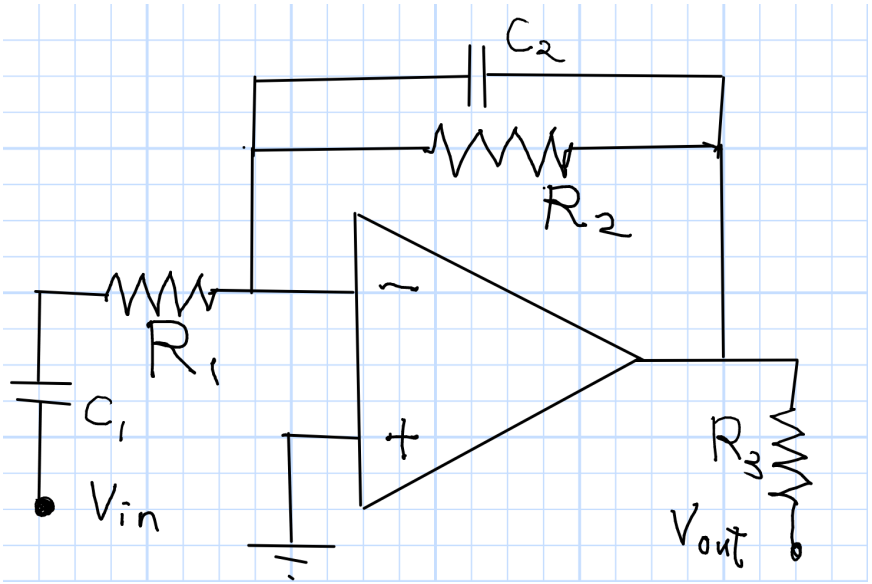


Filter Design

- Goals
  - 50 Ohm Input & Output
  - Gain of 30 dB in Band
  - Bandpass (3dB)
    - $f_1 = 500\text{kHz}$ to $f_2 = 500\text{MHz}$
- Impedance Matching
  - $R_1 = R_3 = 50\text{Ohms}$
- Gain
  - 50 Ohm Load

$$H = -\frac{Z_2}{2Z_1}$$

$$\frac{R_2}{R_1} = 10^{30/20} \times 2 = 63.2$$



- Bandpass
  - Low Freq. $Z_1 \rightarrow -> \infty$

$$R_1 C_1 = \frac{1}{2\pi f_1} = 6.3\text{nF}$$

  - High Freq. $Z_2 \rightarrow -> 0$

$$R_2 C_2 = \frac{1}{2\pi f_2} = 101\text{fF}$$

Approximations (1)

- Low Frequency

$$X_{C2} \gg R_2 \qquad X_{C1} \gg R_1$$

$$H \approx \frac{R_2}{2jX_{C1}} = -\frac{R_2 j 2\pi f C_1}{2}$$

- In Band

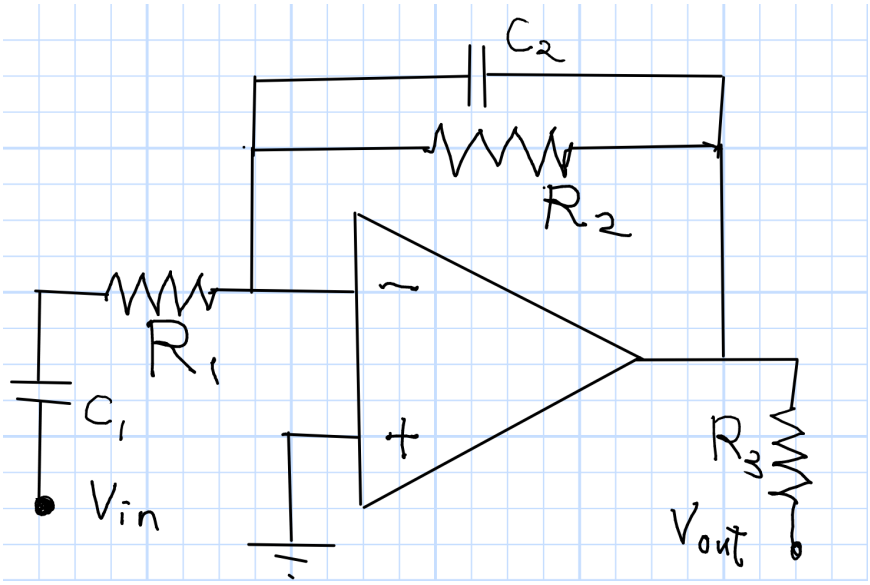
$$X_{C2} \gg R_2 \qquad X_{C1} \ll R_1$$

$$H \approx -\frac{R_2}{2R_1}$$

- High Frequency

$$X_{C2} \ll R_2 \qquad X_{C1} \ll R_1$$

$$H \approx \frac{jX_{C2}}{2R_1} = -\frac{1}{2j2\pi f C_2 R_1}$$



$R_1 = R_3 = 50$ Ohms

$R_2 = 3.16$kOhms

$C_1 = 6.37$nF, $C_2 = 100$fF

Approximations (2)

- Low Frequency

$$H \approx -\frac{R_2 j 2\pi f C_1}{2} = -\frac{j\,(2\pi R_2 C_1)\,f}{2} = -\frac{R_2}{2R_1} j\,(2\pi R_1 C_1)\,f =$$

$$-\frac{R_2}{2R_1} \times j\frac{f}{f_1} \qquad (-90^\circ)$$

- In Band

$$H \approx -\frac{R_2}{2R_1} \qquad (180^\circ)$$

- High Frequency

$$H \approx -\frac{1}{2j2\pi f C_2 R_1} = -\frac{R_2}{2R_1} \times \frac{1}{j2\pi f C_2 R_2} = \frac{R_2}{2R_1} \times j\frac{f_2}{f} \qquad (90^\circ)$$

Approximations (3)

- Low Frequency

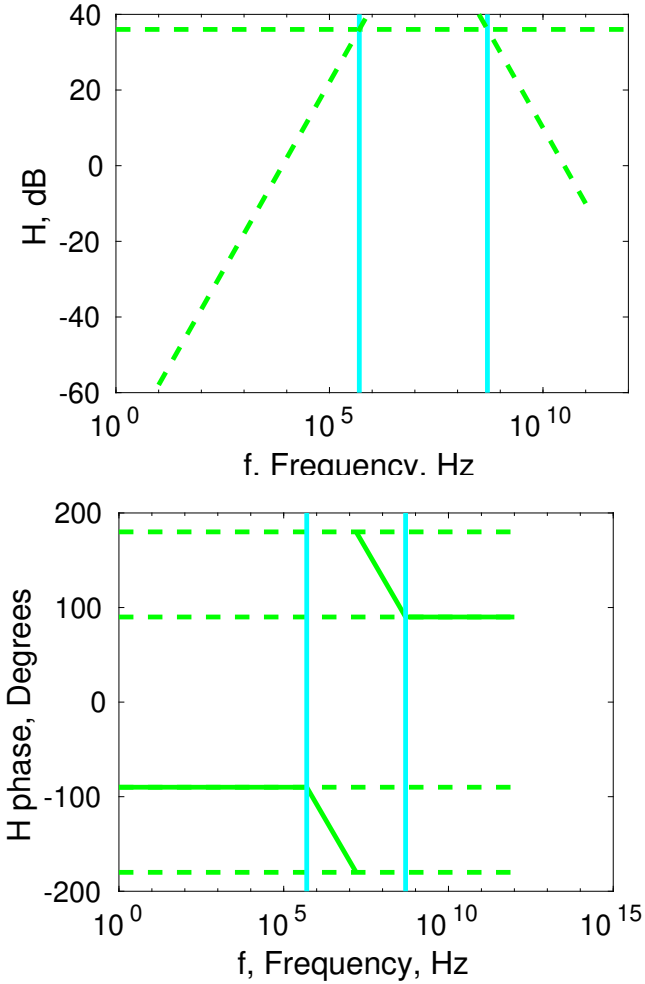
$$H \approx -\frac{R_2}{2R_1} \times j\frac{f}{f_1}$$

- In Band

$$H \approx -\frac{R_2}{2R_1}$$

- High Frequency

$$H \approx \frac{R_2}{2R_1} \times j\frac{f_2}{f}$$

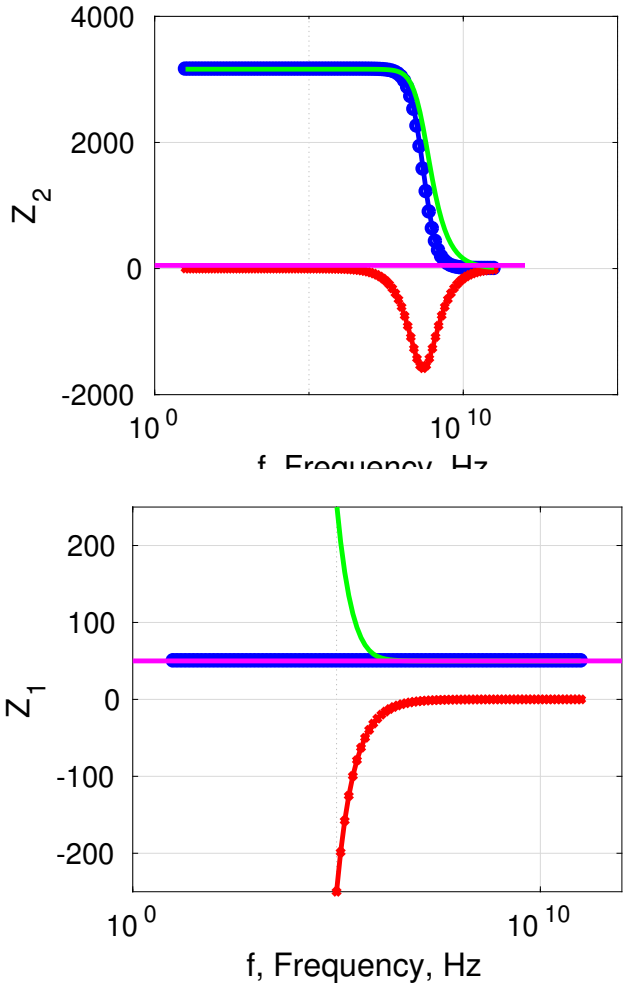


Exact Solution

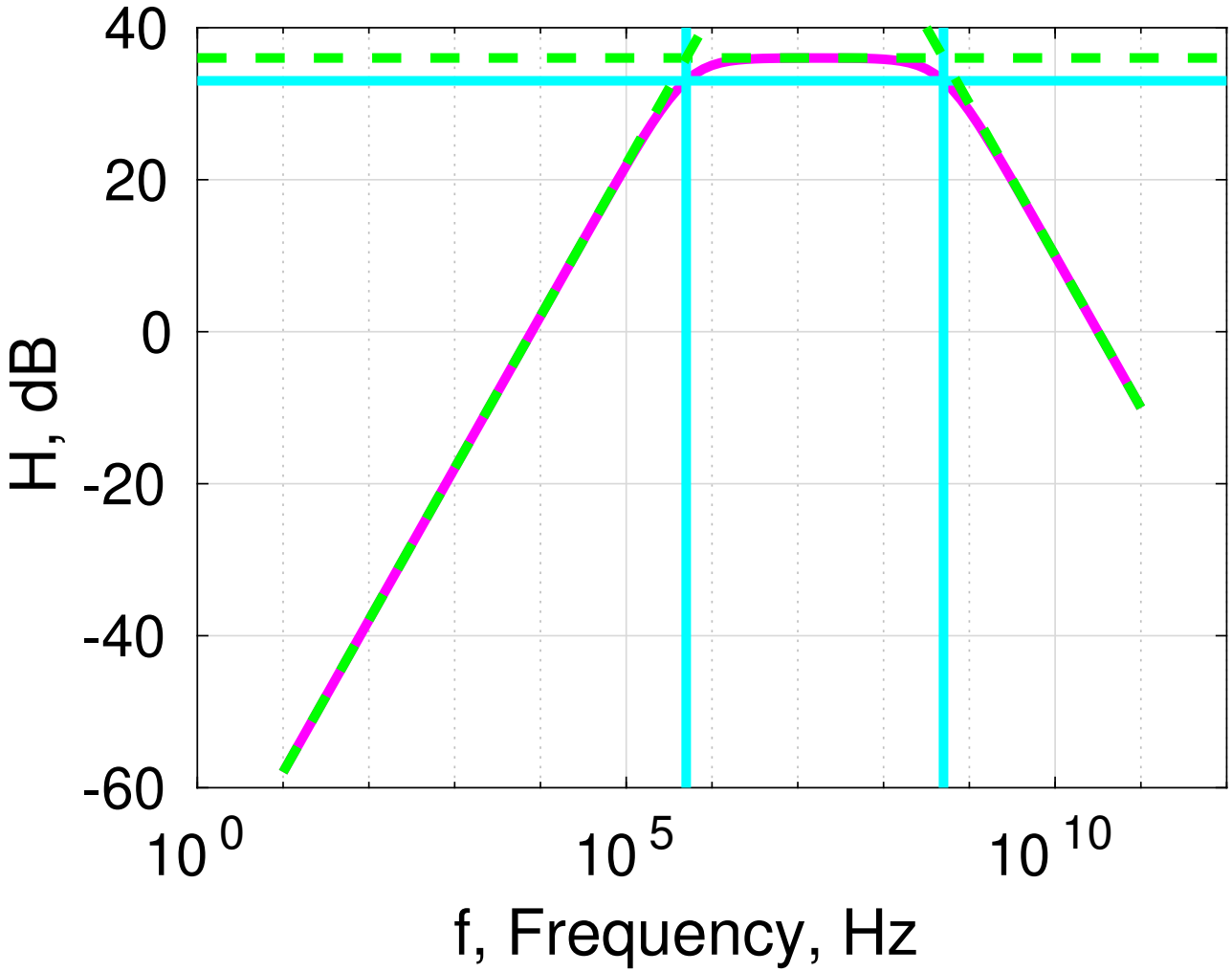
$$Z_2 = R_2 \parallel \frac{1}{j2\pi f C_2}$$

$$Z_1 = R_1 + \frac{1}{j2\pi f C_1}$$

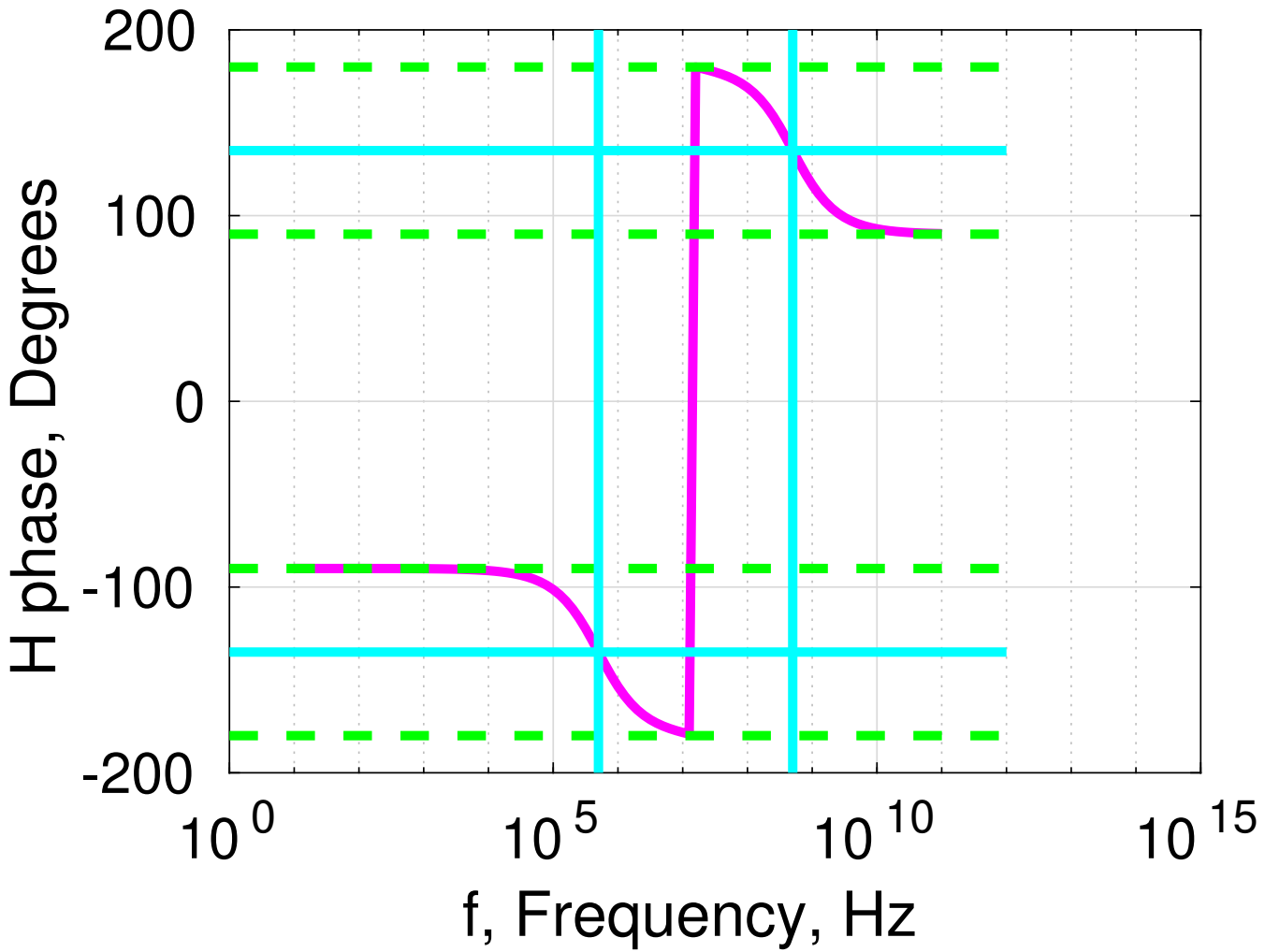
$$H = -\frac{Z_2}{Z_1}$$



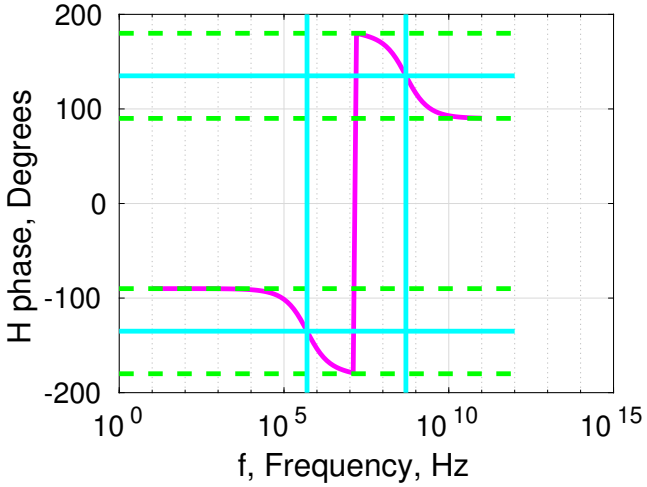
Transfer Function Magnitude

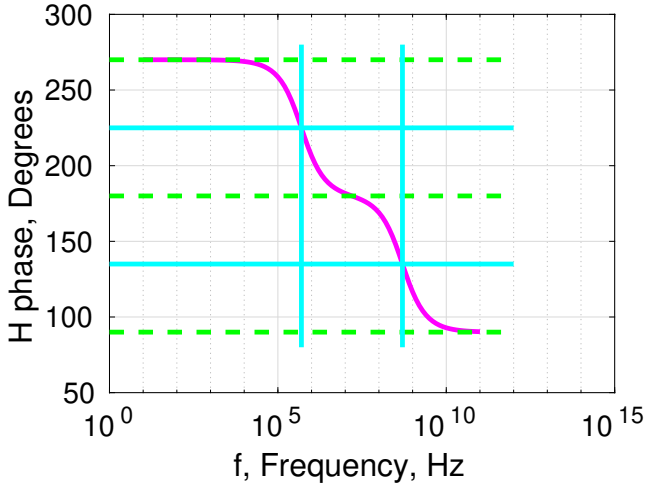


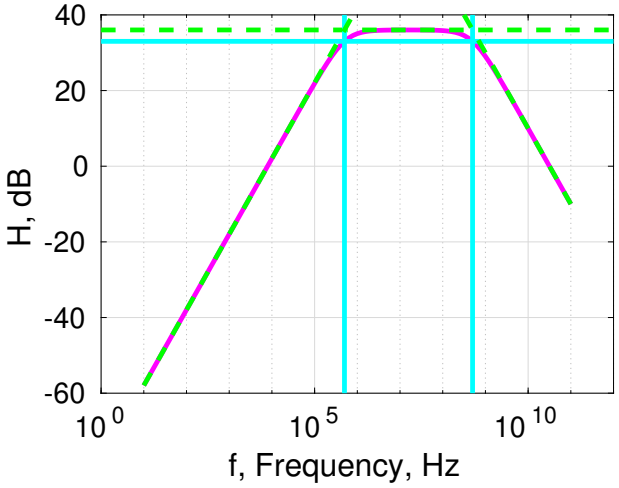
Transfer Function Phase

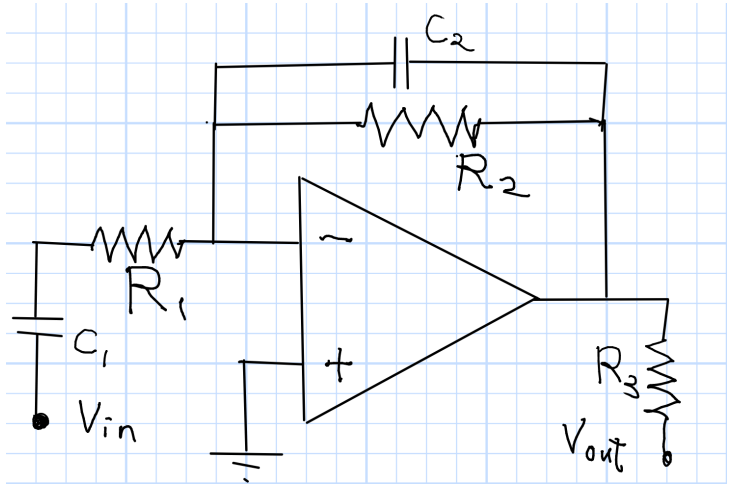


Phase Unwrapping





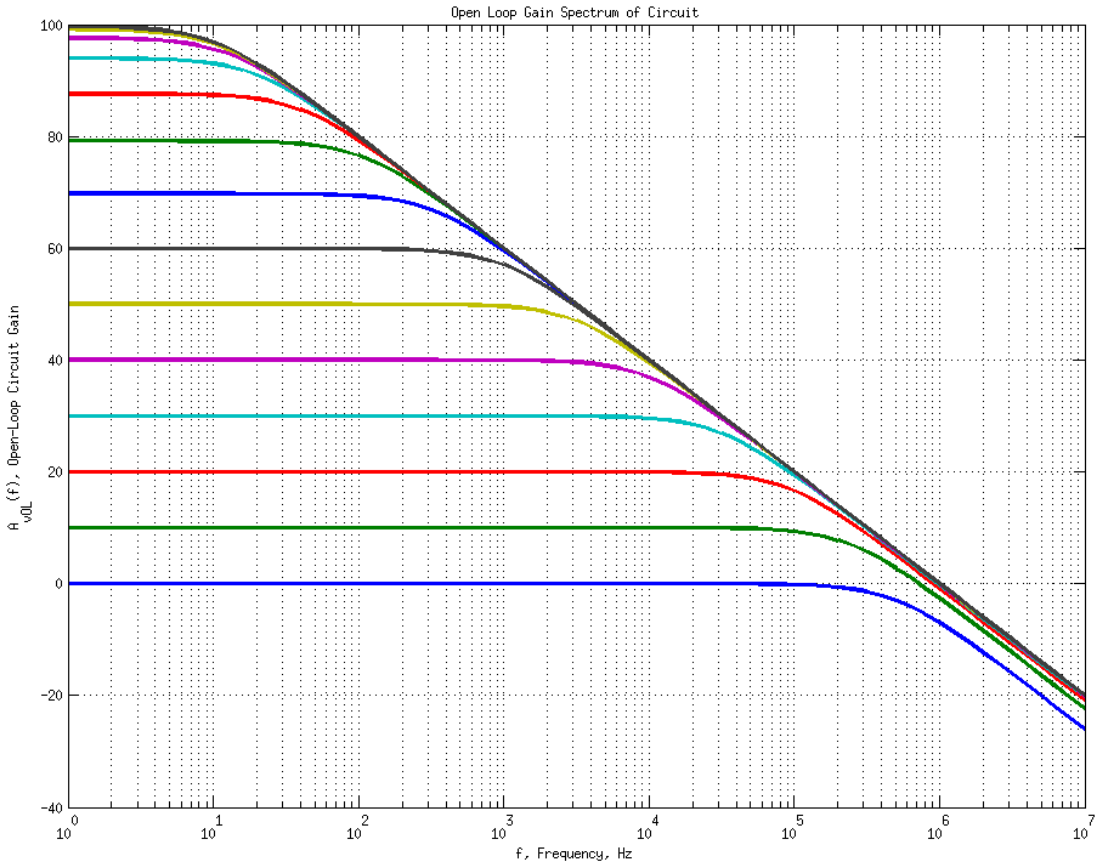




Second Half Review

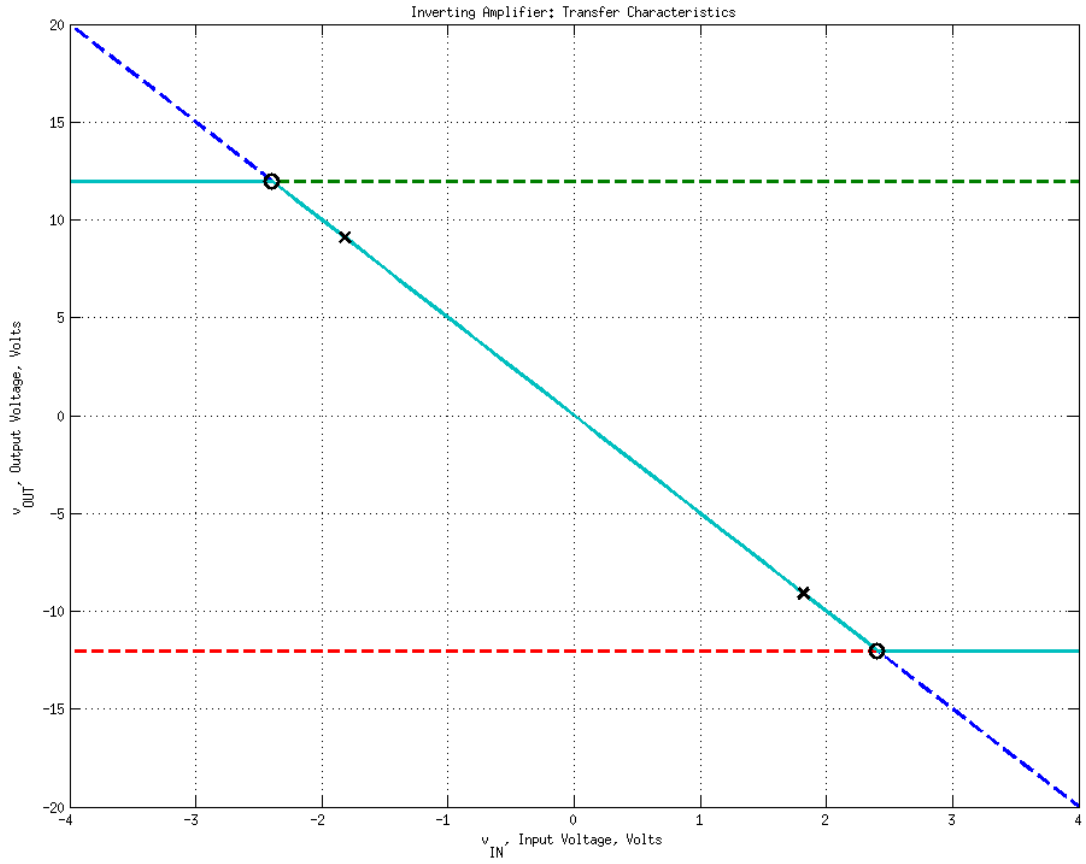
- Remember Op–Amps
- Capacitors; $i = C\frac{dv}{dt}$, $Z = \frac{1}{j\omega C}$
- Inductors; $v = L\frac{di}{dt}$, $Z = j\omega L$
- First–Order Circuits; Time Constants, Steady State, Transient
- Second–Order Circuits; All that plus Resonance
- AC Circuits; Impedance, Eli, Power, RMS, Scopes and Meters
- Thévenin and Norton, Node and Mesh Again
- Fourier Series, Transfer Functions, Filters, Bode Plots

Closed–Loop Gain Spectrum



$A_{CL}f_{0CL} = A_{OL}f_{0OL}$ Gain–Bandwidth Product

Voltage Limits Example



±12–Volt Power Rails. Blue dash is ideal. Cyan solid is actual.

R, L, and C

Devices

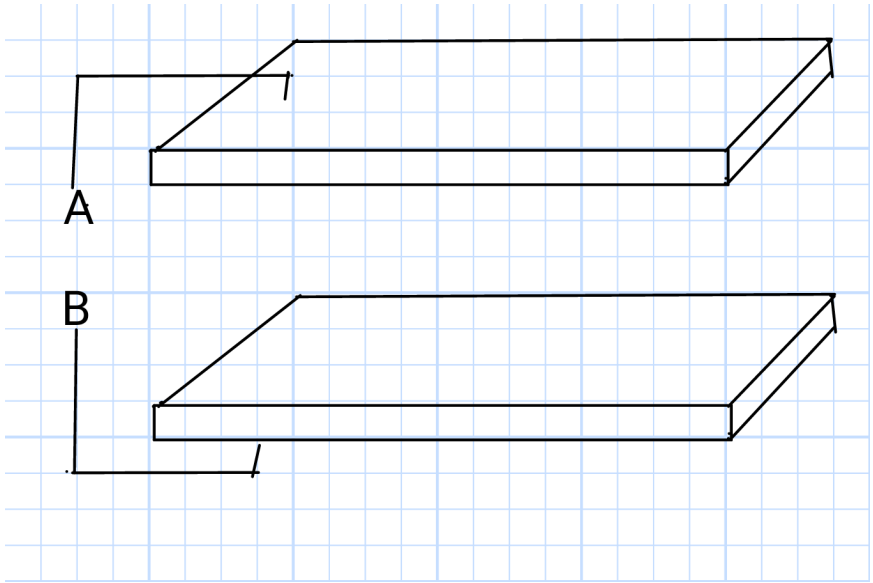
Resistors	Capacitors	Inductors
$v = iR$	$v = \frac{1}{C}\int i\ dt$	$i = \frac{1}{L}\int v\ dt$
R in Ohms	C in Farads	L in Henries
	Voltage Continuous	Current Continuous
	Open to DC	Short to DC

Circuits

RC or RL	LC	RLC
First Order DE	Second Order DE	2nd with Loss
Negative Exponentials	Sinusoids	Lossy Sinusoids

We can do interesting things with time–varying sources.

Capacitors



$$C = \frac{\epsilon A}{d}$$

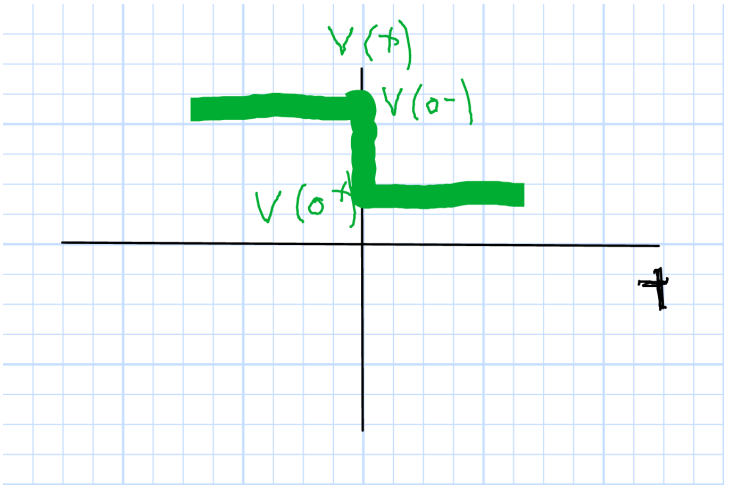
$$\epsilon = \epsilon_r \epsilon_0$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{F/m}$$

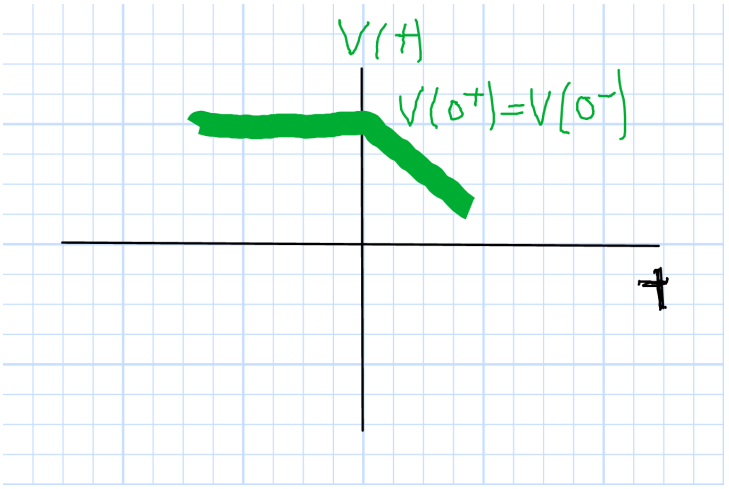
Parallel/Series Summary

	Series	Parallel
Voltage Sources	$v = \sum v_n$	Contradictory
Current Sources	Contradictory	$i = \sum i_n$
Resistors	$R = \sum R_n$	$\frac{1}{R} = \sum \frac{1}{R_n}$
Inductors	$L = \sum L_n$	$\frac{1}{L} = \sum \frac{1}{L_n}$
Capacitors	$\frac{1}{C} = \sum \frac{1}{C_n}$	$C = \sum C_n$

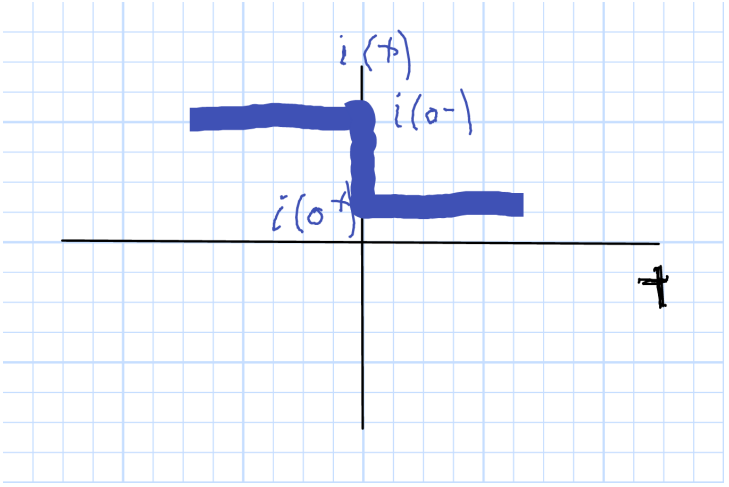
General Boundary Conditions



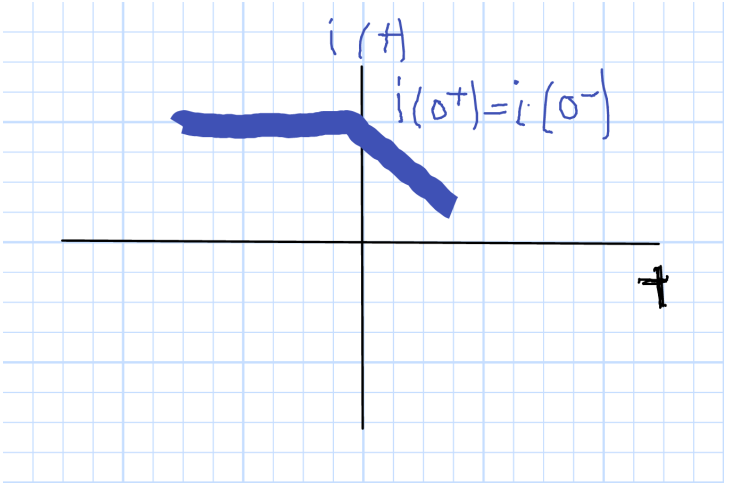
Not Valid for Capacitors



OK

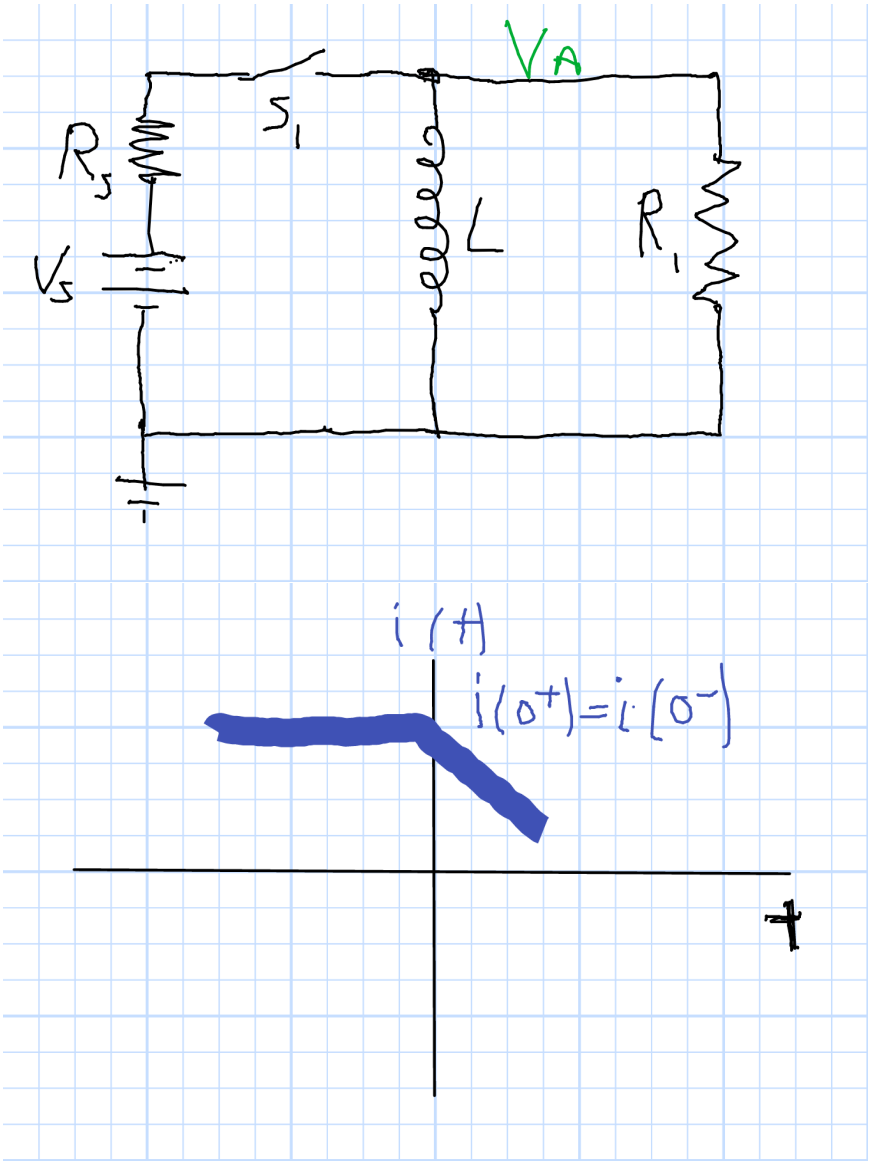


Not Valid for Inductors



OK

First–Order Transients



- From Previous Page

$$v_A = k_1 e^{st} + k_2$$

$$s = -\frac{L}{R_T} \qquad k_2 = 0$$

- Result

$$v_A = k_1 e^{-\frac{L}{R_T}t}$$

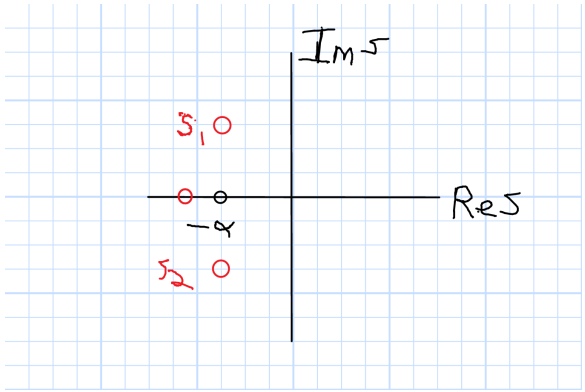
- Boundary Condition:
  $i(0) = 0,\ v_A(0) = v_T$

$$v_A = v_T e^{\frac{L}{R_T}t}$$

- Time Constant

$$v_A = v_T e^{-t/\tau} \qquad \tau = \frac{R_T}{L}$$

Damping in the Complex Plane



Underdamped

$\zeta < 1$

$\alpha < \omega_0$

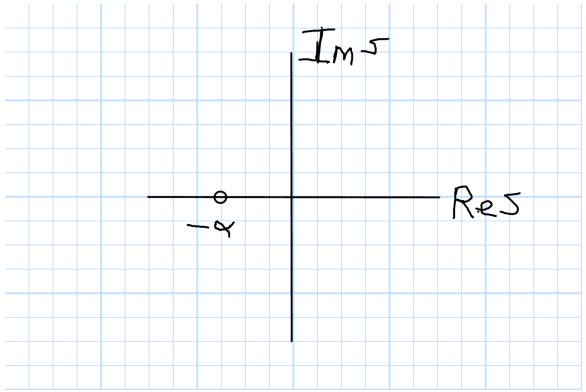
$\Re s < 0$

$\Im s$ Symmetric

Solutions:

Damped Sinusoids

Frequency Shifted



Critically Damped

$\zeta = 1$

$\alpha = \omega_0$

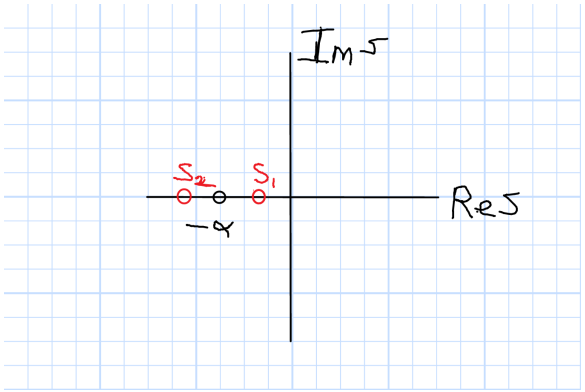
$\Re s < 0$

$\Im s = 0$

Degenerate

Exponential

$+e^{-t/\tau}$



Overdamped

$\zeta > 1$

$\alpha > \omega_0$

$\Re s < 0$

$\Im s = 0$

Exponentials

Damping

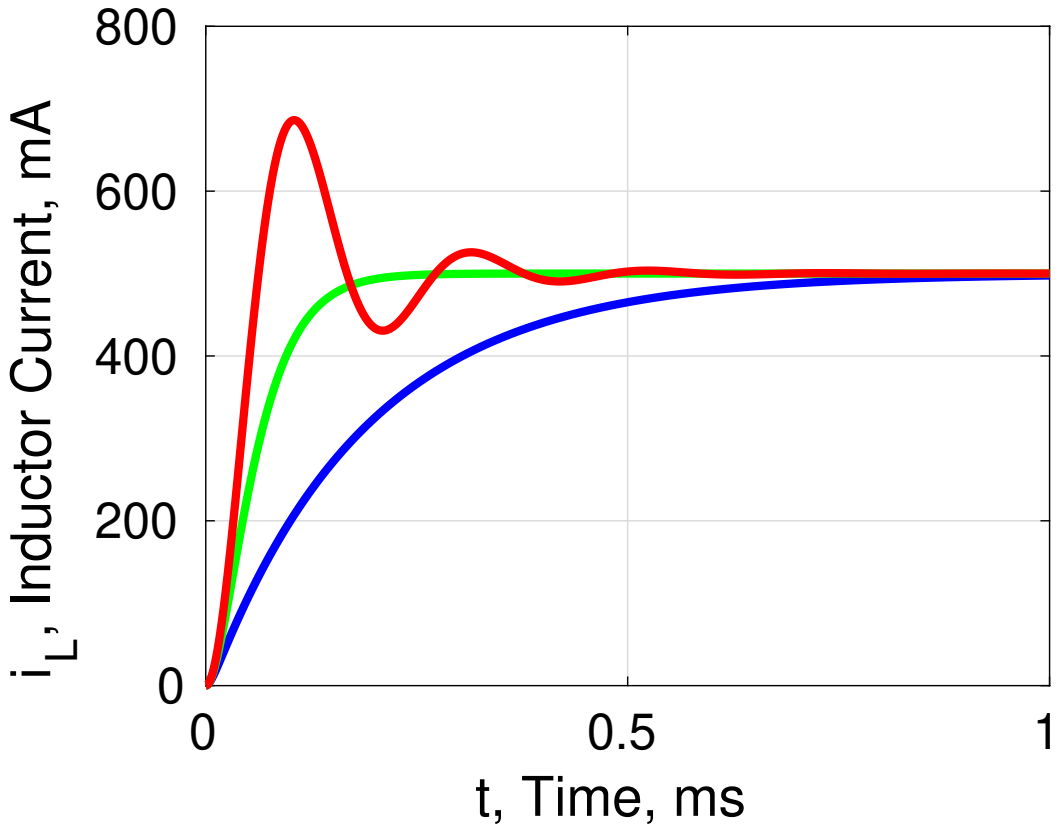
$$\omega_0 = 2\pi \times 5\text{kHz}$$

$$C = 1\mu\text{F}$$

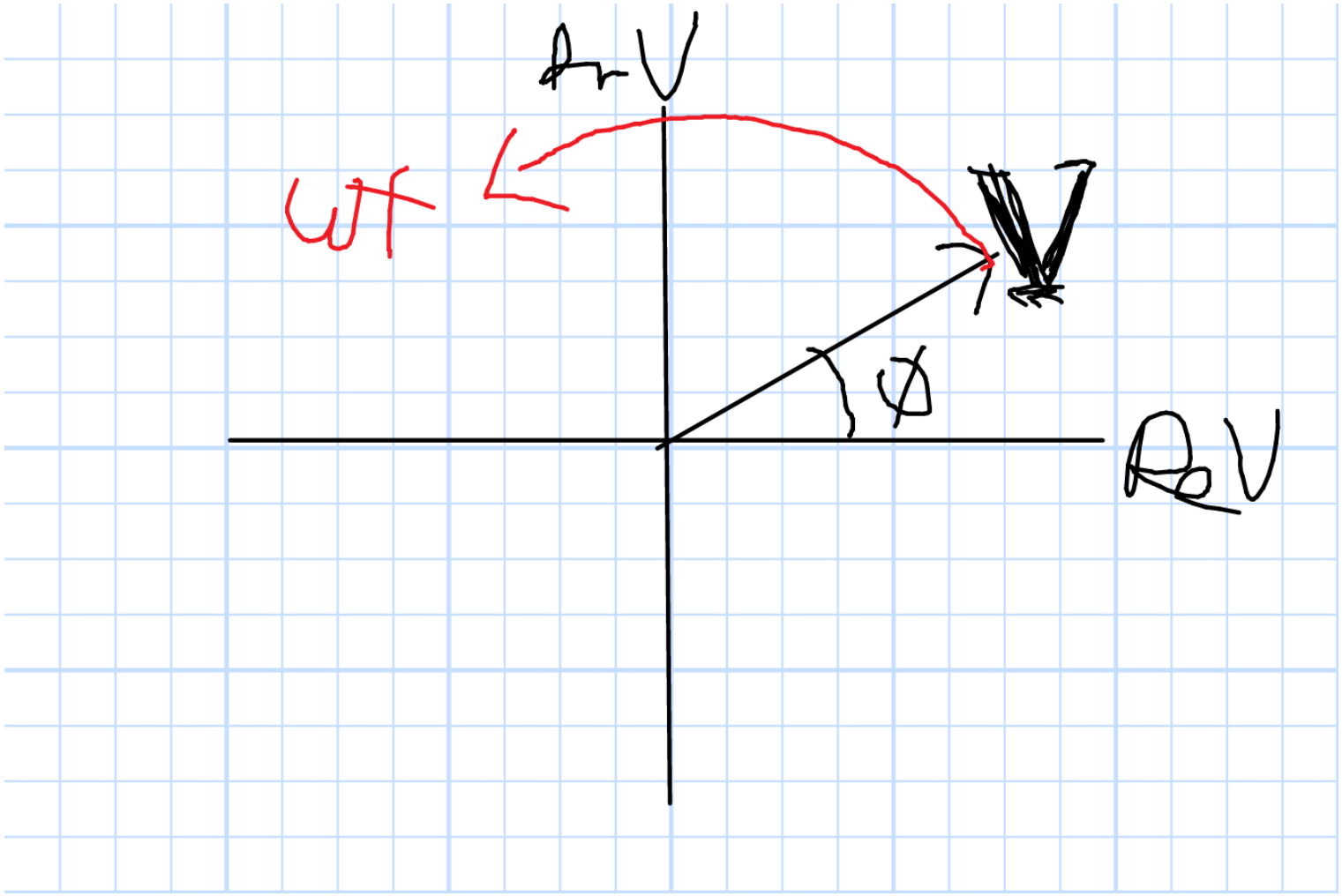
$$L = 1\text{mH}$$

$$\alpha = \omega_0\zeta \qquad \alpha = \frac{1}{2RC}$$

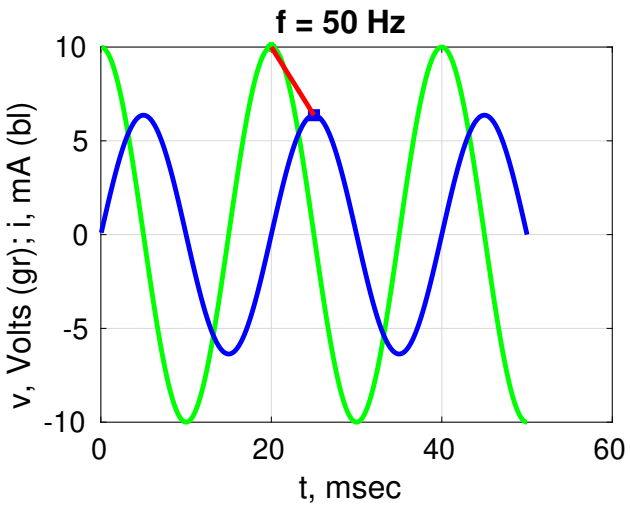
$\zeta = 0.3$	$R = 53.0\Omega$
$\zeta = 1$	$R = 15.9\Omega$
$\zeta = 3$	$R = 5.3\Omega$



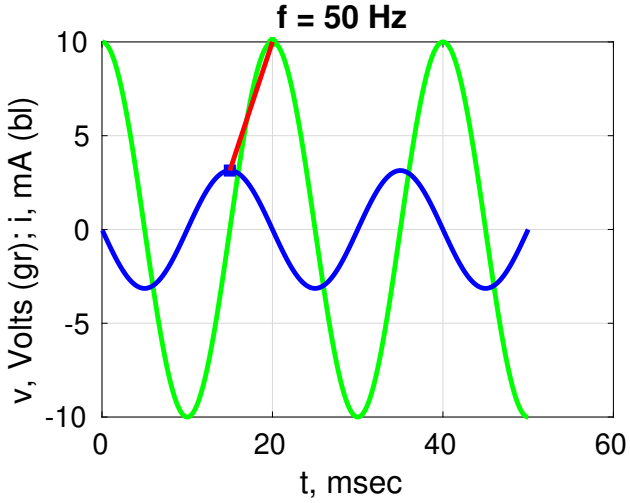
Phasor Graphical Representation



Eli the Ice Man



500mH Inductor



1μF Capacitor

- E for Electromotive Force (Voltage)
- Inductor (ELI)
  E (Voltage) Leads I (Current):
  Peak of Voltage (Green) Before Current (Blue)
- Capacitor (ICE)
  I (Current) Leads E (Voltage):
  Peak of Current (Blue) Before Voltage (Green)

AC Circuits

- Voltage Sources, Current Sources, Resistors

$$v = iR$$

- Voltage Sources, Current Sources, Impedances

$$\mathbf{V} = \mathbf{IZ} = \mathbf{I}R + j\mathbf{I}X$$

- Everything We Learned Before Works
  - Series and Parallel Circuits
  - Voltage and Current Dividers
  - Node and Mesh Analysis
  - Even Op–Amps
- Just Remember the Numbers are Complex

$$i = \text{Re}\left(\mathbf{I}e^{j\omega t}\right) \qquad v = \text{Re}\left(\mathbf{V}e^{j\omega t}\right)$$

Resistive and Reactive Power

- Power Equations

$$v = \text{Re}\left(Ve^{j\omega t}\right) \quad i = \text{Re}\left(Ie^{j\omega t}\right) \qquad p = vi$$

- Complex Notation

$$v = \frac{V}{2}e^{j\omega t} + \frac{V^*}{2}e^{-j\omega t} \qquad i = \frac{I}{2}e^{j\omega t} + \frac{I^*}{2}e^{-j\omega t}$$

$$p = \frac{VI^*}{4} + \frac{V^*I}{4} + \frac{VI}{4}e^{j2\omega t} + \frac{V^*I^*}{4}e^{-j2\omega t}$$

$$p = \text{Re}\left(\frac{VI^*}{2}\right) + \frac{|VI|}{2}\cos\left(2\omega t + \phi\right)$$

$$\phi = \text{Angle}\left(\frac{VI}{2}\right)$$

Voltage Readings

- Equations

$$v = \text{Re}\left(V e^{j\omega t}\right) = 20\text{V}\cos\omega t$$

$$v_{max} = |V| \qquad v_{min} = -|V|$$

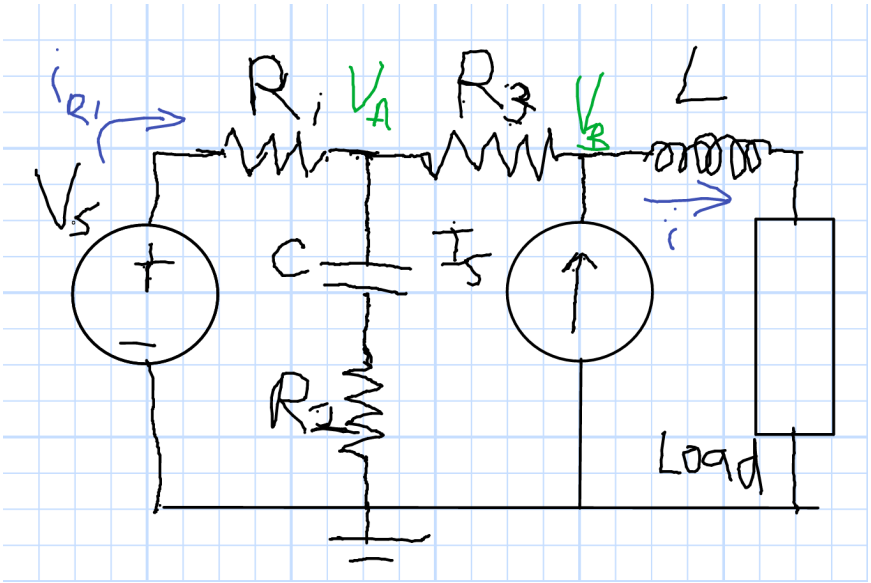
- Volt Meter Measures RMS

$$v_{rms} = \frac{|V|}{\sqrt{2}} = 14\text{V}$$

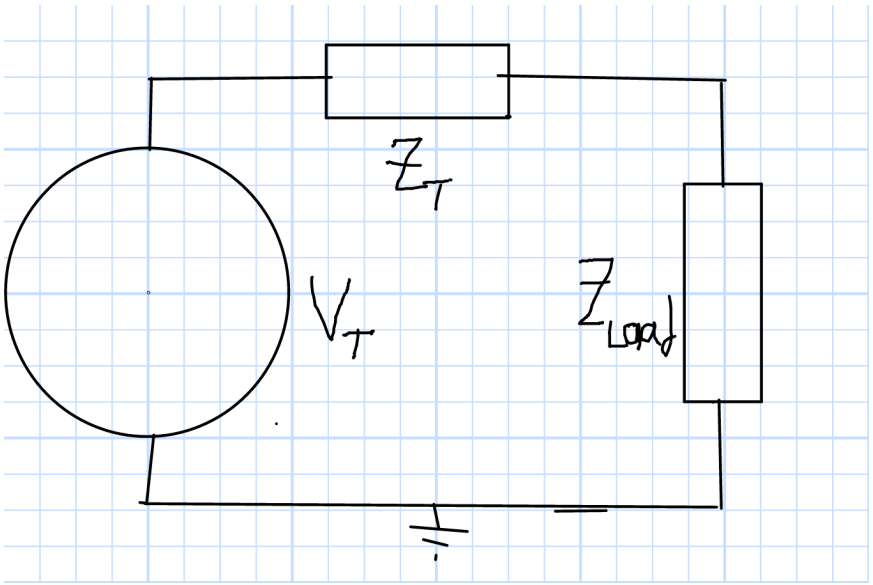
- Oscilloscope Measures Peak–to–Peak

$$v_{pp} = 2\,|V| = 40\text{V}$$

Thévenin Equivalent



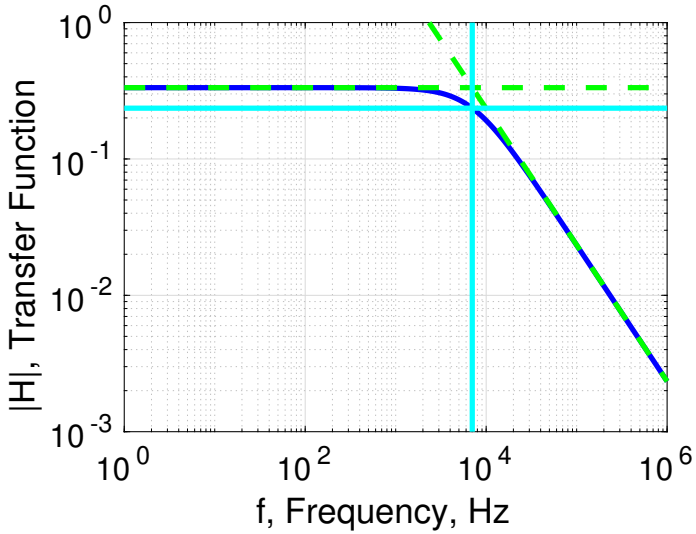
$R_1 = 1\text{k}\Omega,\ R_2 = 3\text{k}\Omega,$
$R_3 = 2\text{k}\Omega,\ C = 10\text{nF},$
$L = 80\text{mH},$
$f = 10\text{kHz}$
$V_S = 5\text{V},\ I_S = 10\text{mA}$

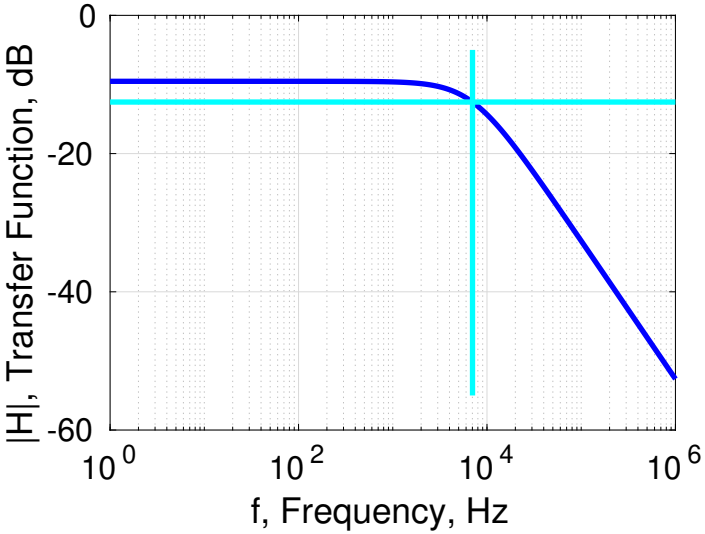


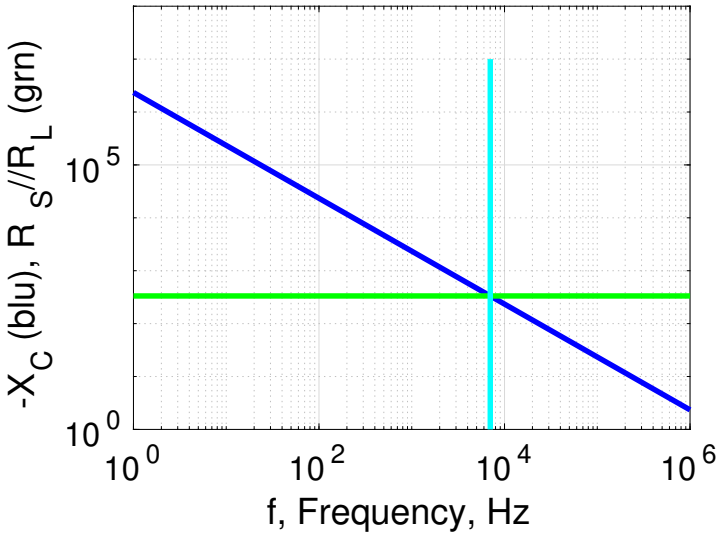
$$V_T = V_{oc} = 31.8\text{V}\angle -2.3^\circ$$

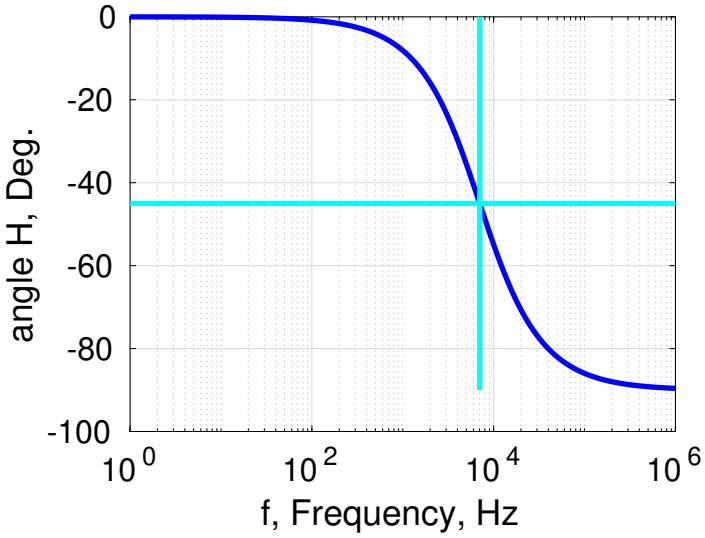
$$Z_T = 5.7\text{k}\Omega\angle -61^\circ$$

Transfer Function

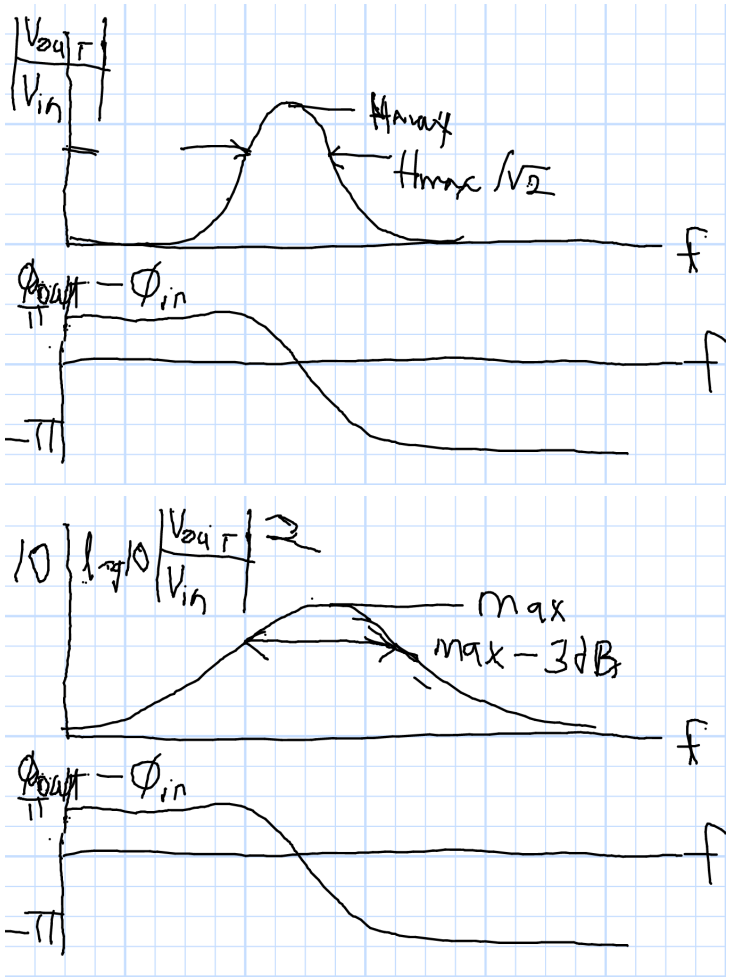


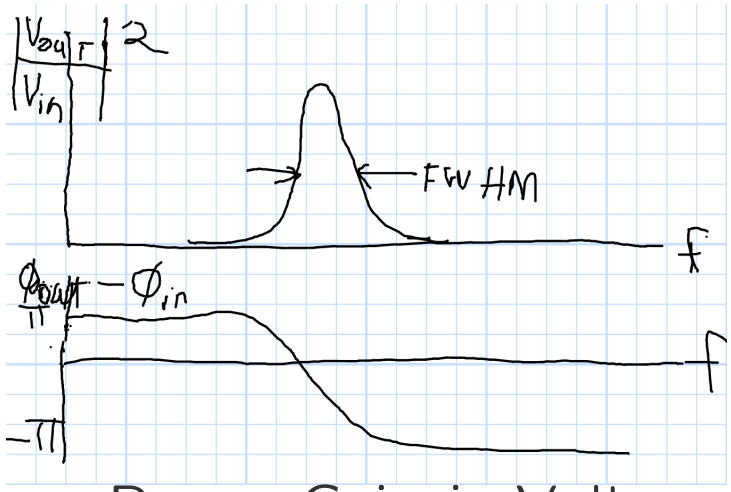






Half–Power Point





- Power Gain is Voltage Gain Squared

$$A_P = |A_V|^2 = \left|\frac{V_{out}}{V_{in}}\right|^2$$

- in deciBels

$$10\log_{10} A_P = 20\log_{10}|A_V|$$

Happy Winter Break



Thanks!