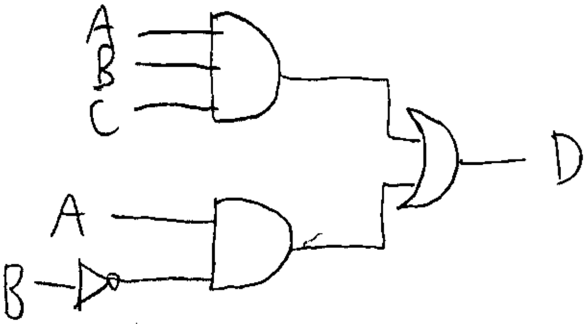
P.1

EECE 2412 - HW 10 Solutions

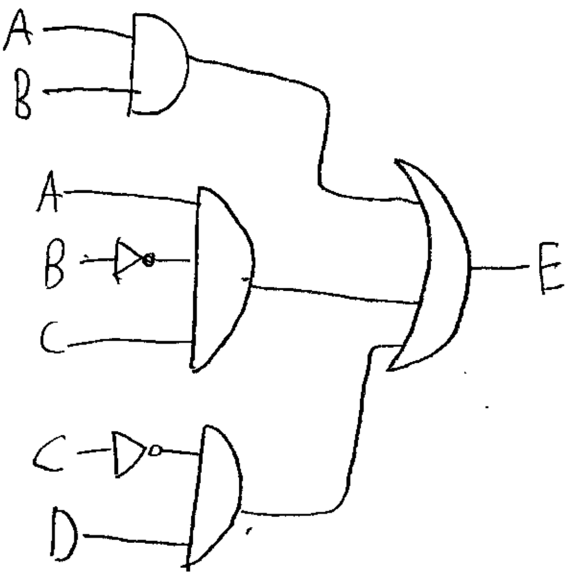
Problem 1

a) $D = ABC + A\bar{B}$



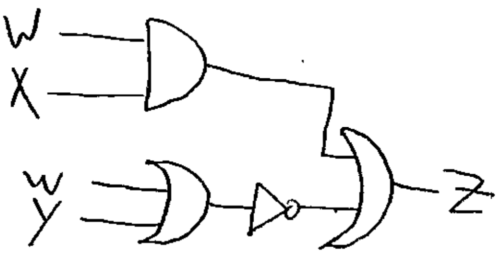
A	B	C	D
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

b) $E = AB + A\bar{B}C + \bar{C}D$



A	B	C	D	E
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

c) $Z = WX + \overline{(W+Y)}$



W	X	Y	Z
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Hint: find which inputs result in "1", then fill the rest with "0"

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(P.2)

Problem 2

$NM_H = V_{OH} - V_{IH} = 4.5V - 3V = \boxed{1.5V = NM_H}$

$NM_L = V_{IL} - V_{OL} = 1.5V - 1V = \boxed{0.5V = NM_L}$

Problem 3

a) $$t_{PHL} = \frac{C_L \cdot V_{DD}}{\left(\frac{W}{L}\right)_N \cdot KP_N \cdot (V_{DD} - V_{toN})^2} = \frac{2\times10^{-12} \cdot 5}{\left(\frac{3}{1}\right) \cdot 50\times10^{-6}(5-1)^2}$$

$$t_{PHL} = 4.167\times10^{-9}s = \boxed{4.167\,ns = t_{PHL}}$$

$$t_{PLH} = \frac{C_L V_{DD}}{\left(\frac{W}{L}\right)_P \cdot KP_P \cdot (V_{DD} - |V_{toP}|)^2} = \frac{2\times10^{-12} \cdot 5}{\left(\frac{6}{1}\right) \cdot 25\times10^{-6}(5-1)^2}$$

$$\boxed{t_{PLH} = 4.167ns}$$

b) same equations, but with $\left(\frac{W}{L}\right)_N = 3$ (no change)

$\left(\frac{W}{L}\right)_P = 60$ (new)

$t_{PHL} = 4.167ns$

$t_{PLH} = 0.4167ns$

c) same equations with $\left(\frac{W}{L}\right)_N = 30$

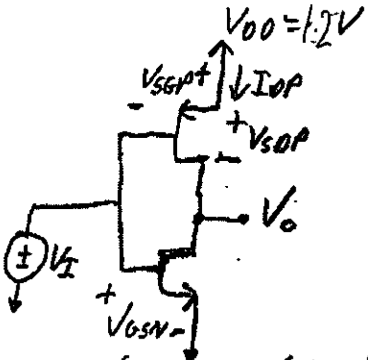
$\left(\frac{W}{L}\right)_P = 6$

$t_{PHL} = 0.4167ns$

$t_{PLH} = 4.167ns$

(P3)

Problem 4



a) Conditions at the trip point:

$V_I = \frac{V_{DD}}{2} = 0.6\ V$

$V_{GSN} = V_I - 0 = 0.6\,V$

It is required that $V_o = \frac{V_{DD}}{2} = 0.6V$, which implies that $I_{DP} = I_{DN}$

$V_{DSN} = V_o - 0V = 0.6\,V > V_{GSN} - V_{toN} = 0.6V - 0.3V = 0.3\ V$

$V_{DSN} > V_{GSN} - V_{toN}$ & $V_{GSN} > V_{toN}$ → The NMOS is in saturation at the trip point

$V_{SGP} = 1.2V - V_I = 0.6\ V > |V_{toP}| = 0.4V$

$V_{SGP} - |V_{toP}| = 0.6\ V - |-0.4V| = 0.2\ V$

$V_{DSP} = 1.2\ V - V_o = 0.6\ V > V_{SGP} - |V_{toP}|$ → The PMOS is in saturation

Equating the currents at the trip point, using the equations for the saturation region:

$I_{DP} = I_{DN}$

$$\left(\frac{W_P}{L_{min}}\right)\cdot\left(\frac{KP_P}{2}\right)\cdot\left(V_{SGP} - |V_{toP}|\right)^2\cdot\left(1 + \lambda\cdot V_{SDP}\right)$$

$$= \left(\frac{W_N}{L_{min}}\right)\left(\frac{KP_N}{2}\right)\cdot\left(V_{GSN} - V_{toN}\right)^2\cdot\left(1 + \lambda\cdot V_{DSN}\right)$$

↳

$$W_P\cdot(30\times10^{-6})\cdot(0.6 - 0.4)^2\cdot(1 + 0.05\cdot 0.6)$$

$$= W_N\cdot(90\times10^{-6})\cdot(0.6 - 0.3)^2\cdot(1 + 0.05\cdot 0.6)$$

↳ $$\boxed{\frac{W_P}{W_n} = 6.75}$$

(P.4) ... Prob.1 cont.:

b) The peak current occurs at the trip point (Fig. 6-33 in the textbook)

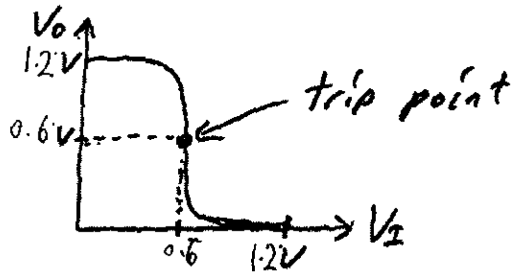
$$I_{DN} = 0.05\times10^{-3} = \left(\frac{W_N}{L_{min}}\right)\cdot\left(\frac{KP_N}{2}\right)\cdot\left(V_{GSN}-V_{THN}\right)^2\cdot\left(1+\lambda\cdot V_{DSN}\right)$$

$$0.05\times10^{-3} = \left(\frac{W_n}{0.12\times10^{-6}}\right)\cdot\left(\frac{90\times10^{-6}}{2}\right)\cdot\left(0.6-0.3\right)^2\cdot\left(1+0.05\times0.6\right)$$

↳ $\boxed{W_N = 1.44\ \mu m}$

$$W_P = \left(\frac{W_P}{W_N}\right)\cdot W_N = 6.75\cdot1.44\times10^{-6} = \boxed{9.72\ \mu m = W_P}$$

c)



Problem 5

Using equation 6.7 in the textbook:

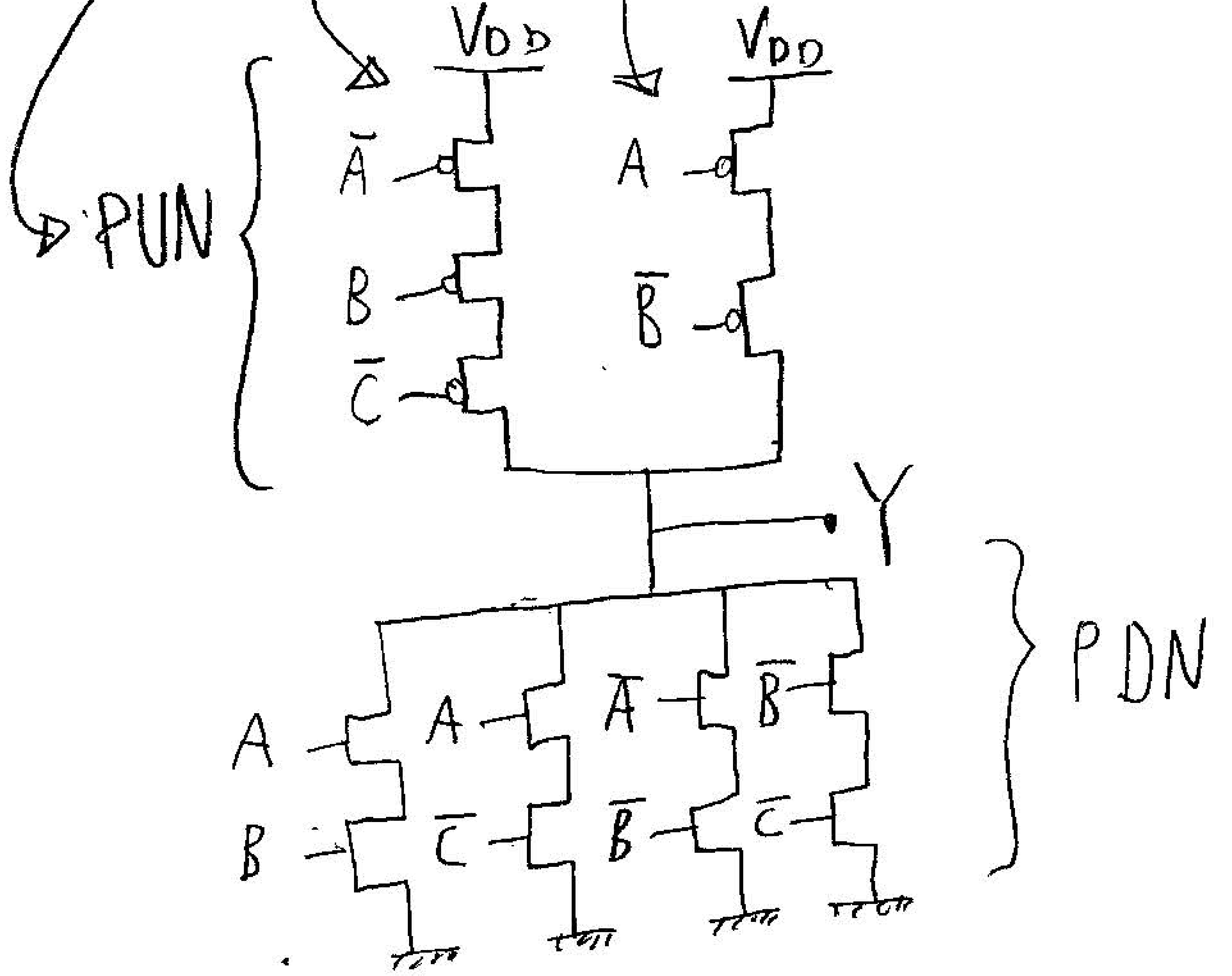
$$P_{dynamic} = f\cdot C_L\cdot(V_{DD})^2 = (100\times10^6\,Hz)\cdot(100\times10^{-15}F)\cdot(3V)^2$$

$\boxed{P_{dynamic} = 90\,\mu W}$

Problem 6

$$Y = A\bar{B}C + \bar{A}B \longrightarrow \bar{Y} = \overline{A\bar{B}C + \bar{A}B} = \overline{A\bar{B}C} \cdot \overline{\bar{A}B}$$

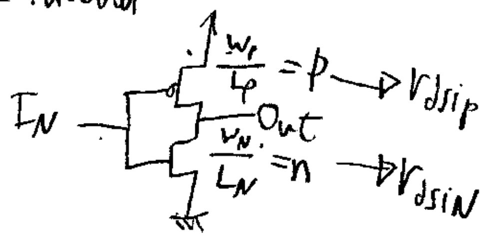
$$\bar{Y} = (\bar{A}+B+\bar{C}) \cdot (A+\bar{B}) = \underline{AB + A\bar{C} + \bar{B}\bar{A} + B\bar{C}}$$



(other solutions are possible)

Problem 7

basic inverter



$$\left.\begin{array}{l} t_{PHL}, t_{PLH} \propto r_{ds} \\ r_{ds} \propto \frac{1}{\left(\frac{W}{L}\right)} \end{array}\right\} t_{PHL}, t_{PLH} \propto \frac{1}{\left(\frac{W}{L}\right)}$$

the series resistance of the transistors in the gate should sum up to an equivalent resistance $\leq r_{ds1p}$ (or $\leq r_{ds1N}$) for all input combinations, where the series resistances of the transistors between the output Y and V_{DD} (or ground) are added when the transistors are "turned on".

$\Downarrow$ $\frac{W}{L}$ ratios of the transistors in the gate

$M_1 = \frac{W}{L} = p$, $M_2, M_3, M_4 : \frac{W}{L} = 2p$

$\rightarrow$ when "on": Two transistors in series each having a resistance $r_{ds\,2,3,4} = \frac{r_{ds1p}}{2}$

$M_5, M_6 : \frac{W}{L} = 2n$

$M_7, M_8 = \frac{W}{L} = 4n$

$\rightarrow$ Worst case M_5 and (M_6 or (M_8 and M_7)) are "on":

if M_5, M_6 on $\rightarrow r = \frac{r_{ds1n}}{2} + \frac{r_{ds1n}}{2} = r_{ds1n}$

if M_5, M_7, M_8 on $\rightarrow r = \frac{r_{ds1n}}{2} + \frac{r_{ds1n}}{4} + \frac{r_{ds1n}}{4}$
$= r_{ds1n}$

if ALL are ON:

$$\frac{r_{ds1n}}{2} + \left(\frac{r_{ds1n}}{2} \,//\left(\frac{r_{ds1n}}{4} + \frac{r_{ds1n}}{4}\right)\right) = \frac{3}{4} r_{ds1n} < r_{ds1n}$$

$M_5 + (M_6 \,// (M_7 + M_8))$