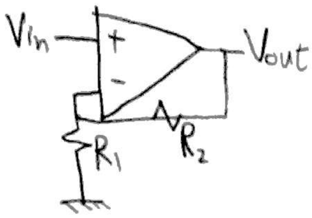
① **Prob 1**

Non-inverting amplifier

gain $= A_V = 1 + \frac{R_2}{R_1}$



Case 1:-

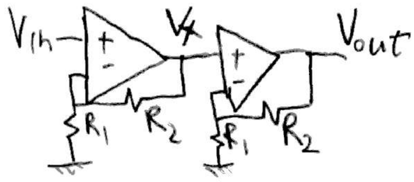
$$A_V = 100 = 1 + \frac{R_2}{R_1} \rightarrow \frac{R_2}{R_1} = 99$$

$$\text{Gain-BW product} = 10^6 = A_{oL} \cdot F_o$$

$$A_{oL}(f) = \frac{A_{oL}}{1 + \frac{jF}{F_o}} = \frac{1}{\frac{1}{A_{oL}} + \frac{jF}{A_{oL} F_o}} \longrightarrow \text{Assume } A_{oL} = \infty \rightarrow A_{oL}(f) = \frac{GBW}{jF}$$

$$\therefore V_{out} = \left(V_{in} - V_{out}\left(\frac{R_1}{R_1 + R_2}\right)\right) A_{oL}(f) \rightarrow \frac{V_{out}}{V_{in}}(f) = \frac{100}{1 + \frac{100 jf}{GBW \rightarrow 10^6}} = \boxed{\frac{100}{1 + \frac{jf}{10^4}}} \rightarrow \boxed{\therefore f_{3\text{-}dB} = 10^4\ Hz}$$

Case 2:-



$$A_V = 10 = 1 + \frac{R_2}{R_1} \rightarrow \frac{R_2}{R_1} = 9$$

$$\therefore \frac{V_{out}}{V_{in}}(f) = \frac{V_x(f)}{V_{in}(f)} \times \frac{V_{out}}{V_x}(f)$$

$$V_x = \left(V_{in} - V_x\left(\frac{R_1}{R_1 + R_2}\right)\right) A_{oL}(f) \qquad V_{out} = \left(V_x - V_{out}\left(\frac{R_1}{R_1 + R_2}\right)\right) A_{oL}(f)$$

$$\therefore \frac{V_x}{V_{in}}(f) = \frac{10}{1 + \frac{10 jF}{GBW}} = \frac{10}{1 + \frac{jf}{10^5}} \qquad \frac{V_{out}}{V_x}(f) = \frac{10}{1 + \frac{10 jf}{GBW}} = \frac{10}{1 + \frac{jf}{10^5}}$$

$\therefore f_{3\text{-}dB}$ of each stage $= 10^5\ Hz$

$$\boxed{\therefore \frac{V_{out}}{V_{in}}(f) = \frac{100}{\left(1 + \frac{jf}{10^5}\right)^2}}$$

to get the OVERALL 3-dB BW, we set $\left|\frac{V_{out}}{V_{in}}(f)\right|$ to $\frac{100}{\sqrt{2}}$

$$\therefore \left|\frac{V_{out}}{V_{in}}(f)\right| = \frac{100}{1 + \left(\frac{f}{10^5}\right)^2} = \frac{100}{\sqrt{2}} \rightarrow f_{3\text{-}dB} = \sqrt{\sqrt{2} - 1} \times 10^5$$

$$\therefore \boxed{f_{3\text{-}dB} \simeq 64.4\ KHz}$$

②

Prob. 2

a) $v_d = v_1 - v_2 = 120\mu V - 80\mu V$

$\boxed{v_d = 40\ \mu V}$

b) $v_{cm} = \frac{v_1 + v_2}{2} = \frac{120\mu V + 80\mu V}{2}$

$\boxed{v_{cm} = 100\ \mu V}$

c) $A_d = 10^{\frac{A_d(dB)}{20}} = 10^{\frac{80}{20}} = 10^4$

$CMRR = 20 \log \frac{|A_d|}{|A_{cm}|} = 100\ dB$

$10^{\frac{100}{20}} = \frac{|A_d|}{|A_{cm}|}$

$\rightarrow |A_{cm}| = \frac{10^4}{10^5} = \boxed{0.1 = |A_{cm}|}$

$\rightarrow A_{cm} = \pm 0.1$

d) $v_o = A_d \cdot v_d + A_{cm} \cdot v_{cm}$

$= (10^4)(40\mu V) \pm (0.1)\cdot(100\mu V)$

$= 0.4 \pm 10^{-5}\ V$

$\boxed{v_o = 0.40001\ V}$ or $\boxed{v_o = 0.39999\ V}$

③

Prob. 3

a) KCL at the inverting input of the circuit in Fig. P2.43:

$$0 = \frac{v_s - (-v_i)}{R_1} + \frac{v_i}{R_{in}} + \frac{v_o - (-v_i)}{R_2}$$

$$0 = v_s + v_i + \frac{R_1}{R_{in}} \cdot v_i + \frac{R_1}{R_2} \cdot v_o + \frac{R_1}{R_2} \cdot v_i$$

$$v_i \cdot \left(1 + \frac{R_1}{R_{in}} + \frac{R_1}{R_2}\right) = -\left(v_s + \frac{R_1}{R_2} \cdot v_o\right)$$

$$\hookrightarrow ①: \quad v_i = -\frac{v_s}{1 + \frac{R_1}{R_{in}} + \frac{R_1}{R_2}} - \frac{R_1 \cdot v_o}{R_2 + \frac{R_1 R_2}{R_{in}} + R_1}$$

KCL at the output:

$$0 = \frac{v_o - A_{OL} \cdot v_i}{R_o} + \frac{v_o - (-v_i)}{R_2}$$

$$0 = v_o - A_{OL} \cdot v_i + \frac{R_o}{R_2} \cdot v_o + \frac{R_o}{R_2} \cdot v_i$$

$$②: v_o \cdot \left(1 + \frac{R_o}{R_2}\right) = v_i \cdot \left(A_{OL} - \frac{R_o}{R_2}\right)$$

substituting eqn. ① in to ②:

$$v_o \cdot \left(1 + \frac{R_o}{R_2}\right) = -\frac{v_s \cdot \left(A_{OL} - \frac{R_o}{R_2}\right)}{1 + \frac{R_1}{R_{in}} + \frac{R_1}{R_2}} - \frac{v_o \cdot R_1 \cdot \left(A_{OL} - \frac{R_o}{R_2}\right)}{R_1 + R_2 + \frac{R_1 R_2}{R_{in}}}$$

$$v_o \cdot \left(1 + \frac{R_o}{R_2} + \frac{R_1 \cdot \left(A_{OL} - \frac{R_o}{R_2}\right)}{R_1 + R_2 + \frac{R_1 R_2}{R_{in}}}\right) = -\frac{v_s \cdot \left(A_{OL} - \frac{R_o}{R_2}\right)}{1 + \frac{R_1}{R_{in}} + \frac{R_1}{R_2}}$$

$$\boxed{A_{vs} = \frac{v_o}{v_s} = \frac{-\left(A_{OL} - \frac{R_o}{R_2}\right)}{\left(1 + \frac{R_1}{R_{in}} + \frac{R_1}{R_2}\right) \cdot \left(1 + \frac{R_o}{R_2} + \frac{R_1 \cdot (A_{OL} - R_o/R_2)}{R_1 + R_2 + R_1 \cdot R_2 / R_{in}}\right)}}$$

(4) ... Prob. 1 a) continued:

$$A_{vs} = \frac{-\left(10^5 - \frac{25}{10000}\right)}{\left(1+\frac{1000}{10^6}+\frac{10000}{1000}\right)\cdot\left(1+\frac{25}{10000}+\frac{1000\cdot\left(10^5-\frac{25}{10000}\right)}{1000+10000+\frac{(1000)\cdot(10000)}{10^6}}\right)}$$

$$\boxed{A_{vs} = -9.998896}$$

with $A_{OL} = \infty$: $\boxed{A_{vs}|_{ideal} = 10}$

b) KVL at the input:

$$0 = -v_s + R_1 \cdot i_s - v_i \longrightarrow (1): \; v_i = R_1 \cdot i_s - v_s$$

KVL in the outer loop:

$$0 = v_i + \left(i_s + \frac{v_i}{R_{in}}\right)\cdot R_2 + \left(i_s + \frac{v_i}{R_{in}}\right)\cdot R_o + A_{OL}\cdot v_i$$

$$(2): \; 0 = v_i + \left(i_s + \frac{v_i}{R_{in}}\right)\cdot(R_2 + R_o) + A_{OL}\cdot v_i$$

sub. (1) into (2):

$$0 = R_1 \cdot i_s - v_s + \left(i_s + \frac{R_1 \cdot i_s - v_s}{R_{in}}\right)\cdot(R_2+R_o) + A_{OL}\cdot R_1 \cdot i_s - A_{OL}\cdot v_s$$

$$v_s \cdot \left[1 + \frac{R_2+R_o}{R_{in}} + A_{OL}\right] = i_s \cdot \left[1 + R_2 + R_o + \frac{R_1\cdot(R_2+R_o)}{R_{in}} + A_{OL}\cdot R_1\right]$$

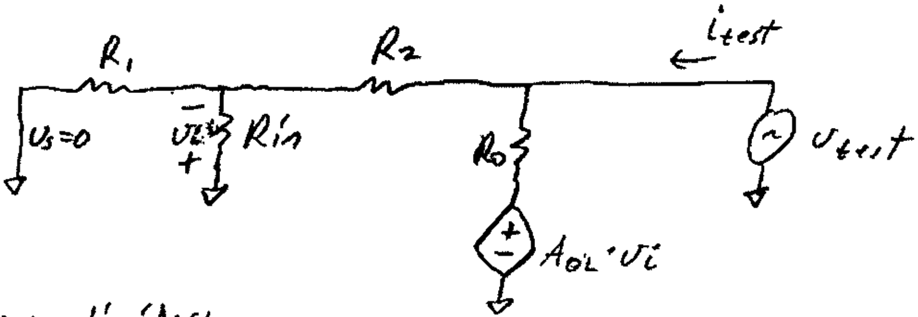
$$\boxed{Z_{in} = \frac{v_s}{i_s} = \frac{1 + R_2 + R_o + \frac{R_1\cdot(R_2+R_o)}{R_{in}} + A_{OL}\cdot R_1}{1 + \frac{R_2+R_o}{R_{in}} + A_{OL}}}$$

$$Z_{in} = \frac{1 + 10000 + 25 + 1000\cdot(10000+25)/10^6 + 10^5\cdot 1000}{1 + (10000+25)/10^6 + 10^5}$$

$$\boxed{Z_{in} = 1.00009\,k\Omega}$$ with $A_{OL} = \infty$: $\boxed{Z_{in}|_{ideal} = 1k\Omega}$

5 ... Prob. 3 cont.:

c)



Voltage divider:

$$① : v_i = -v_{test} \cdot \left(\frac{R_{in}||R_1}{R_2 + R_{in}||R_1} \right)$$

KCL at the output:

$$② : i_{test} = \frac{v_{test} - A_{OL} \cdot v_i}{R_o} + \frac{v_{test}}{R_2 + R_{in}||R_1}$$

Substituting ① into ②:

$$i_{test} = \frac{v_{test}}{R_o} + v_{test} \cdot \frac{A_{OL}}{R_o} \cdot \left(\frac{R_{in}||R_1}{R_2 + R_{in}||R_1} \right) + \frac{v_{test}}{R_2 + R_{in}||R_1}$$

$$\boxed{Z_o = \frac{v_{test}}{i_{test}} = \frac{1}{\frac{1}{R_o} + \frac{A_{OL}}{R_o} \cdot \left(\frac{R_{in}||R_1}{R_2 + R_{in}||R_1} \right) + \frac{1}{R_2 + R_{in}||R_1}}}$$

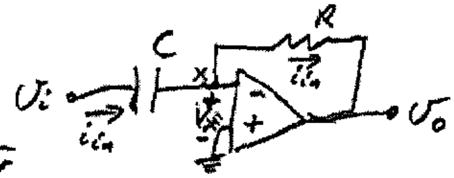
$$R_{in}||R_1 = \frac{1}{\frac{1}{10^6} + \frac{1}{1000}} = 999.000999 \ \Omega$$

$$Z_o = \frac{1}{\frac{1}{25} + \frac{10^5}{25} \cdot \left(\frac{999}{10000 + 999} \right) + \frac{1}{10000 + 999}}$$

$$\boxed{Z_o = 2.752 \times 10^{-3} \ \Omega}$$

with $A_{OL} = \infty$: $\boxed{Z_{o\,ideal} = 0}$

⑥



a) Capacitor impedance: $Z_c = \frac{1}{sC} = \frac{1}{j\omega C}$

①: $i_{in} = \frac{v_i - v_x}{Z_c} = j\omega C(v_i - v_x) = j\omega C\left(v_i + \frac{v_o}{A_{OL}}\right)$ (sub. $v_x = -\frac{v_o}{A_{OL}}$)

($v_x \neq 0$ with finite gain $\rightarrow v_o = (v_+ - v_-)\cdot A_{OL} = -v_x \cdot A_{OL}$)

②: $i_{in} = \frac{v_x - v_o}{R} = \frac{-\frac{v_o}{A_{OL}} - v_o}{R}$

Setting ① equal to ②:

$j\omega C\cdot\left(v_i + \frac{v_o}{A_{OL}}\right) = \frac{-\frac{v_o}{A_{OL}} - v_o}{R} \rightarrow j\omega RC \cdot v_i = -\frac{j\omega RC}{A_{OL}}\cdot v_o - \frac{v_o}{A_{OL}} - v_o$

$$\rightarrow \boxed{A_{CL}(j\omega) = \frac{v_o}{v_i} = \frac{-j\omega RC}{1 + \frac{1}{A_{OL}} + j\frac{\omega RC}{A_{OL}}}}$$

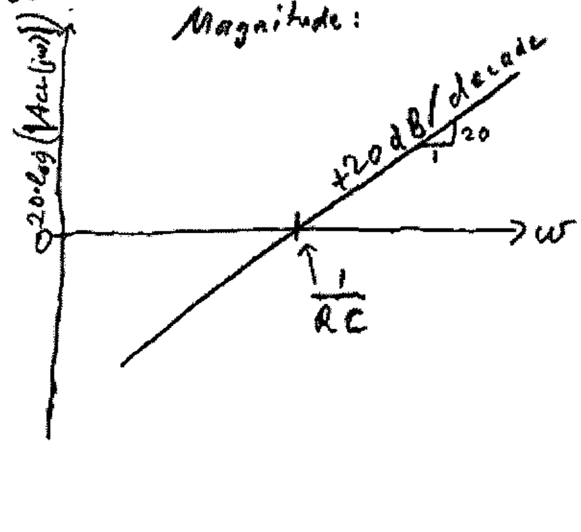
b) with $A_{OL} = \infty$:

$$\boxed{A_{CL}(j\omega)\Big|_{A_{OL}=\infty} = -j\omega RC}$$

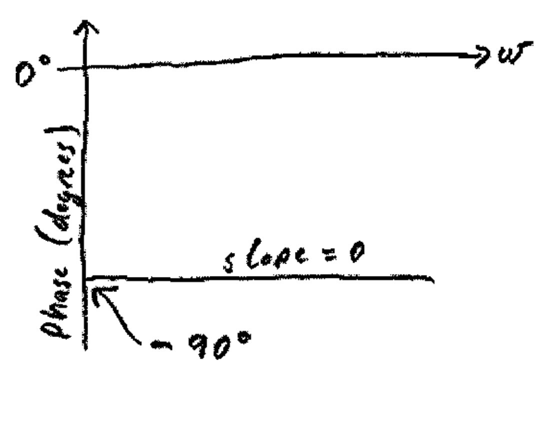
FYI: This is a differentiator circuit (same equation as in eq. 2.56 in the book)

Bode Plot:

Magnitude:



Phase of $A_{CL}(j\omega)$:



⑦

c) Substituting $\omega = 2\pi f$ and the given values into the equation from part a):

$$A_{CL}(j2\pi\cdot 500) = \frac{v_o(j2\pi\cdot 500)}{v_i(j2\pi\cdot 500)} = \frac{-j\cdot 2\pi\cdot 500\cdot 10\times10^3\cdot 20\times10^{-6}}{1+\frac{1}{10^4}+j\cdot\frac{2\pi\cdot 500\cdot 10\times10^3\cdot 20\times10^{-6}}{10^4}}$$

$$A_{CL}(j2\pi\cdot 500) = 627.02\angle -93.6^\circ$$

[Just for comparison A_{CL} at 500 Hz with ideal op-amp:
$$A_{CL}(j2\pi\cdot 500)\Big|_{A_{OL}=\infty} = -j2\pi\cdot f\cdot R\cdot C = 628.32\angle -90.0^\circ]$$

Output at 500 Hz:

$$\begin{aligned} v_o(j2\pi\cdot 500) &= A_{CL}(j2\pi\cdot 500)\cdot v_{in}(j2\pi\cdot 500) \\ &= (627.02\angle -93.6^\circ)\cdot(2\,mV\angle 0^\circ) \\ &= 1.254\angle -93.6^\circ\ [V] \end{aligned}$$

$$v_o(t) = 1.254\,V\cdot\sin(2\pi\cdot 500\cdot t - 93.6^\circ)$$

$\boxed{\text{Output Amplitude} = 1.254\,V}$

d) Closed-loop voltage gain in decibels $= 20\log_{10}(|A_{CL}|) = 20\log_{10}(627.02) = \boxed{55.95\,dB}$

Problem 5

a) From the analysis in example 2.10 (page 96) in the book:

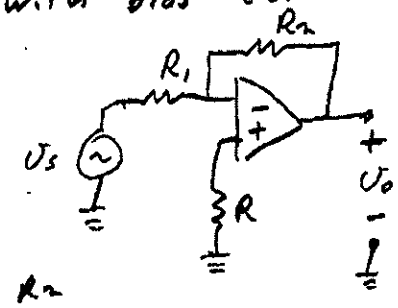
$$V_o = -\left(1+\frac{R_2}{R_1}\right)\cdot V_{off} \rightarrow V_{off,max} = \frac{\pm 100mV}{-\left(1+\frac{R_2}{R_1}\right)} = \frac{\pm 100mV}{-\left(1+\frac{100\times10^3}{10\times10^3}\right)}$$

$$\boxed{V_{off,max} = \pm 9.091\,mV}$$

b) As in example 2.10, $V_o = R_2 \cdot I_B \rightarrow I_B = \frac{V_o}{R_2}$

$$\rightarrow I_{B,max} = \frac{\pm 100mV}{100\times10^3\,\Omega} = \boxed{\pm 1.0\,\mu A = I_{B,max}}$$

c) Circuit with bias current cancellation:

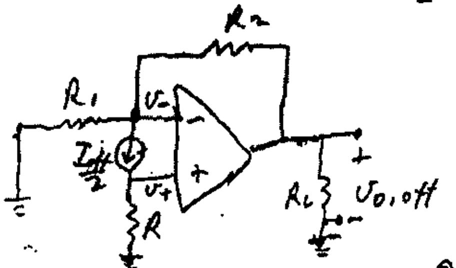


requires:

$$\boxed{R = R_1 // R_2 = \frac{R_1 R_2}{R_1+R_2}}$$

↳ shown on the slide entitled "Example DC offset Analysis" in lecture 7.

d)



Summing point constraint: ① $V_- = V_+ = \frac{I_{off}}{2}\cdot R$

KCL at V_-: ② $\frac{V_-}{R_1} + \frac{I_{off}}{2} + \frac{V_x - V_{o,off}}{R_2} = 0$

Sub. ① into ②, and multiplying all terms by R_2:

same solution as with the alternative analysis in the lecture slides.

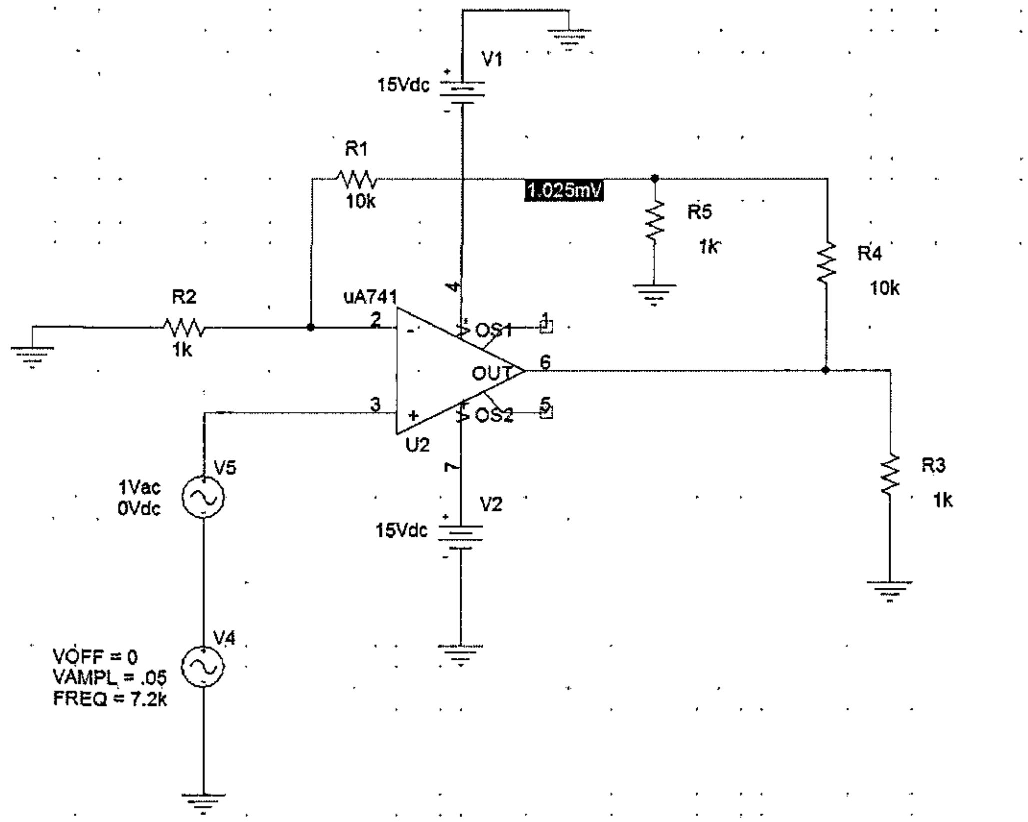
$$\frac{R_2}{R_1}\cdot R\cdot\frac{I_{off}}{2} + R_2\cdot\frac{I_{off}}{2} + R\cdot\frac{I_{off}}{2} - V_{o,off} = 0$$

$$\rightarrow V_{o,off} = \frac{I_{off}}{2}\cdot\left[R_2 + \frac{RR_2 + RR_1}{R_1}\right] = \frac{I_{off}}{2}\cdot\left[R_2 + \frac{R}{R_1}\cdot(R_1+R_2)\right]$$

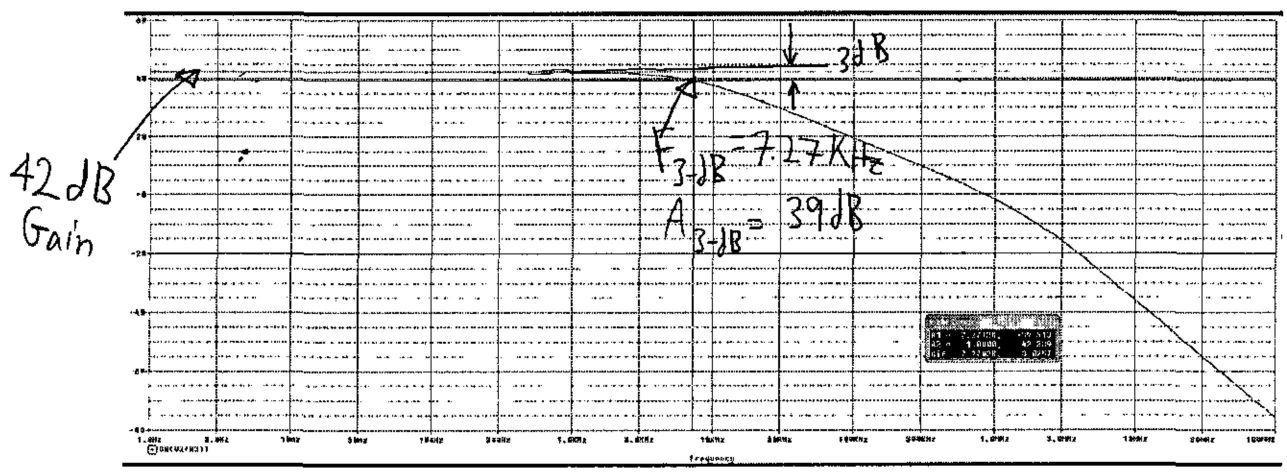
$$= \frac{I_{off}}{2}\cdot\left[R_2 + \frac{R_1R_2}{R_1+R_2}\cdot\frac{1}{R_1}\cdot(R_2+R_1)\right] = \frac{I_{off}}{2}\cdot\left[R_2+R_2\right] = R_2\cdot I_{off}$$

$$\rightarrow I_{off,max} = \frac{V_{o,off,max}}{R_2} = \frac{\pm 100mV}{100\times10^3\,\Omega} = \boxed{\pm 1.0\,\mu A = I_{off,max}}$$

(9)



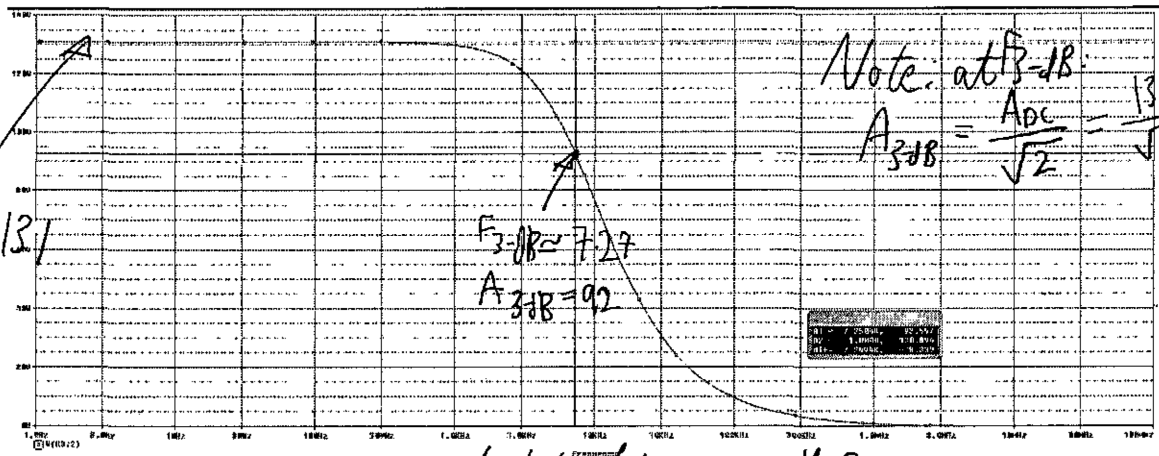
a)



Voltagegain in dB vs. Freq.

(10)

a) Cont.

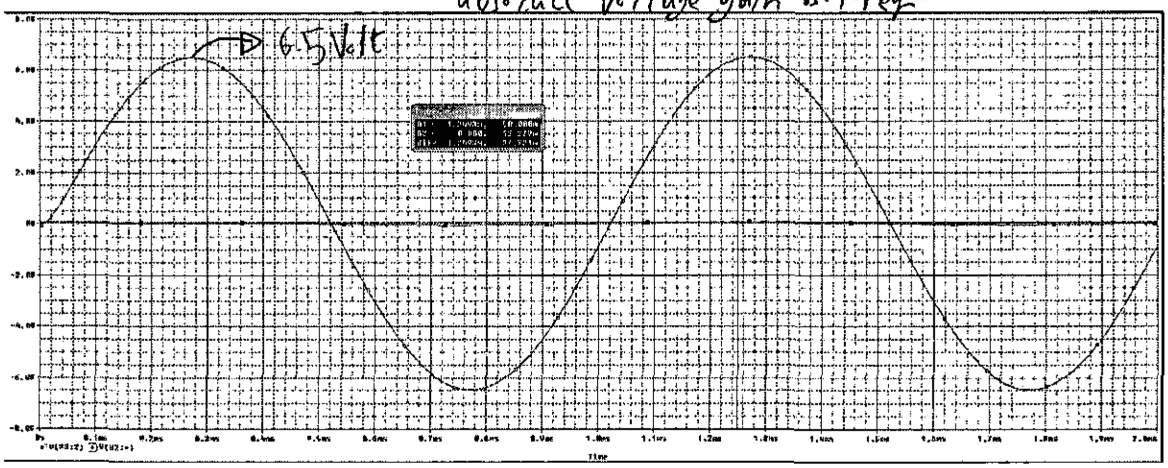


DC gain ≈ 131

absolute Voltage gain Vs. Freq.

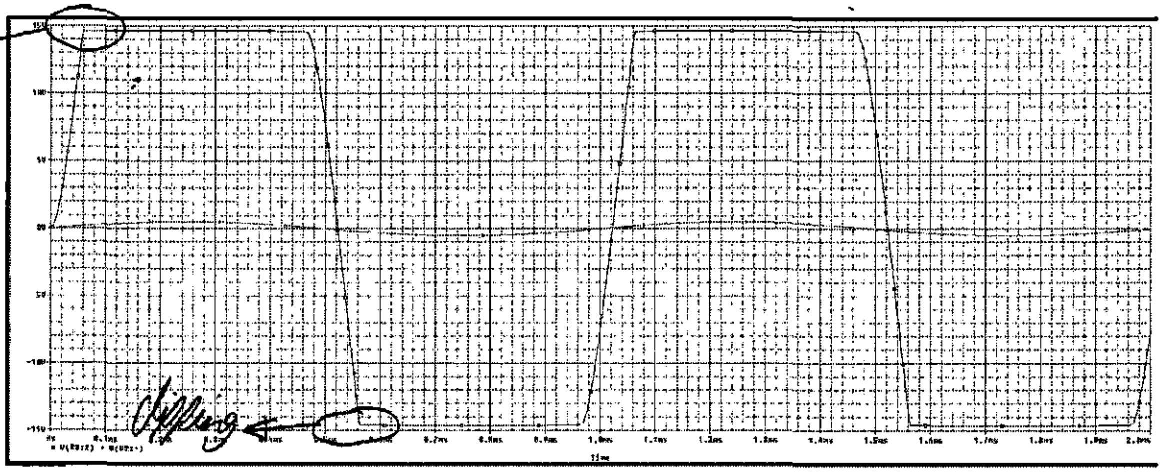
b) $A_V = \frac{6.5^V}{0.05V} = 130$

"as expected"



c)

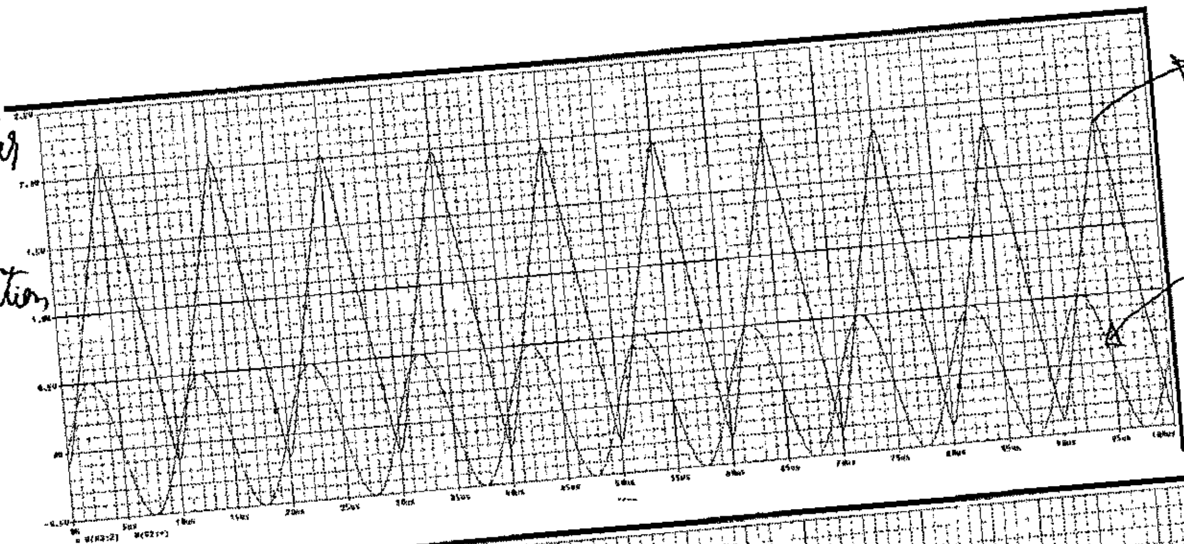
clipping



clipping at ± 14.5 Volt "due to supply limitation"

(11) d)

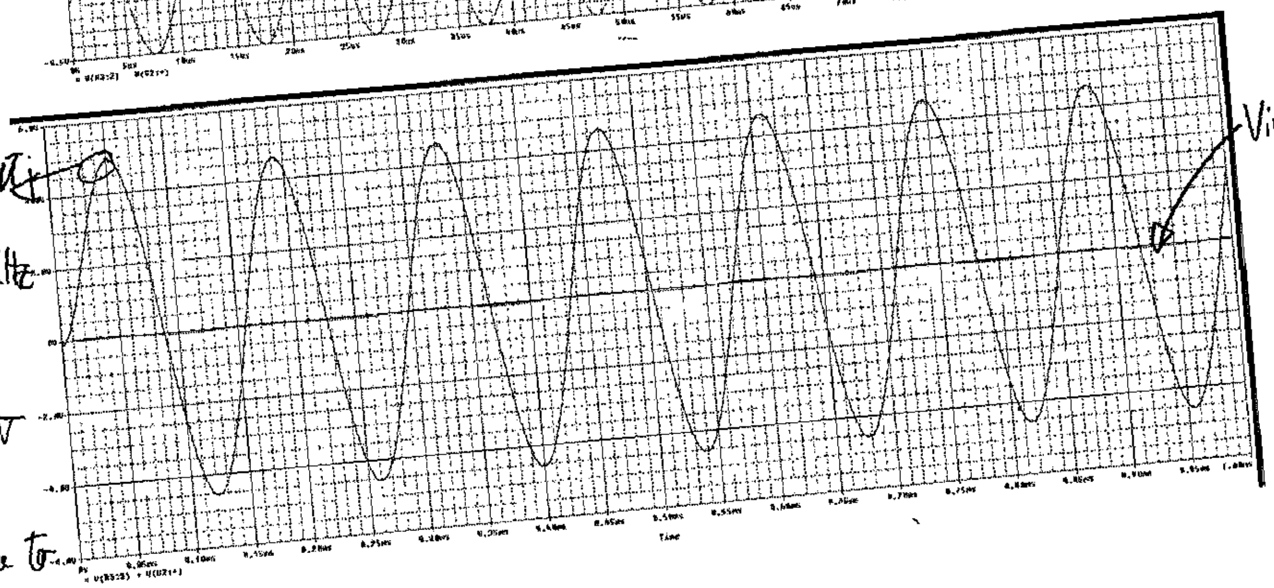
output is Triangular wave due to Slewrate limitation



Vout

Vin

e)



$V_{in} = 0.05V$

$V_{out} = 5.$ ~~Volt~~

$F_{in} = 7.2\,KHz$

$\simeq F_{3dB}$

$gain = \frac{5\,V}{0.05\,V}$

$= 100$

"very close to theoritical prediction of 92"