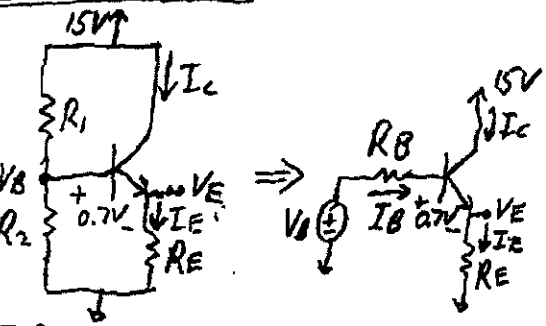
(1)

EECE 2412 : HW 7 Solutions

Prob. 1

DC circuit:



$V_B = 15V \cdot \left(\frac{R_1}{R_1+R_2}\right) = 15V \cdot \left(\frac{10k\Omega}{20k\Omega}\right)$

$V_B = 7.5V$

$R_B = R_1 \| R_2 = 5k\Omega$

KVL: $-V_B + I_B \cdot R_B + 0.7V + I_E \cdot R_E = 0$

$\hookrightarrow 0 = -V_B + \left(\frac{I_C}{\beta}\right) \cdot R_B + 0.7V + \left(\frac{\beta+1}{\beta}\right) \cdot I_c \cdot R_E$

$0 = -7.5V + (I_{CQ}/100) \cdot (5000) + 0.7V + \left(\frac{101}{100}\right) \cdot I_{CQ} (1000)$

$\hookrightarrow \boxed{I_{CQ} = 6.42 \text{ mA}}$

$r_\pi = \frac{\beta \cdot V_T}{I_{CQ}} = \frac{100 \cdot (26 mV)}{6.42 mA} = \boxed{405\Omega = r_\pi}$ (at room temp.)

Small-signal equivalent circuit: see Fig. 4.36 in the book

Using the small-signal analysis results in section 4.8 of the book:

$R_L' = R_L \| R_E = 1k\Omega \| 500\Omega = 333\Omega$

$A_v = \frac{R_L'(\beta+1)}{r_\pi + R_L'(\beta+1)} = \frac{333(101)}{405 + 333(101)} = \boxed{0.988 = A_v}$

$Z_{in} = R_B \| [r_\pi + R_L' \cdot (\beta+1)] = \frac{1}{\frac{1}{5000} + \frac{1}{405+333(101)}} = \boxed{4.36k\Omega = Z_{in}}$

$A_i = A_v \cdot \frac{Z_{in}}{R_L} = (0.988) \cdot \frac{4360}{500} = \boxed{8.62 = A_i}$ ← equation 1.4 on page 20

$R_s' = R_B \| R_s = 5k\Omega \| 1k\Omega = 833\Omega$

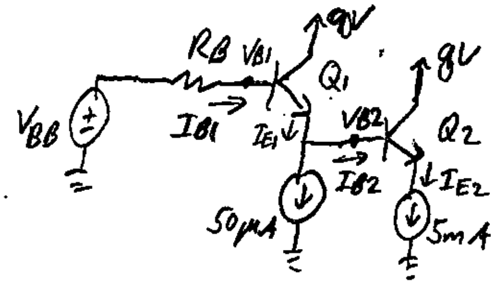
$Z_o = R_E \| \left[\frac{(R_s' + r_\pi)}{\beta+1}\right] = 1k\Omega \| \left(\frac{833+405}{101}\right) = \boxed{12.1\Omega = Z_o}$

$G = A_v \cdot A_i = (0.988) \cdot (8.62) = \boxed{8.52 = G}$

(p.2)

$\boxed{\text{Problem 2}}$

a) DC equivalent circuit:



$$V_{BB} = 9V \cdot \frac{R_2}{R_1+R_2} = 4.17\ V$$

$$R_B = R_1 // R_2 = 57.4\ k\Omega$$

$$\boxed{I_{E2} = 5mA} \qquad I_{E2} = (1+\beta)\cdot I_{B2} \rightarrow I_{B2} = \frac{I_{E2}}{1+\beta_2} = \frac{5mA}{1+100}$$

$$I_{B2} = 49.50\ \mu A$$

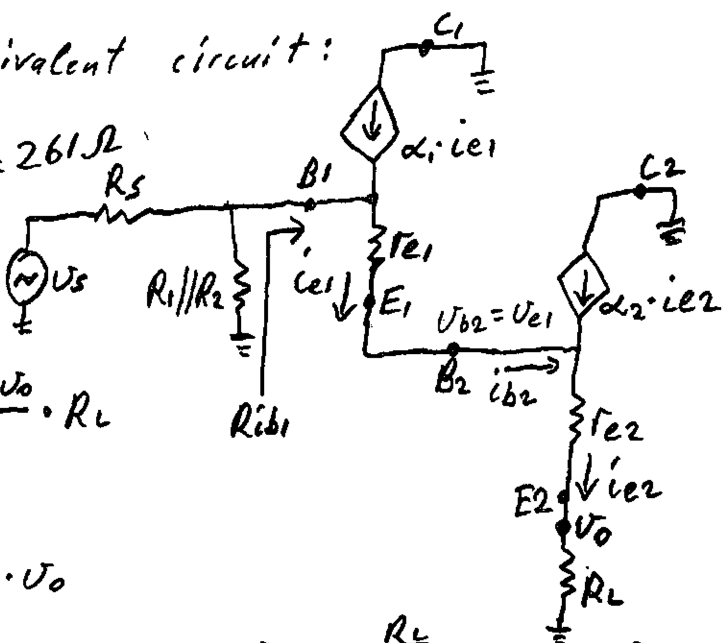
$$I_{E1} = 50\mu A + I_{B2} = \boxed{99.50\ \mu A = I_{E1}}$$

$$I_{B1} = \frac{I_{E1}}{1+\beta_1} = \frac{99.5\mu A}{1+50} = 1.951\ \mu A$$

$$V_{B1} = V_{BB} - I_{B1}\cdot R_B = 4.17V - (1.951\times10^{-6})(57.4\times10^{3}) = \boxed{3.05V = V_{B1}}$$

$$V_{B2} = V_{B1} - V_{BE1} = 3.05\ V - 0.7V = \boxed{2.35\ V = V_{B2}}$$

b) Small-signal equivalent circuit:



$$r_{e1} = \frac{V_T}{I_{EQ1}} = \frac{26\times10^{-3}V}{99.5\times10^{-6}A} = 261\Omega$$

$$r_{e2} = \frac{V_T}{I_{EQ2}} = \frac{26mV}{5mA} = 5.2\Omega$$

$$v_o = i_{e2}\cdot R_L = \frac{v_{b2}-v_o}{r_{e2}}\cdot R_L$$

$$i_{e2} = \frac{v_{b2}-v_o}{r_{e2}} \quad \text{sub.}$$

$$v_o = \frac{R_L}{r_{e2}}\cdot v_{b2} - \frac{R_L}{r_{e2}}\cdot v_o$$

$$v_o\left(1+\frac{R_L}{r_{e2}}\right) = \frac{R_L}{r_{e2}}\cdot v_{b2} \longrightarrow \frac{v_o}{v_{b2}} = \frac{\frac{R_L}{r_{e2}}}{1+\frac{R_L}{r_{e2}}} = \frac{R_L}{r_{e2}+R_L}$$

$$\boxed{\frac{v_o}{v_{e1}} = \frac{v_o}{v_{b2}} = \frac{R_L}{r_{e2}+R_L} = \frac{2000}{5.2+2000} = 0.9974}$$

(p.3) ... Problem 2 b) continued:

$$i_{b2} = i_{e2} - \alpha \cdot i_{e2} = (1-\alpha)\cdot i_{e2} = (1-\alpha)\cdot\left(\frac{v_{b2}}{r_{e2}+R_L}\right)$$

$$i_{e2} = \frac{v_{b2}}{r_{e2}+R_L} \quad \text{sub.}$$

$$\boxed{R_{ib2} = \frac{v_{b2}}{i_{b2}} = \frac{r_{e2}+R_L}{1-\alpha_2}} = \frac{r_{e2}+R_L}{1-\frac{\beta_2}{\beta_2+1}} = \frac{(1+\beta_2)\cdot(r_{e2}+R_L)}{\beta_2+1-\beta_2} = (1+\beta_2)\cdot(r_{e2}+R_L)$$

$$\alpha = \frac{\beta}{\beta+1}$$

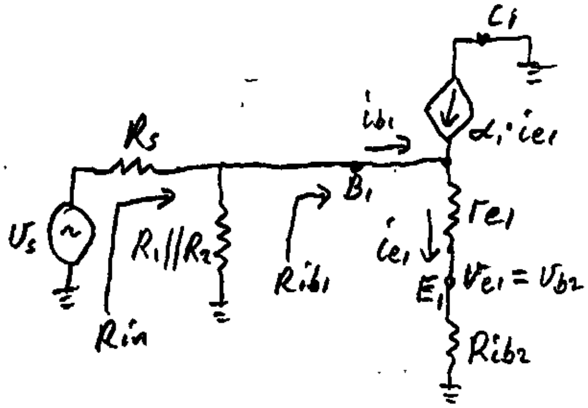
$$\boxed{R_{ib2} = (1+\beta_2)\cdot(r_{e2}+R_L)}$$

$$R_{ib2} = (1+100)\cdot(5.2\,\Omega + 2000\,\Omega) = \boxed{202.5\text{ k}\Omega = R_{ib2}}$$

c)

$$v_{e1} = i_{e1}\cdot R_{ib2} = \frac{v_{b1}-v_{e1}}{r_{e1}}\cdot R_{ib2}$$

($r_{e1} = 261\,\Omega$ [from part b])



$$\boxed{\frac{v_{e1}}{v_{b1}} = \frac{R_{ib2}}{r_{e1}+R_{ib2}}} = \frac{202.5\times10^3}{261+202.5\times10^3}$$

$$\boxed{\frac{v_{e1}}{v_{b1}} = \frac{v_{b2}}{v_{b1}} = 0.9987}$$

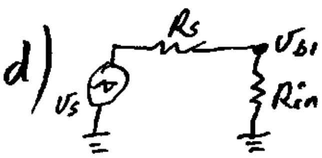
$$i_{b1} = i_{e1} - \alpha_1 i_{e1} = (1-\alpha_1)\cdot\frac{v_{b1}}{r_{e1}+R_{ib2}}$$

$$\boxed{R_{ib1} = \frac{v_{b1}}{i_{b1}} = \frac{r_{e1}+R_{ib2}}{(1-\alpha_1)} = (1+\beta_1)\cdot(r_{e1}+R_{ib2})}$$

$$R_{ib1} = (1+50)\cdot(261\,\Omega + 202.5\text{ k}\Omega) = \boxed{10.34\text{ M}\Omega = R_{ib2}}$$

$$R_{in} = R_{ib1}||R_1||R_2 = \frac{1}{\frac{1}{10.34\text{ M}\Omega}+\frac{1}{1.1\text{ M}\Omega}+\frac{1}{1.2\text{ M}\Omega}} = \boxed{544\text{ k}\Omega = R_{in}}$$

d)



$$\boxed{\frac{v_{b1}}{v_s} = \frac{R_{in}}{R_s+R_{in}} = \frac{544}{120+544} = 0.819\text{ V/V}}$$

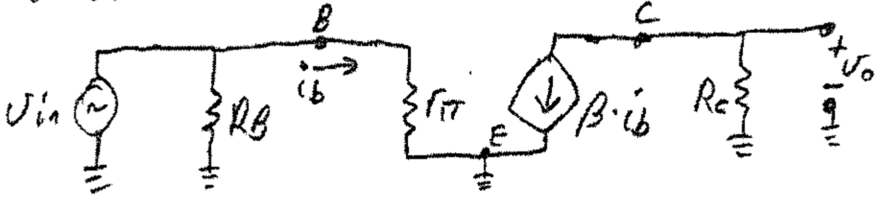
e)

$$\frac{v_o}{v_s} = \frac{v_{b1}}{v_s}\cdot\frac{v_{e1}}{v_{b1}}\cdot\frac{v_o}{v_{e1}} = 0.819\times0.9987\times0.9974 = \boxed{0.816\text{ V/V} = \frac{v_o}{v_s}}$$

(p.4)

Problem 3

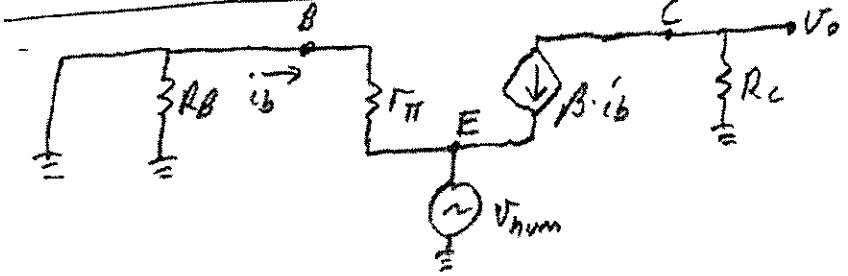
The equivalent circuit is identical for both circuits when $v_{hum} = 0$ during small-signal analysis.



$$i_b = \frac{v_{in}}{r_\pi} \quad \text{Sub.}$$

$$v_o = -\beta \cdot i_b \cdot R_C = -\beta \cdot \frac{v_{in}}{r_\pi} \cdot R_C \rightarrow \boxed{\frac{v_o}{v_{in}} = -\frac{\beta \cdot R_C}{r_\pi}}$$

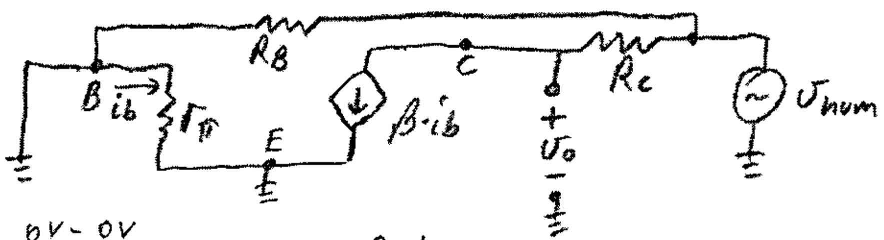
(a) With $v_{in} = 0$:



$$i_b = -\frac{v_{hum}}{r_\pi} \quad \text{Sub.}$$

$$v_o = -\beta \cdot i_b \cdot R_C = \beta \cdot \frac{v_{hum}}{r_\pi} \cdot R_C \rightarrow \boxed{\frac{v_o}{v_{hum}} = \frac{\beta \cdot R_C}{r_\pi}}$$

(b) With $v_{in} = 0$:



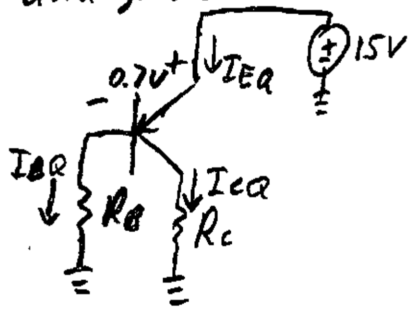
$$i_b = \frac{0V - 0V}{r_\pi} = 0 \rightarrow \beta \cdot i_b = 0$$

$$\text{KVL: } 0 = -v_o - \cancel{\beta \cdot i_b}^{\,0} + v_{hum} \rightarrow v_o = v_{hum} \rightarrow \boxed{\frac{v_o}{v_{hum}} = 1}$$

... Prob. 3 cont.:

DC analysis:

(a)



$0 = -15V + 0.7V + I_{BQ} \cdot R_B$

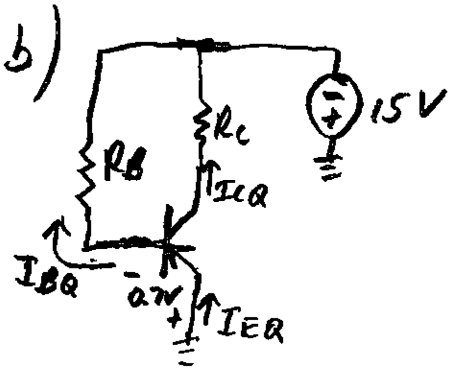
$\hookrightarrow I_{BQ} = \dfrac{14.3V}{1\times10^6 \Omega} = 14.3\mu A$

$I_{CQ} = \beta \cdot I_{BQ} = 1.43\,mA$

$r_\pi = \dfrac{\beta \cdot V_T}{I_{CQ}} = \dfrac{100 \cdot 26mV}{1.43mA} = 1.82k\Omega$

$\boxed{\dfrac{v_o}{v_{in}} = \dfrac{-\beta \cdot R_C}{r_\pi} = -258.5}$ $\boxed{\dfrac{v_o}{v_{hum}} = \dfrac{\beta \cdot R_C}{r_\pi} = 258.5}$

(b)



$0 = 0.7V + I_{BQ} \cdot R_B - 15V$

$\hookrightarrow I_{BQ} = \dfrac{14.3V}{R_B} = 14.3\mu A$

$I_{CQ} = \beta \cdot I_{BQ} = 1.43\,mA$

$r_\pi = \dfrac{\beta \cdot V_T}{I_{CQ}} = 1.82k\Omega$

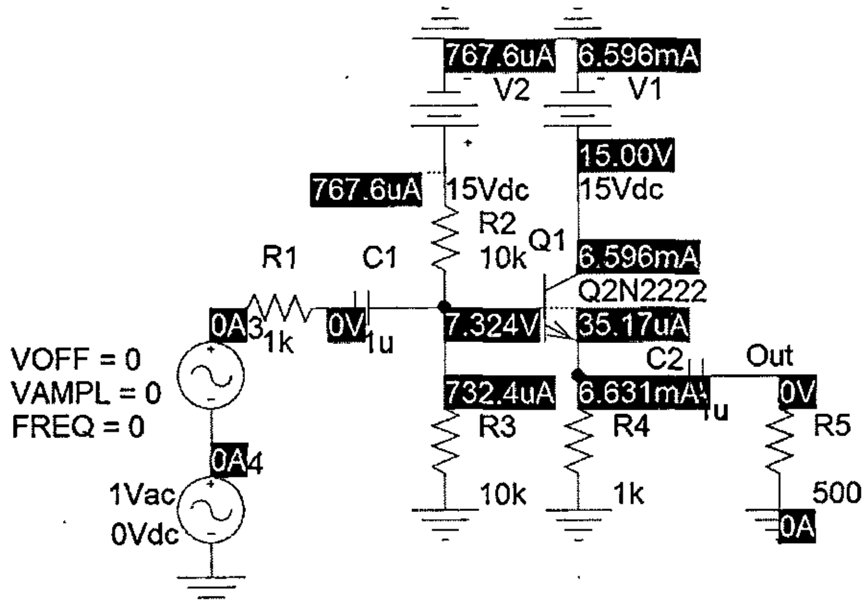
$\boxed{\dfrac{v_o}{v_{in}} = \dfrac{-\beta \cdot R_C}{r_\pi} = -258.5}$ $\boxed{\dfrac{v_o}{v_{hum}} = 1}$

Circuit (b) is better because it has a lower gain from v_{hum} to v_o, while having the same gain from v_{in} to v_o.

(6)

Problem 4

a)

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****      OPERATING POINT INFORMATION      TEMPERATURE =   27.000 DEG C

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**** BIPOLAR JUNCTION TRANSISTORS

NAME         Q_Q1
MODEL        Q2N2222
IB           3.52E-05
IC           6.60E-03
VBE          6.93E-01
VBC         -7.68E+00
VCE          8.37E+00
BETADC       1.88E+02
GM           2.50E-01
RPI          7.96E+02
RX           1.00E+01
RO           1.24E+04
CBE          1.40E-10
CBC          3.20E-12
CJS          0.00E+00
BETAAC       1.99E+02
CBX/CBX2     0.00E+00
FT/FT2       2.77E+08
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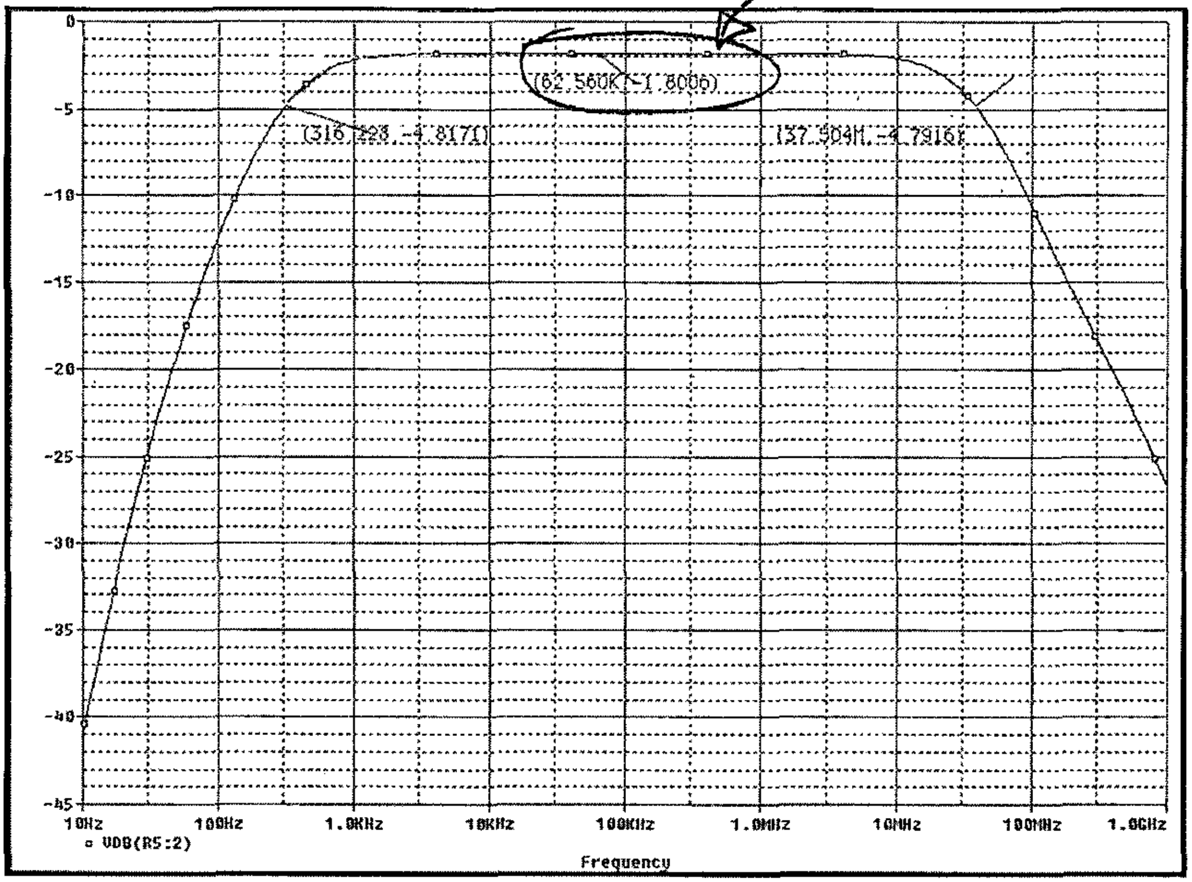
$R_L' = R_L \parallel R_E = 333\,\Omega$

$$A_V = \frac{R_L'(\beta+1)}{r_\pi + R_L'(\beta+1)} = 0.988\,V/V$$

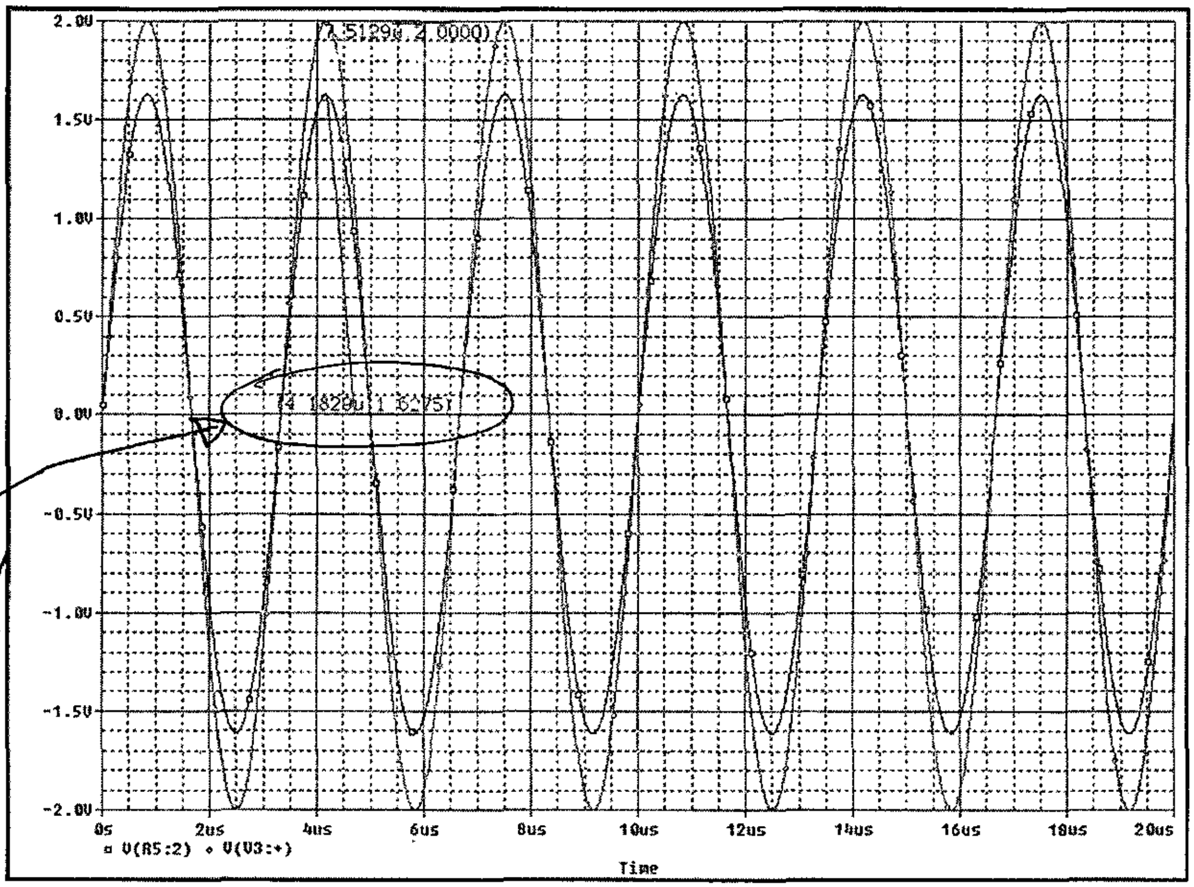
(7)

$-1.8006\,dB = 0.81277\,V/V$

b)



c)

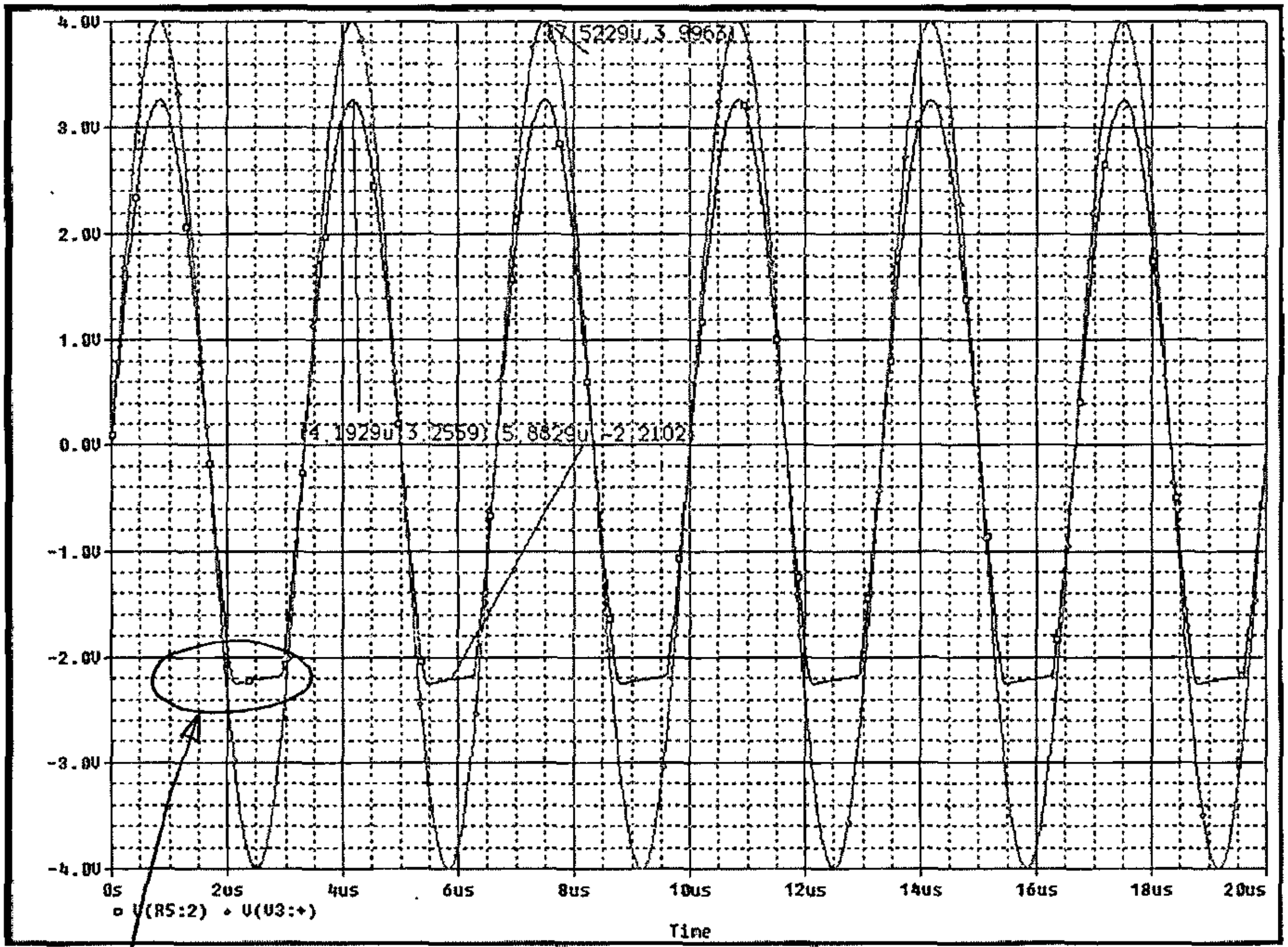


$gain = \frac{1.62}{2} = 0.81\,V/V$

it agrees with Part B

⑧

d)



distortion
happens due to limiting the currents in the load resistance (R_4) leading to decrease in gain.