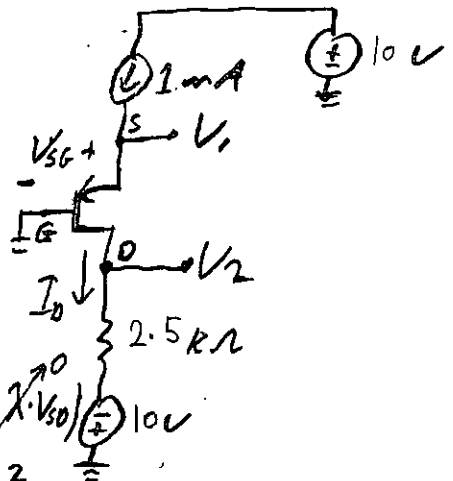


P.1 EECE 2412: HW 9 Solutions

Problem 1

- a) Assuming that the PMOS transistor is operating in the saturation region:



$$I_D = 1 \text{ mA} = \frac{1}{2} \left(\frac{\mu}{L} \right) \mu_p \cdot C_{ox} (V_{SG} - |V_{t0}|)^2 \cdot (1 + \lambda V_{SD})$$

$$1 \text{ mA} = \frac{1}{2} (1 \times 10^{-3} \frac{\text{A}}{\text{V}^2}) \cdot (V_{SG} - 1 - 1 \text{ V})^2$$

$$\hookrightarrow V_{SG} = 2.41 \text{ V} \quad (\text{or } V_{SG} = 0.41 \text{ V} \rightarrow \text{ignore})$$

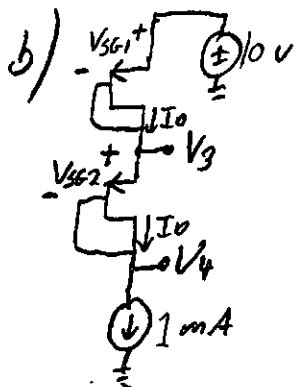
$$\text{KVL: } 0 = -V_{SG} + V_1 \rightarrow V_1 = V_{SG} = \boxed{2.41 \text{ V} = V_1}$$

$$\text{KVL: } 0 = 10 \text{ V} - I_D \cdot 2.5 \text{ k}\Omega + V_2$$

$$\hookrightarrow V_2 = (1 \text{ mA}) \cdot (2.5 \text{ k}\Omega) - 10 \text{ V} = \boxed{-7.5 \text{ V} = V_2}$$

$$\text{check: } ①: V_{SG} = 2.41 \text{ V} > |V_{t0}| = 1 \text{ V}$$

$$②: V_{SD} = V_1 - V_2 = 9.91 \text{ V} > V_{SG} - |V_{t0}| = 1 \text{ V} \quad \left. \vphantom{\begin{matrix} ②: V_{SD} = V_1 - V_2 = 9.91 \text{ V} > V_{SG} - |V_{t0}| = 1 \text{ V} \end{matrix}} \right\} \begin{matrix} \text{The PMOS} \\ \text{is in} \\ \text{saturation} \end{matrix}$$



Both transistors are diode-connected ($V_G = V_D$)

$\hookrightarrow$ both transistors are in saturation

$$I_D = 1 \text{ mA} = \frac{1}{2} (1 \frac{\text{mA}}{\text{V}^2}) \cdot (V_{SG} - |V_{t0}|)^2$$

$$V_{SG1} = V_{SG2} = V_{SG} = 2.41 \text{ V} \quad [\text{as in part a)]}$$

$$\text{KVL: } 0 = -10 \text{ V} + V_{SG1} + V_3$$

$$\hookrightarrow V_3 = 10 \text{ V} - 2.41 \text{ V} = \boxed{7.6 \text{ V} = V_3}$$

$$\text{KVL: } 0 = -10 \text{ V} + V_{SG1} + V_{SG2} + V_4$$

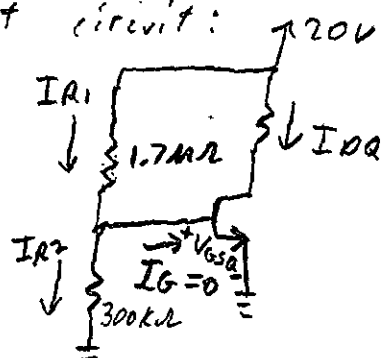
$$\hookrightarrow V_4 = 10 \text{ V} - 2 \cdot 2.41 \text{ V} = \boxed{5.18 \text{ V} = V_4}$$

(p.2) **Problem 2**

DC equivalent circuit:

a) $I_{R1} = I_{R2}$ because $I_G = 0$
(NMOS gate current ≈ 0)

$$V_{GSQ} = 20V \cdot \frac{300k\Omega}{1.7M\Omega + 300k\Omega} = 3V$$



The AC input signal is coupled to the gate with a capacitor (assumed to be a short-circuit in this AC analysis). Applying the superposition principle, the total voltage $V_{GS}(t)$ is:

$$V_{GS}(t) = V_{GSQ} + v_{in}(t) = 3V + \sin(2000\pi t) \cdot V$$

b) - c)

For the saturation region, plot the following equation vs. V_{DS} for the given V_{GS} values:

$$\textcircled{1}: I_D = K \cdot (V_{GS} - V_{th})^2 = 0.5 \frac{mA}{V^2} \cdot (V_{GS} - 1V)^2$$

For the triode region, plot the following equation vs. V_{DS} : $\textcircled{2}: I_D = K \cdot [2(V_{GS} - V_{th}) \cdot V_{DS} - V_{DS}^2]$

The load line equation from KVL is:

$$0 = -20V + I_D \cdot 2k\Omega + V_{DS}$$

→ Note: $I_D = 0 \rightarrow V_{DS} = 20V$

$$I_D = 5mA \rightarrow V_{DS} = 10V$$

Also notice from equation $\textcircled{1}$ that:

$$V_{GS} = 4V \rightarrow I_D = 4.5mA \rightarrow V_{DS} = 20V - 4.5mA \cdot 2k\Omega = 11V$$

$$V_{GS} = 3V \rightarrow I_D = 2mA \rightarrow V_{DS} = 20V - 2mA \cdot 2k\Omega = 16V$$

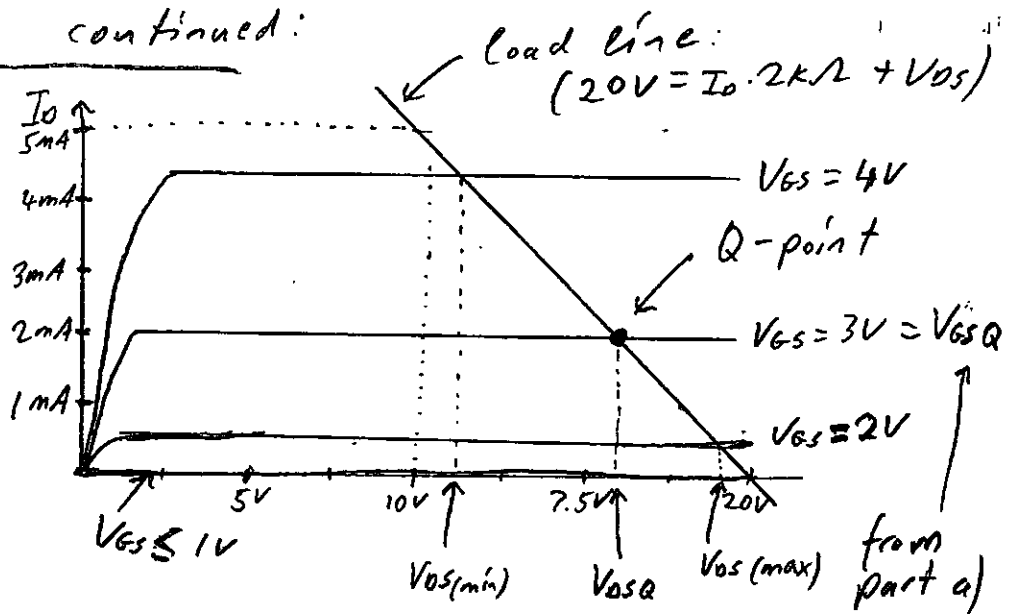
$$V_{GS} = 2V \rightarrow I_D = 0.5mA \rightarrow V_{DS} = 20V - 0.5mA \cdot 2k\Omega = 19V$$

in cutoff: $V_{GS} = 1V \rightarrow V_{GS} > V_{th} = 1V$ is not satisfied $\rightarrow I_D = 0$

(p.3)

... Prob. 2 continued:

b) - c)



d) From the above plot:

$$V_{DS(min)} \approx 11V$$

$$V_{DSQ} \approx 16V$$

$$V_{DS(max)} \approx 19V$$

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Prob. 3

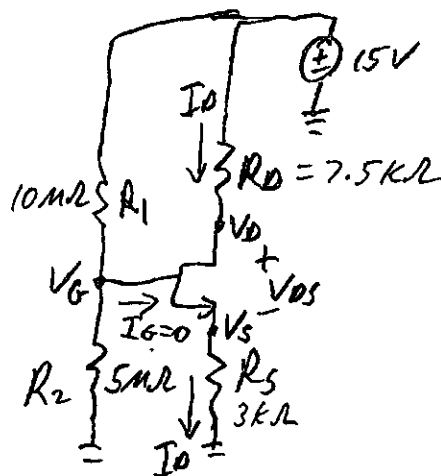
DC equivalent circuit:

a)

$$V_G = 15V \cdot \left(\frac{R_2}{R_1 + R_2} \right) = 5V$$

$$V_S = I_D \cdot R_S = I_D \cdot 3000$$

Assuming that the NMOS operates in the saturation region:



$$I_D = K \cdot (V_{GS} - V_{t0})^2 = 1 \times 10^{-3} \frac{A}{V^2} \cdot (5V - 3000 \cdot \frac{V}{A} \cdot I_D - 1V)^2$$

↳ Solving for I_D : $I_{D1} = 1.778mA$, $I_{D2} = 1mA$

with I_{D1} : $V_{GS} = 5V - 3000 \cdot I_{D1} = -0.33V < V_{t0}$
(the NMOS would not be in saturation)

with I_{D2} : $V_{GS} = 5V - 3000 \cdot I_{D2} = 2V > V_{t0} = 1V$

KVL: $0 = -15V + I_{D2} \cdot R_D + V_{DS} + I_{D2} \cdot R_S$

↳ $V_{DS} = 15V - I_{D2}(7500\Omega + 3000\Omega) = 4.5V$

$V_{DS} = 4.5V > V_{GS} - V_{t0} = 1V$

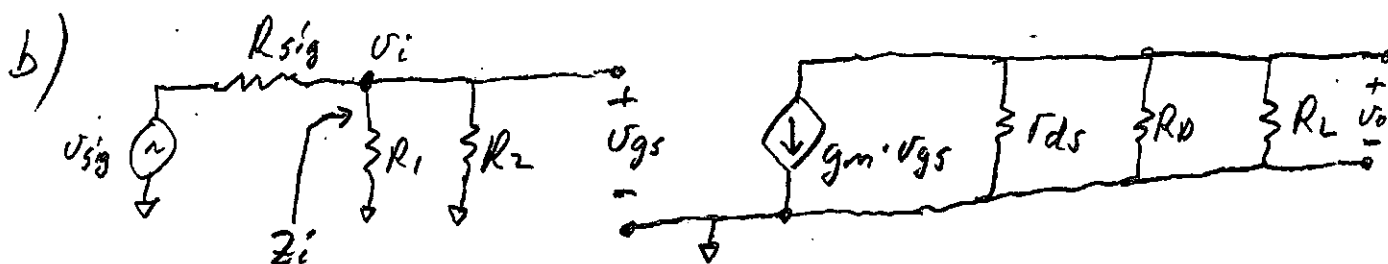
↳ The transistor is in saturation with $I_{D2} = 1mA$

$$I_{DQ} = I_{D2} = 1mA$$

$$g_m = \frac{2 \cdot I_{DQ}}{V_{GSQ} - V_{t0}} = \frac{2 \cdot (1mA)}{2V - 1V} = 2 \frac{mA}{V} = g_m$$

$$r_{ds} = \frac{V_A}{I_{DQ}} = \frac{100V}{1mA} = 100k\Omega = r_{ds}$$

Prob. 3 cont.: small-signal equivalent circuit:



$$v_i = v_{gs}, \quad v_o = -g_m \cdot v_{gs} \cdot (r_{ds} \parallel R_D \parallel R_L)$$

$$v_o = -g_m \cdot v_i \cdot (r_{ds} \parallel R_D \parallel R_L)$$

$$A_v = \frac{v_o}{v_i} = -g_m (r_{ds} \parallel R_D \parallel R_L)$$

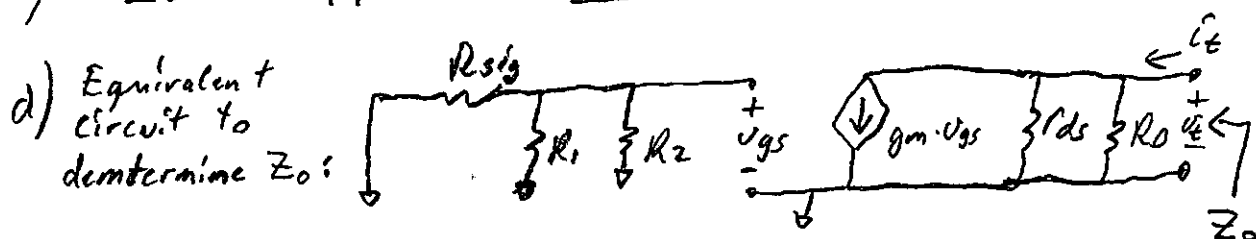
$$A_v = -2 \times 10^{-3} \cdot \left(\frac{1}{100000} + \frac{1}{7500} + \frac{1}{10000} \right)^{-1} = -8.22 = A_v$$

$$v_i = v_{sig} \cdot \left(\frac{R_1 \parallel R_2}{R_{sig} + R_1 \parallel R_2} \right) \rightarrow \frac{v_i}{v_{sig}} = \frac{R_1 \parallel R_2}{R_{sig} + R_1 \parallel R_2}$$

$$A_{vs} = \frac{v_o}{v_{sig}} = \frac{v_i}{v_{sig}} \cdot \frac{v_o}{v_i} = \frac{R_1 \parallel R_2}{R_{sig} + R_1 \parallel R_2} \cdot A_v = \frac{\left(\frac{1}{10 \times 10^6} + \frac{1}{5 \times 10^6} \right)^{-1}}{100 \times 10^3 + \left(\frac{1}{10 \times 10^6} + \frac{1}{5 \times 10^6} \right)^{-1}} \cdot (-8.22)$$

$$A_{vs} = -7.98$$

$$c) \quad Z_i = R_1 \parallel R_2 = 3.33 \text{ M}\Omega = Z_i$$



$$v_{gs} = 0 \rightarrow g_m \cdot v_{gs} = 0$$

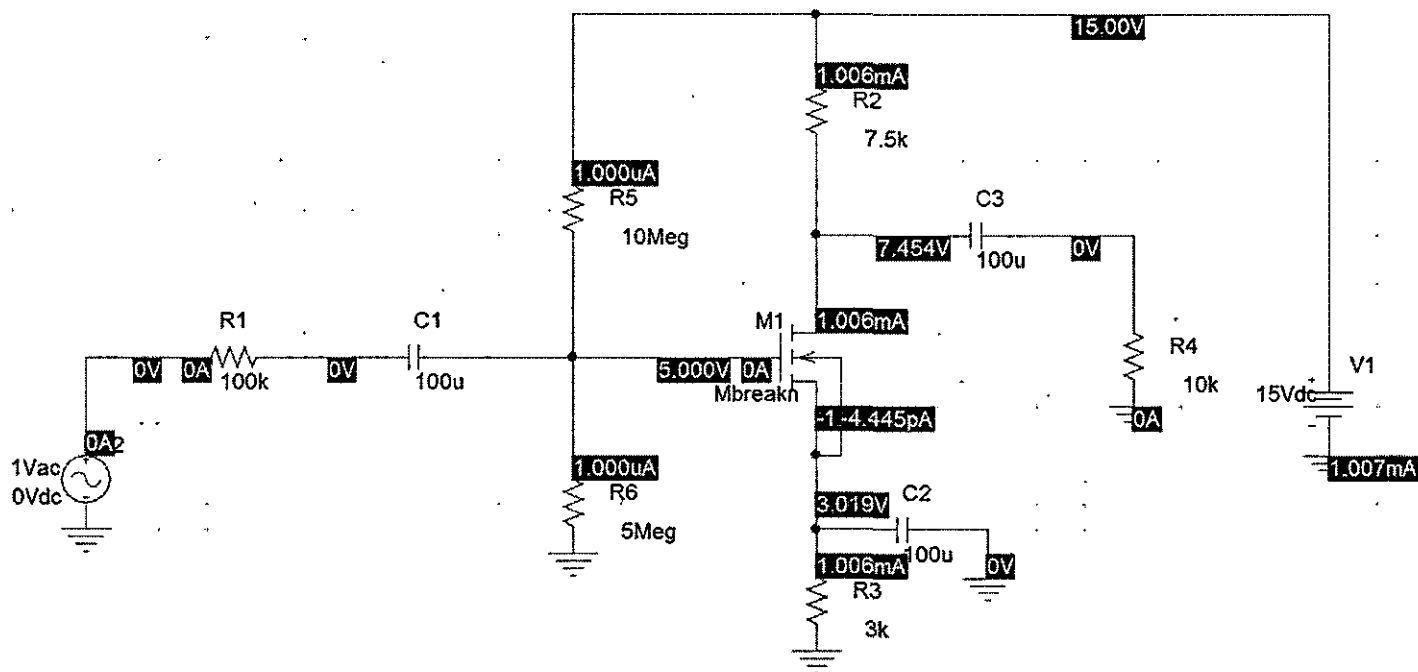
$$i_t = g_m v_{gs} + \frac{v_t}{r_{ds}} + \frac{v_t}{R_D} = \frac{v_t}{r_{ds}} + \frac{v_t}{R_D} = v_t \left(\frac{1}{r_{ds}} + \frac{1}{R_D} \right)$$

$$Z_o = \frac{v_t}{i_t} = \frac{1}{\frac{1}{r_{ds}} + \frac{1}{R_D}} = \frac{1}{\frac{1}{100 \times 10^3} + \frac{1}{7500}} = 6977 \text{ k}\Omega = Z_o$$

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Problem 4

Part a



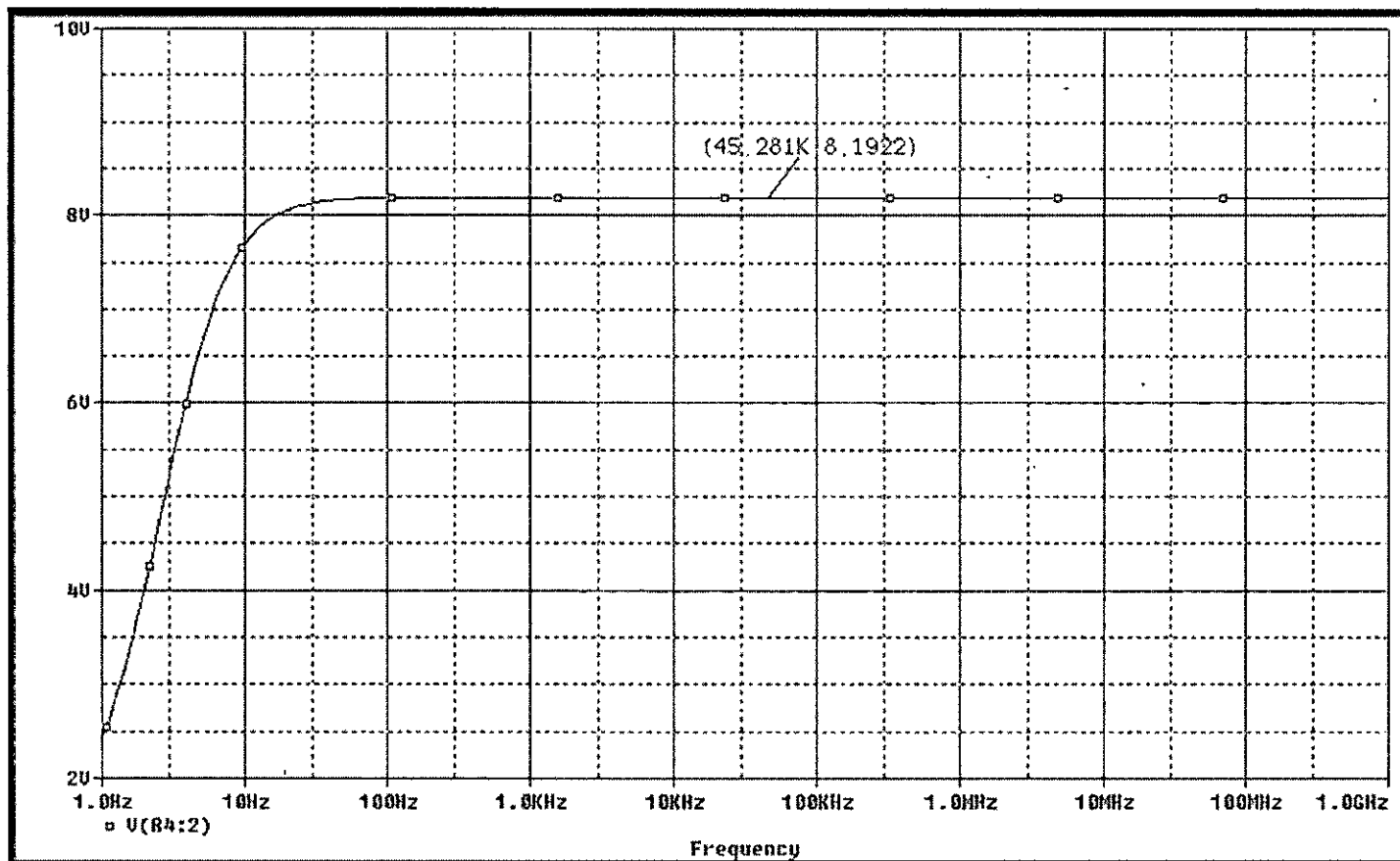
Part b

NAME	M_M1
MODEL	Mbreakn
ID	1.01E-03
VGS	1.98E+00
VDS	4.44E+00
VBS	0.00E+00
VTH	1.00E+00
VDSAT	9.82E-01
Lin0/Sat1	-1.00E+00
if	-1.00E+00
ir	-1.00E+00
TAU	-1.00E+00
GM	2.05E-03
GDS	9.63E-06
GMB	0.00E+00
CBD	0.00E+00
CBS	0.00E+00
CGSOV	0.00E+00
CGDOV	0.00E+00
CGBOV	0.00E+00
CGS	0.00E+00
CGD	0.00E+00
CGB	0.00E+00

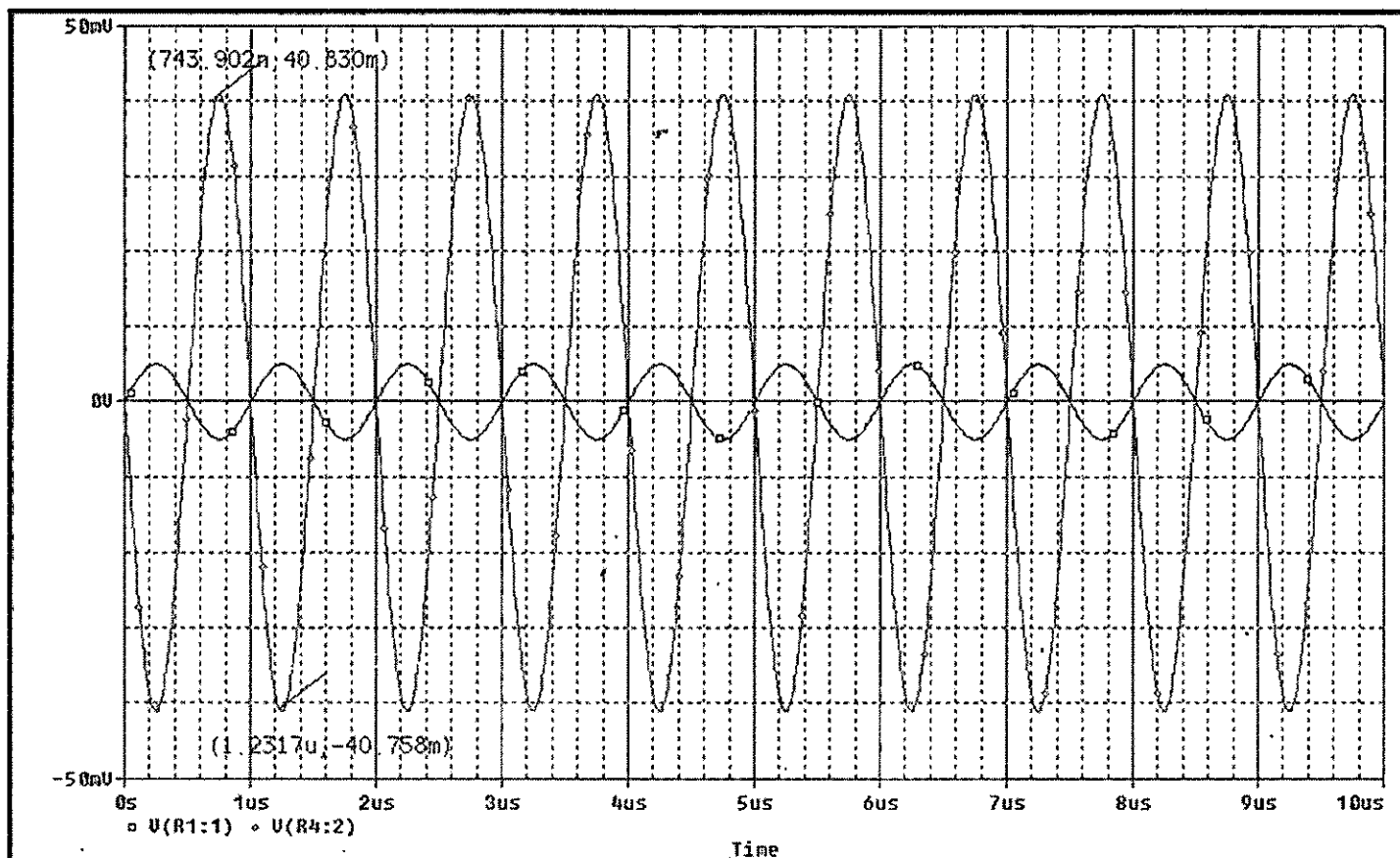
$$\rightarrow r_{ds} = \frac{1}{g_{ds}} = 103.8 K\Omega$$

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Part c



Part d



P.8

Problem 5

$$V_{in} = V_{gs1} + V_{s1}$$

$$V_o = (-g_{m1} V_{gs1} - \frac{V_o - V_{s1}}{r_{ds1}}) R_L$$

$$V_o \left(1 + \frac{R_L}{r_{ds1}}\right) = \left(\frac{V_{s1}}{r_{ds1}} - g_{m1} V_{in} + g_{m1} V_{s1}\right) R_L \quad V_i \quad g_1$$

$$V_o \left(1 + \frac{R_L}{r_{ds1}}\right) = V_{in} (-g_{m1} R_L) + V_{s1} \left(\frac{1}{r_{ds1}} + g_{m1}\right) R_L \quad (1)$$

$$V_{s1} = \left(\frac{1}{g_{m2}} \parallel r_{ds2}\right) \left(g_{m1} V_{gs1} + \frac{V_o - V_{s1}}{r_{ds1}}\right)$$

$$V_{s1} = \left(\frac{1}{g_{m2}} \parallel r_{ds2}\right) \left(g_{m1} V_{in} + \frac{V_o}{r_{ds1}} - V_{s1} \left(g_{m1} + \frac{1}{r_{ds1}}\right)\right)$$

$$V_{s1} \left(1 + \left(\frac{1}{g_{m2}} \parallel r_{ds2}\right) \left(g_{m1} + \frac{1}{r_{ds1}}\right)\right) = \left(\frac{1}{g_{m2}} \parallel r_{ds2}\right) \left(g_{m1} V_{in} + \frac{V_o}{r_{ds1}}\right)$$

$$V_{s1} = \frac{\left(\frac{1}{g_{m2}} \parallel r_{ds2}\right) \left(g_{m1} V_{in} + \frac{V_o}{r_{ds1}}\right)}{1 + \left(\frac{1}{g_{m2}} \parallel r_{ds2}\right) \left(g_{m1} + \frac{1}{r_{ds1}}\right)}$$

Substitute in (1)

$$V_o \left(1 + \frac{R_L}{r_{ds1}}\right) = V_{in} (-g_{m1} R_L) + \frac{R_L \left(\frac{1}{r_{ds1}} + g_{m1}\right) \left(\frac{1}{g_{m2}} \parallel r_{ds2}\right) \left(g_{m1} V_{in} + \frac{V_o}{r_{ds1}}\right)}{1 + \left(\frac{1}{g_{m2}} \parallel r_{ds2}\right) \left(g_{m1} + \frac{1}{r_{ds1}}\right)}$$

$$V_o \left(1 + \frac{R_L}{r_{ds1}} - \frac{R_L}{r_{ds1}} \frac{X}{1+X}\right) = V_{in} \left(-g_{m1} R_L + g_{m1} R_L \frac{X}{X+1}\right)$$

$$V_o \left(1 + \frac{R_L}{r_{ds1}(1+X)}\right) = V_{in} \left(\frac{-g_{m1} R_L}{1+X}\right)$$

$$V_o \left(1 + X + \frac{R_L}{r_{ds1}}\right) = -V_{in} g_{m1} R_L$$

$$\frac{V_o}{V_{in}} = \frac{-g_{m1} R_L}{1 + X + \frac{R_L}{r_{ds1}}} = \frac{-g_{m1} r_{ds1} R_L (1 + g_{m2} r_{ds2})}{r_{ds1} + r_{ds2} + R_L + g_{m1} r_{ds1} r_{ds2} + g_{m2} r_{ds2} (r_{ds1} + R_L)}$$

