Fundamentals of Electronics
Week 8

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EECE–2412
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Oct 2024

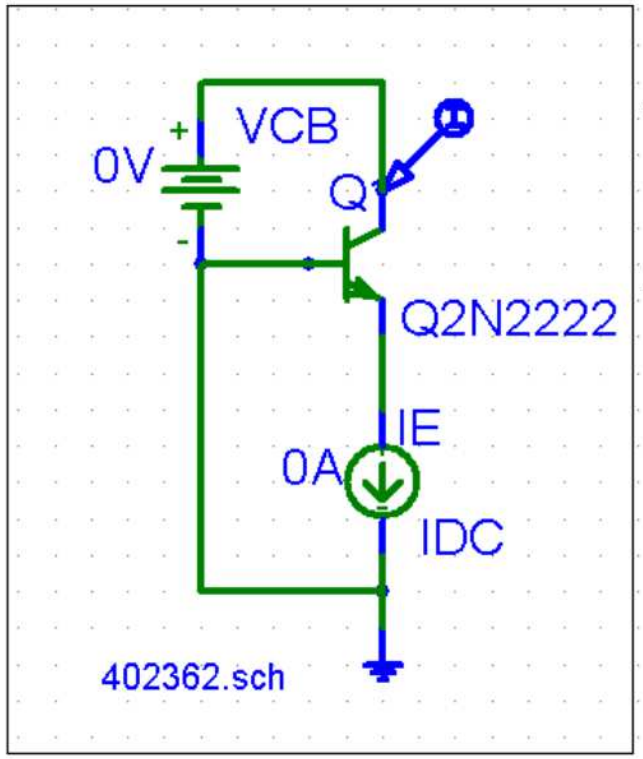
Agenda

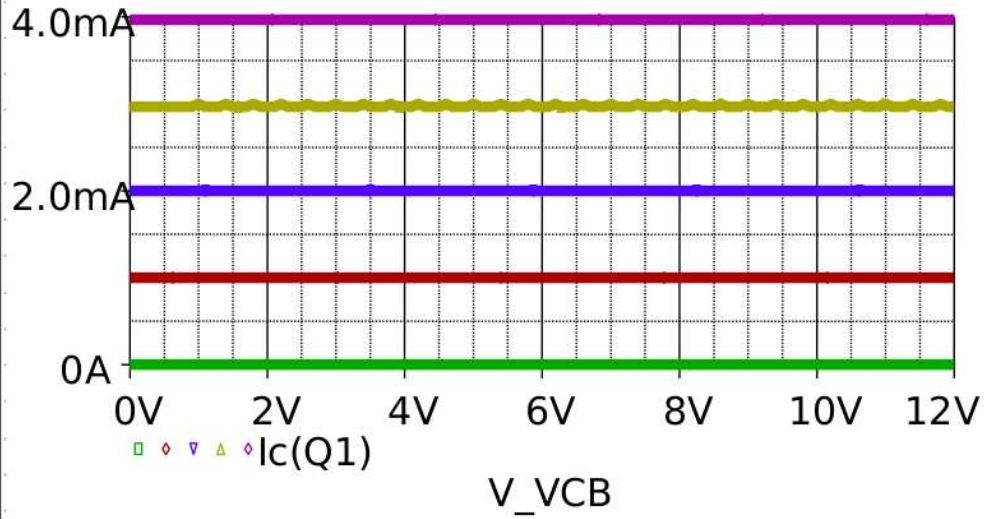
- Graphical Representation
- Early Voltage
- DC Circuits with Transistors
- A Transistor Amplifier
- Small Signal Models
  - Analysis
  - AC Pi and Tee Models

Graphical Analysis Concepts

- Three Connections
- Two Independent Parameters
  "Parametric" in PSPICE
- Multiple Choices for Configuration
  Some more useful than others

Emitter Current as Parameter



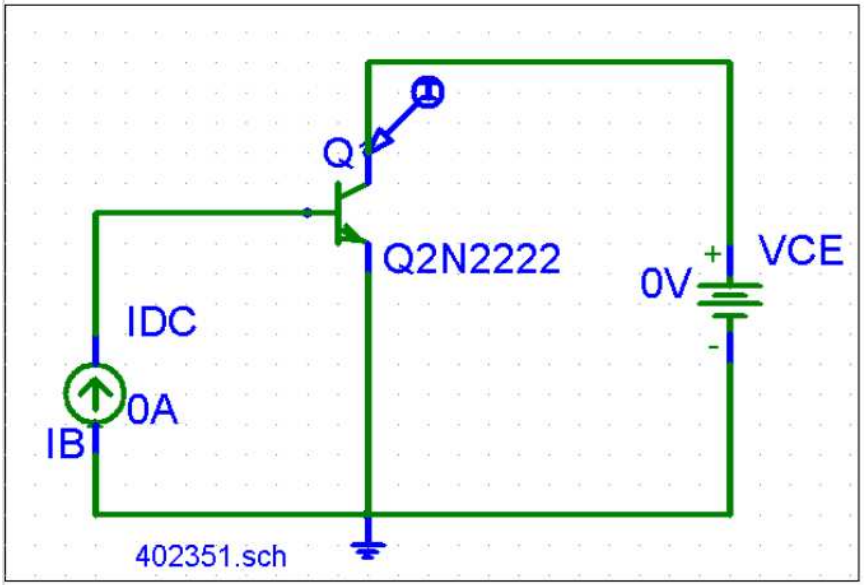


i_E=*[0:1:4]* mA

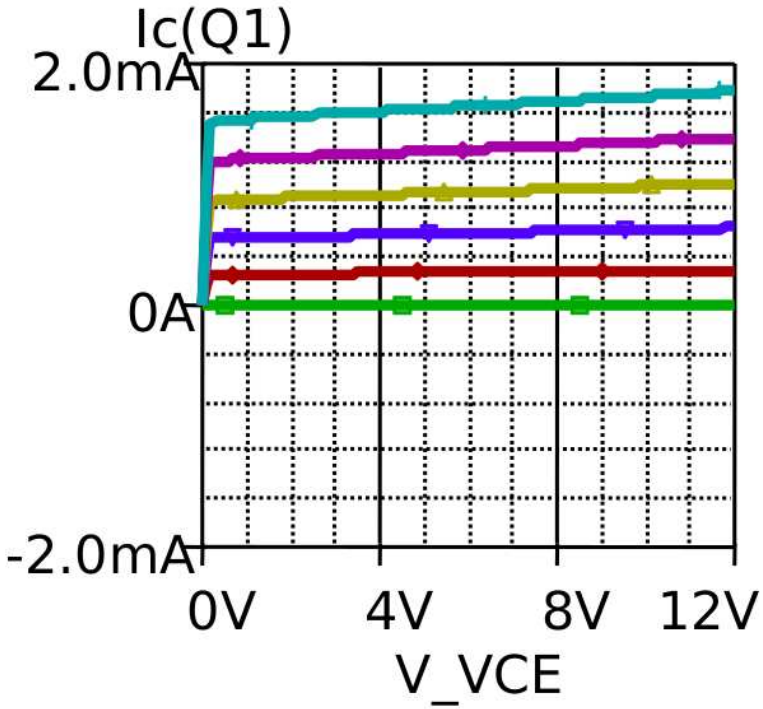
Collector I–V Curves

•Ideal Equation

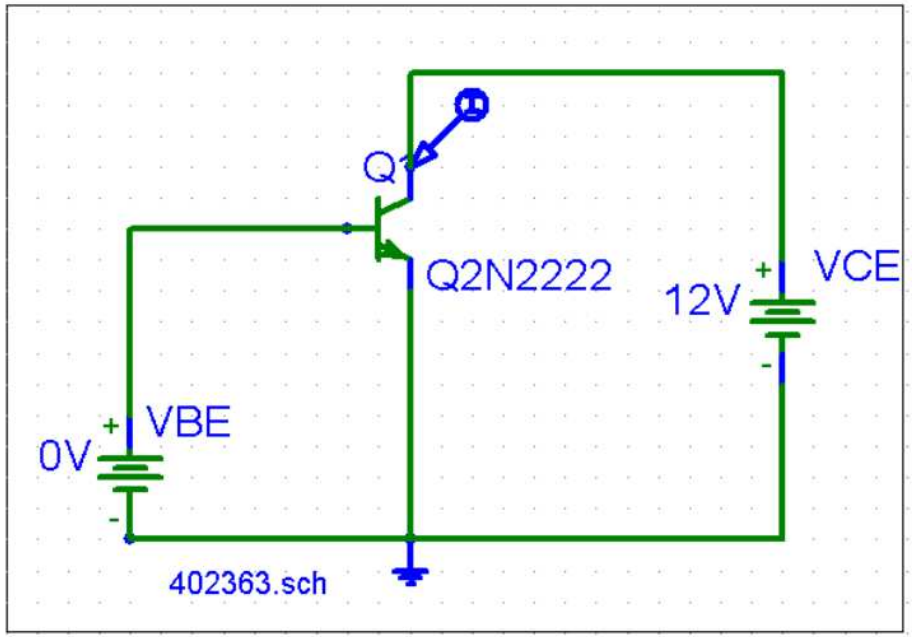
$$I_C = I_S e^{v_{BE}/V_T} = \beta I_B$$

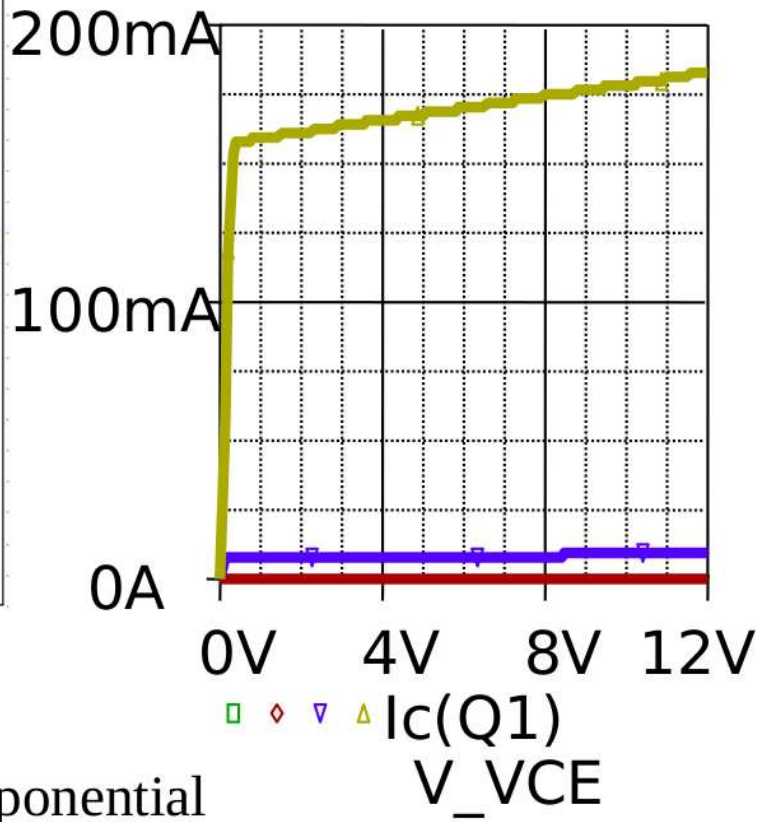


- I-V Curves
  - i_B=[0:2:10] microamperes



Another Set of I–V Curves

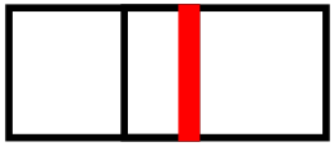




V_{BE}=[0.6,0.7,0.8]

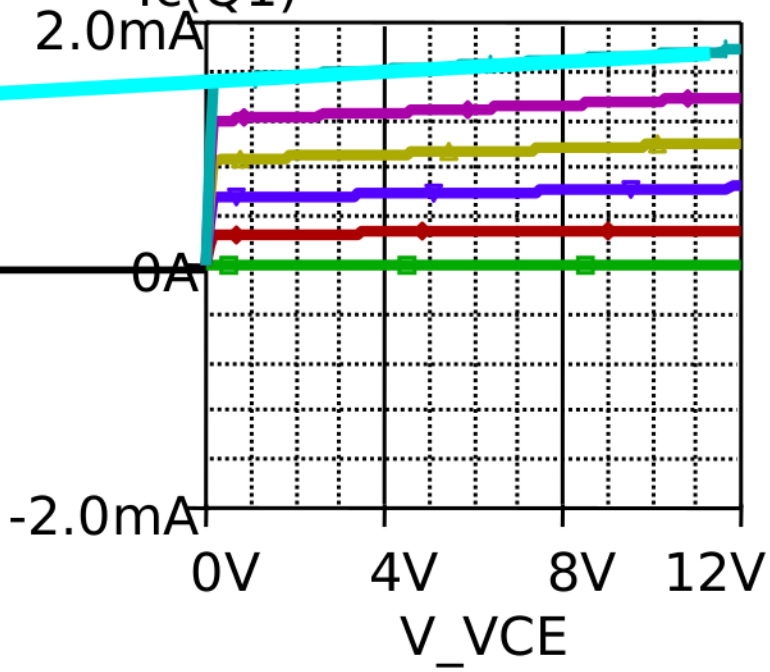
Not Easy to Use because of exponential

The Early Voltage



- V_{CE} (Reverse) Increases the Depletion Width in the CBJ,
- Base Width Decreases,
- Minority Carrer Gradient in Base Increases

- I-V Curves
  - i_B=[0:2:10] microamperes

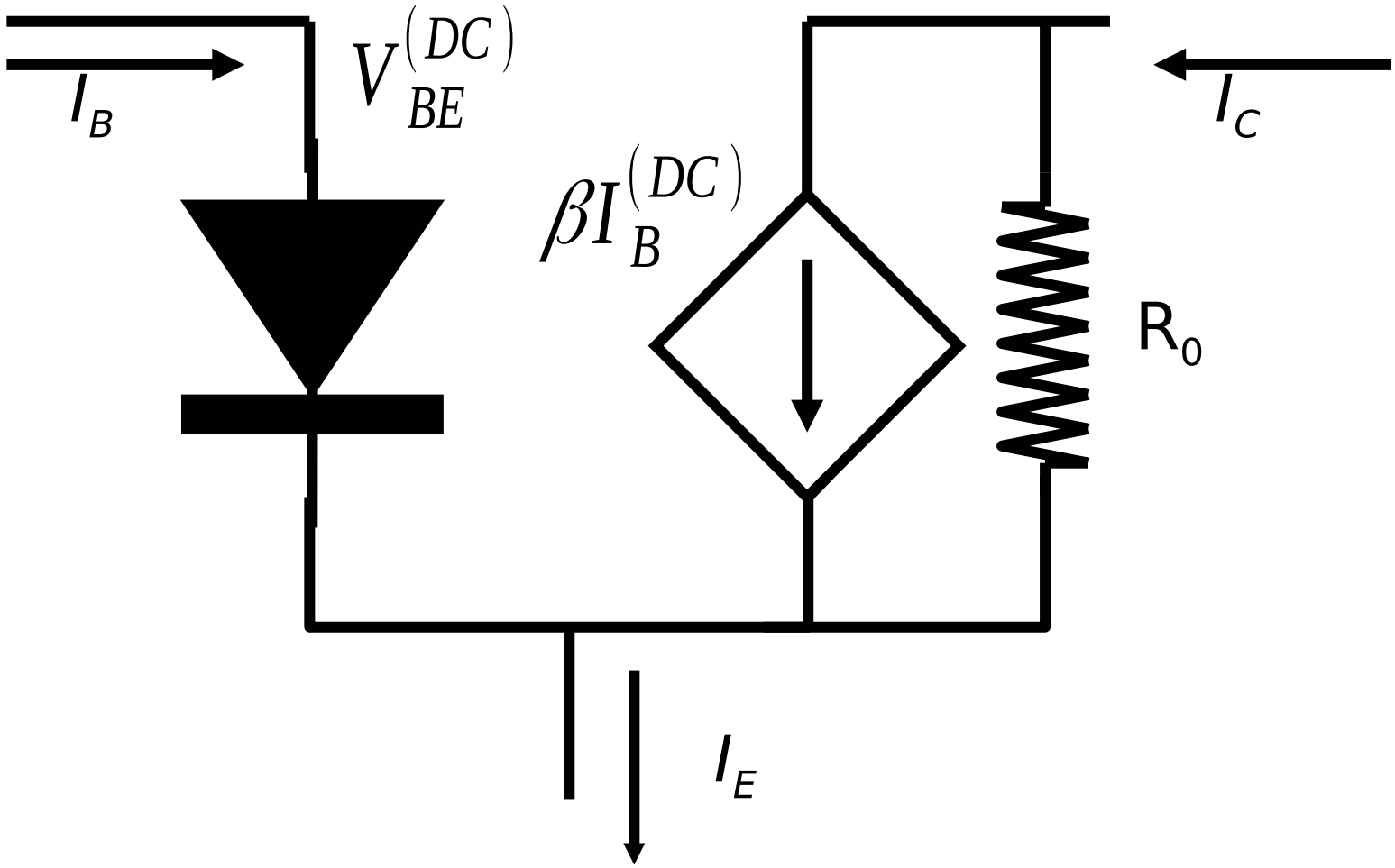


$-V_A$

- Early Voltage = V_A
- Typically 50 to 100 V

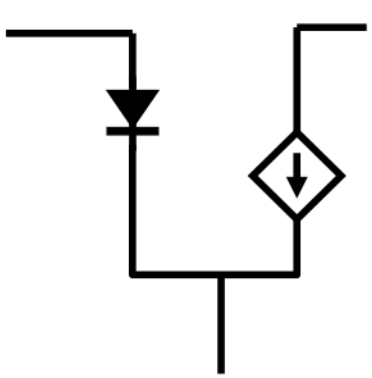
$$i_C = I_S e^{v_{BE}/V_T}\left(1+\frac{v_{CE}}{V_A}\right)$$

The "Early Resistor," R_0



DC Circuits with Transistors

- Transistor



$$i_C = \beta i_B$$

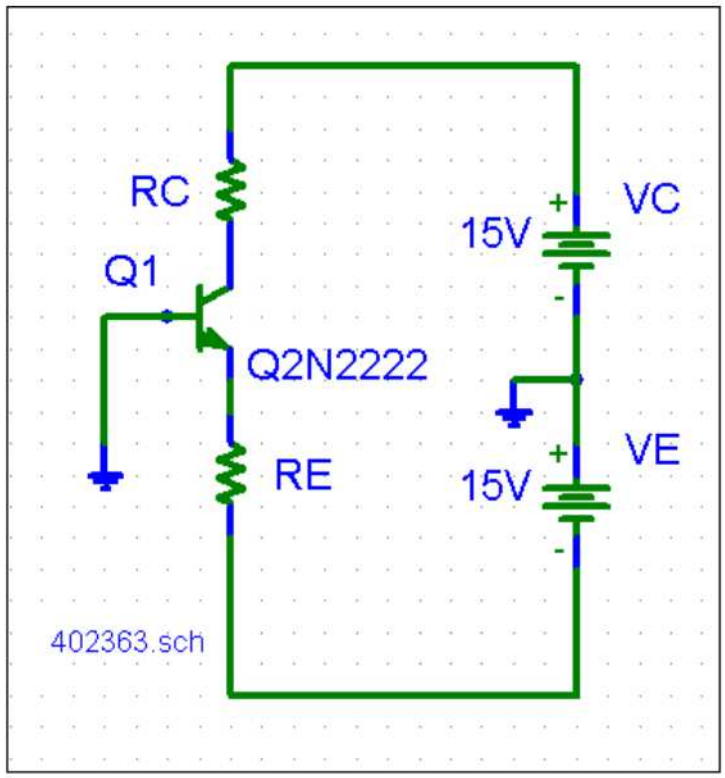
$$i_E = i_C + i_B$$

$$i_B = \frac{I_S}{\beta} e^{v_{BE}/V_T}$$

- External Circuit
  - Usual Rules
  - Thevenin or Norton

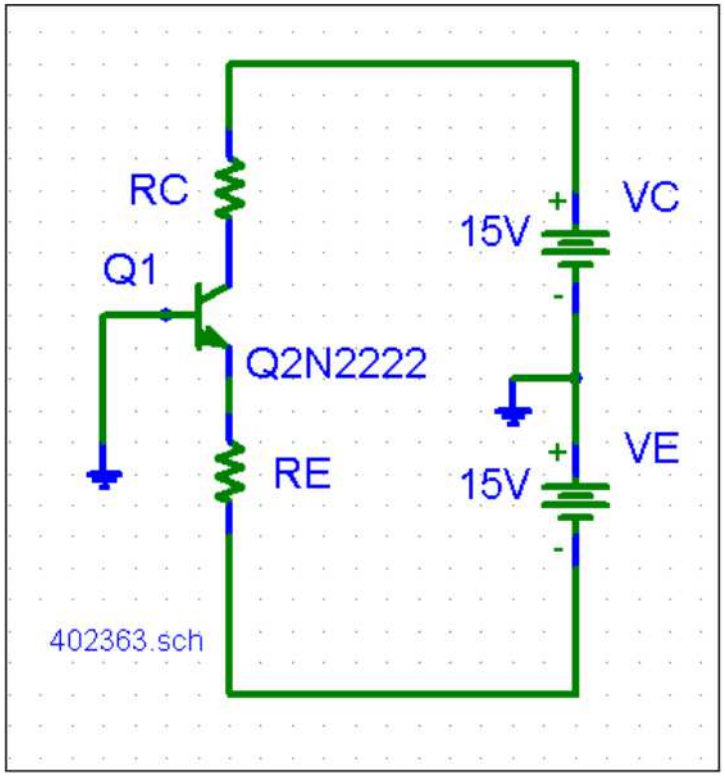
Then Verify Active Region (Fwd EBJ, Rev CBJ)

Example: Ex. 4.1 in Text (1)



- Given
  - v_{BE}=0.7 V at i_c=1mA
  - β=100
- Want
  - 2mA through Collector
  - 5 V at Collector

An Example (2)



1

$$R_C = \frac{15\text{V} - 5\text{V}}{2\text{mA}} = 5\text{k}$$

2

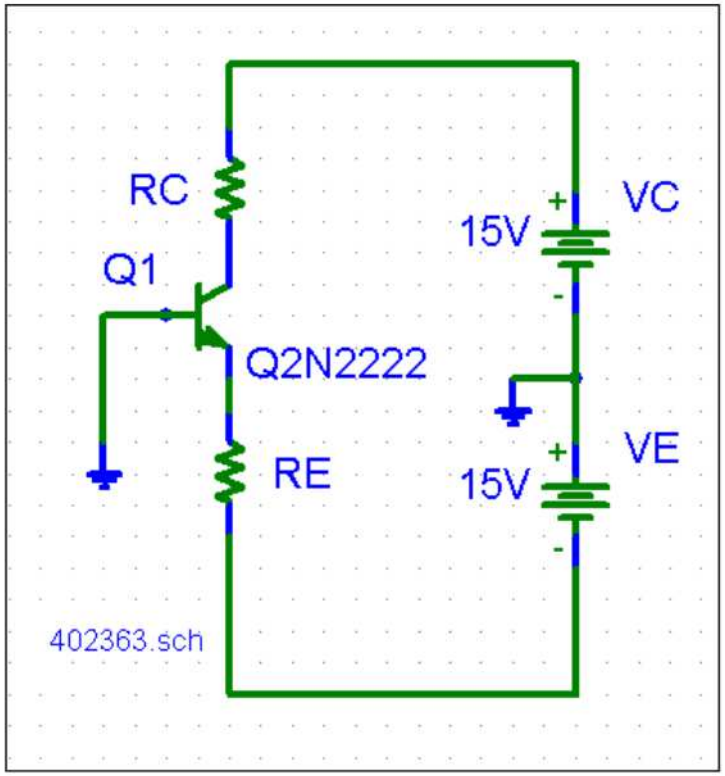
$$V_{BE} = 0.7 + V_T \ln\left(\frac{2}{1}\right) = 0.717\text{V}$$

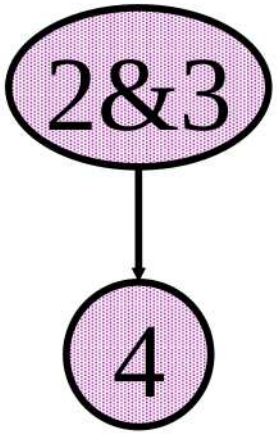
$$V_E = -0.717\text{V}$$

3

$$I_E = \frac{I_C}{\alpha} = I_C \frac{\beta + 1}{\beta} = 2.02\text{mA}$$

An Example (3)





$$V_E = -0.717\text{V}$$

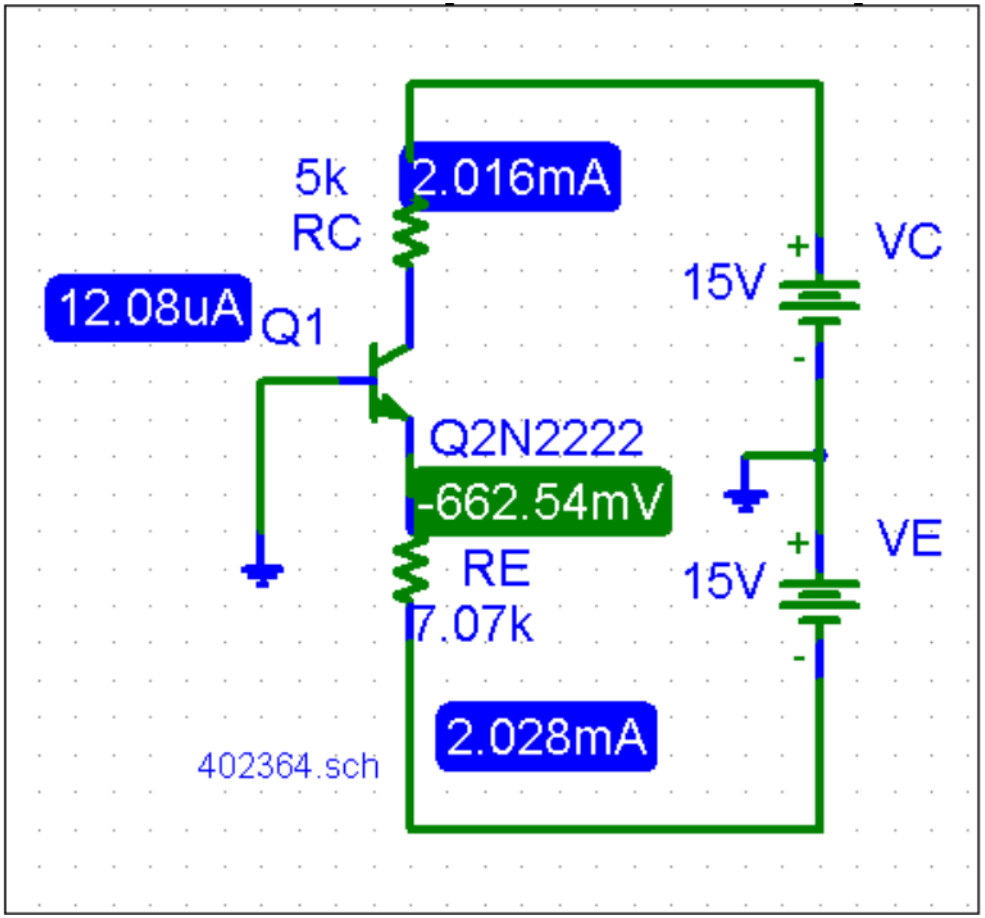
$$I_E = 2.02\text{mA}$$

$$R_E = \frac{-V_E - (-15\text{V})}{I_E}$$

$$= \frac{-0.717 - (-15\text{V})}{2.02\text{mA}}$$

$$= 7.07\text{k}$$

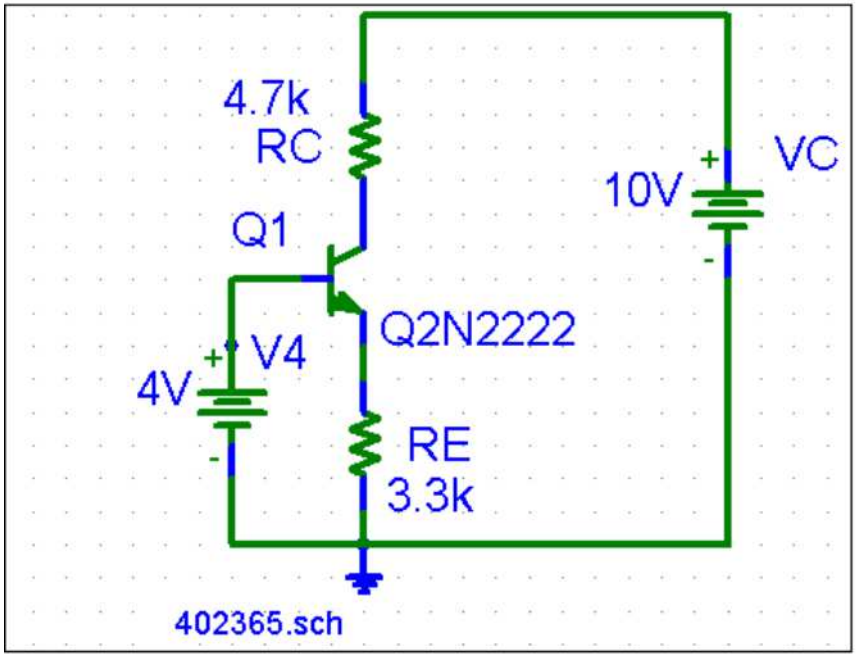
Example: Spice Comparison



Not the Same Transistor so values are not exact.

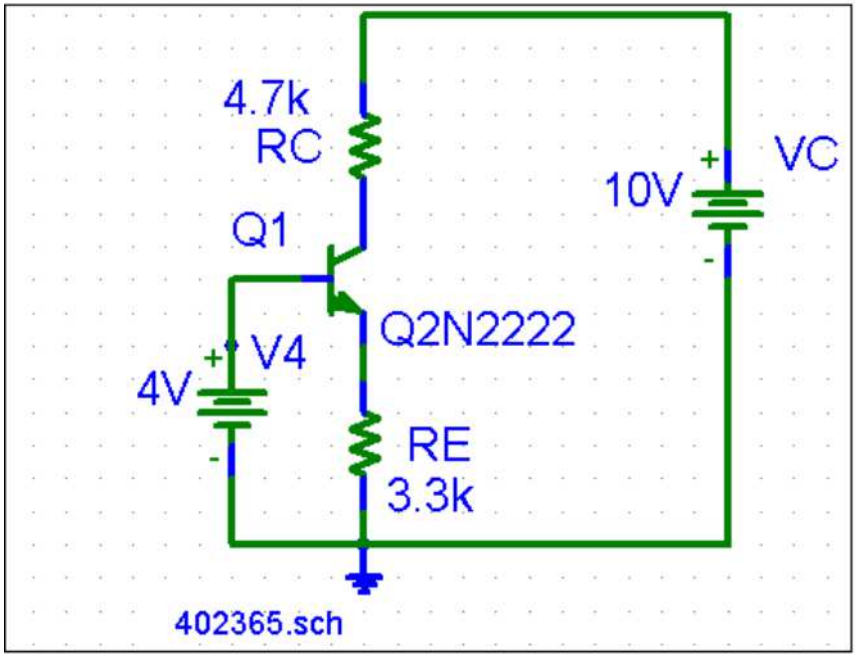
- Given Originally
  - v_{BE}=0.7 V at i_c=1mA
  - β=100
- Wanted
  - 2mA through Collector
  - 5 V at Collector

Another Example: 4.2 (1)



- Given
  - β=100
- Find Voltages and Currents

Another Example: 4.2 (2)



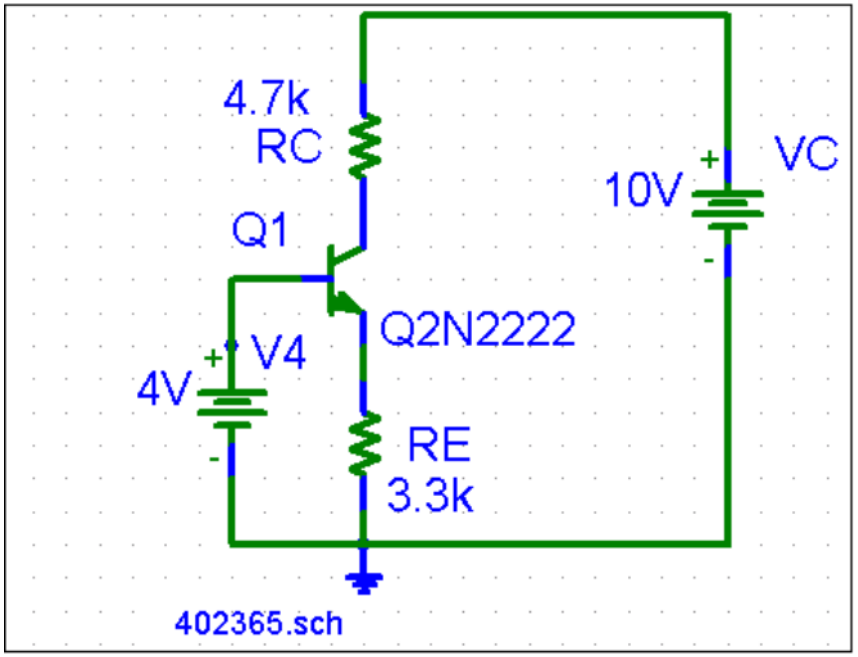
1. Assume $v_{BE} = 0.7\text{V}$

   $v_E = 3.3\text{V}$

2. $i_E = 1\text{mA}$

3. $i_C = \alpha i_E = 0.99\text{mA}$

Another Example: 4.2 (3)



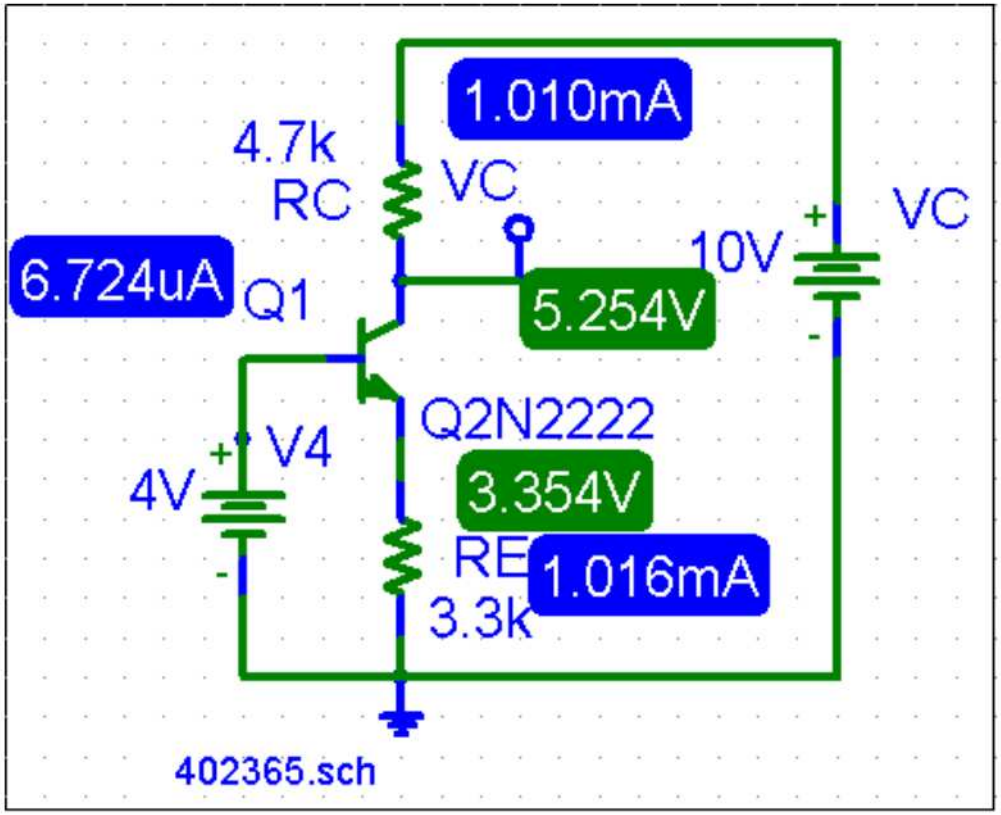
4 4.7 V across RC

$$v_C = 5.3\text{V}$$

5 $$i_B = i_C / \beta = 0.01\text{mA}$$

6 Check for Active Mode
Fwd EBJ (0.7V)
Rev CBJ (1.3V)

Another Example: 4.2 (4)

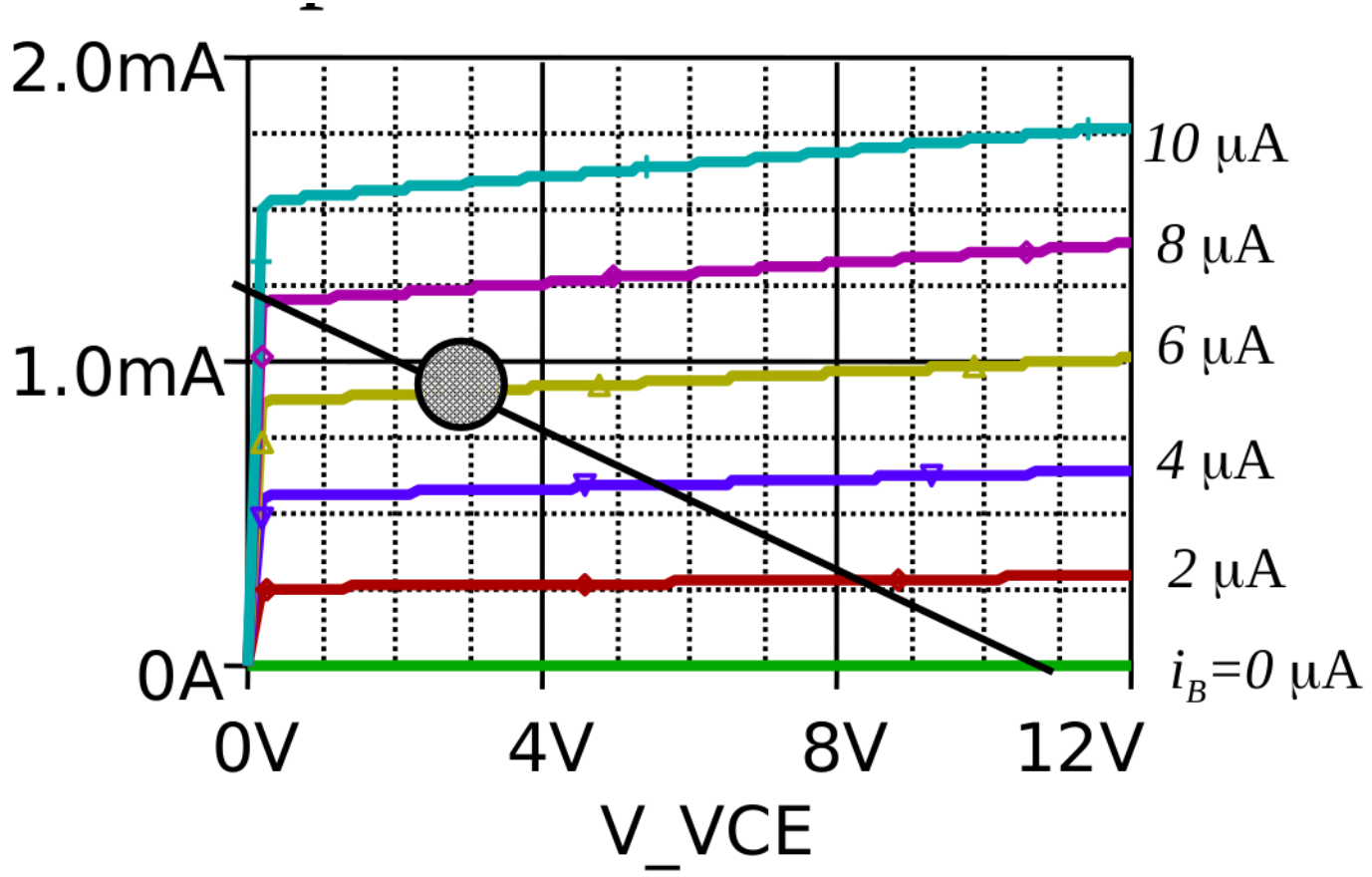


Different Transistor,
But "Not Bad."

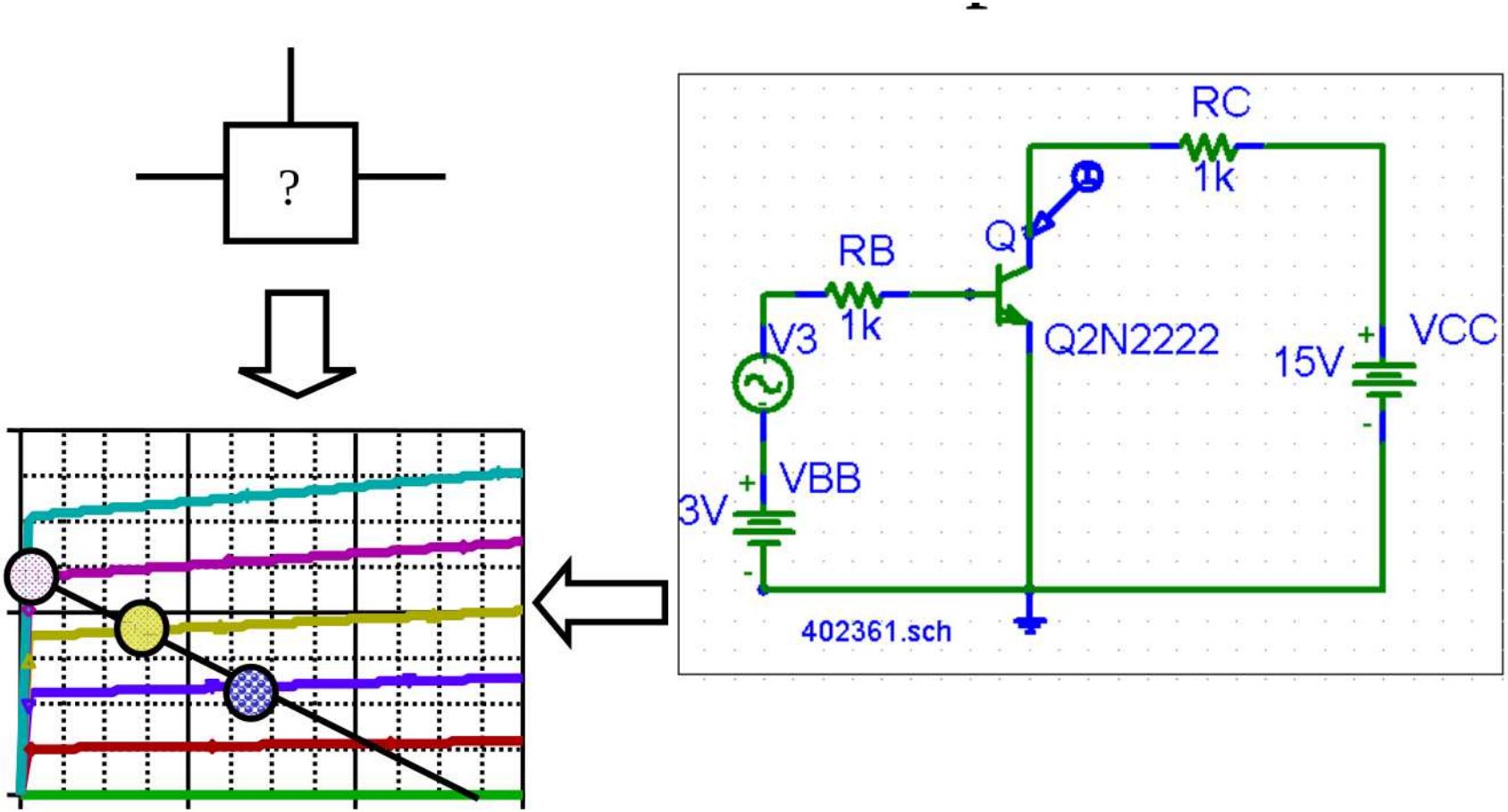
$v_C = 5.3\text{V}$

$i_B = i_C / \beta = 0.01\text{mA}$

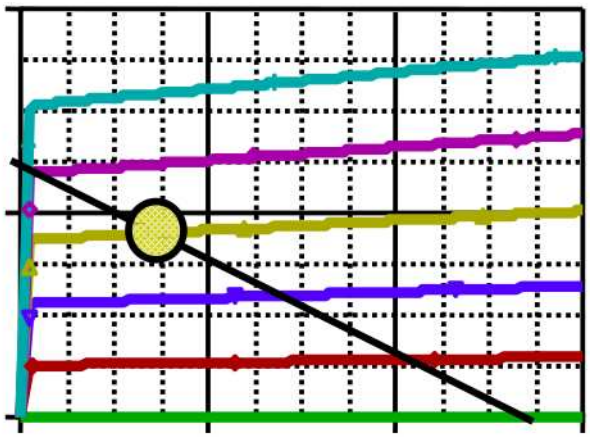
Graphical Methods

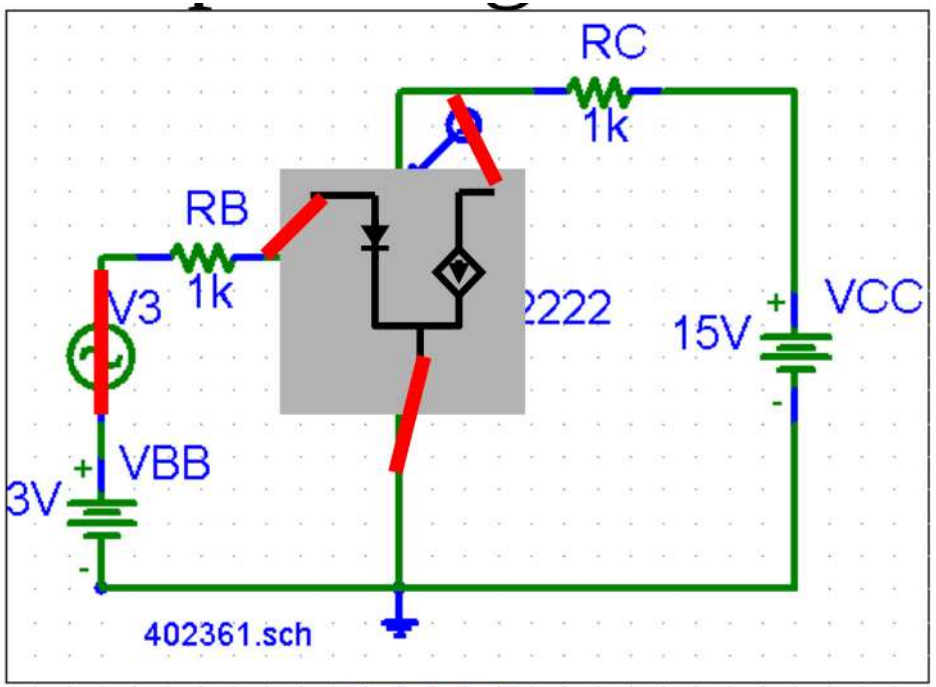


Transistors in Amplifiers



DC Model for Operating Point





$$I_C^{(DC)} = I_S e^{v_{BE}^{(DC)}/V_T}$$

$$I_B^{(DC)} = I_C^{(DC)} / \beta$$

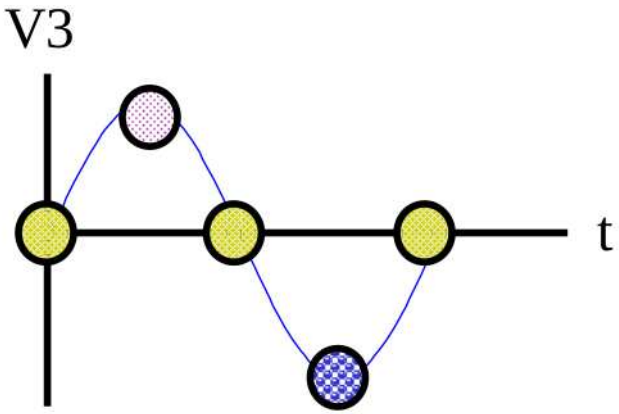
$$I_E^{(DC)} = I_C^{(DC)} / \alpha$$

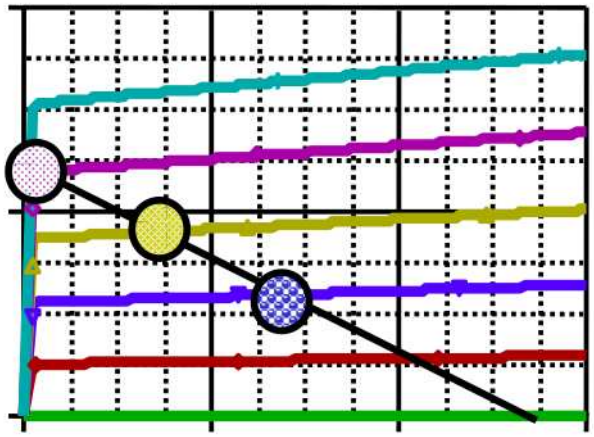
Five Unknowns

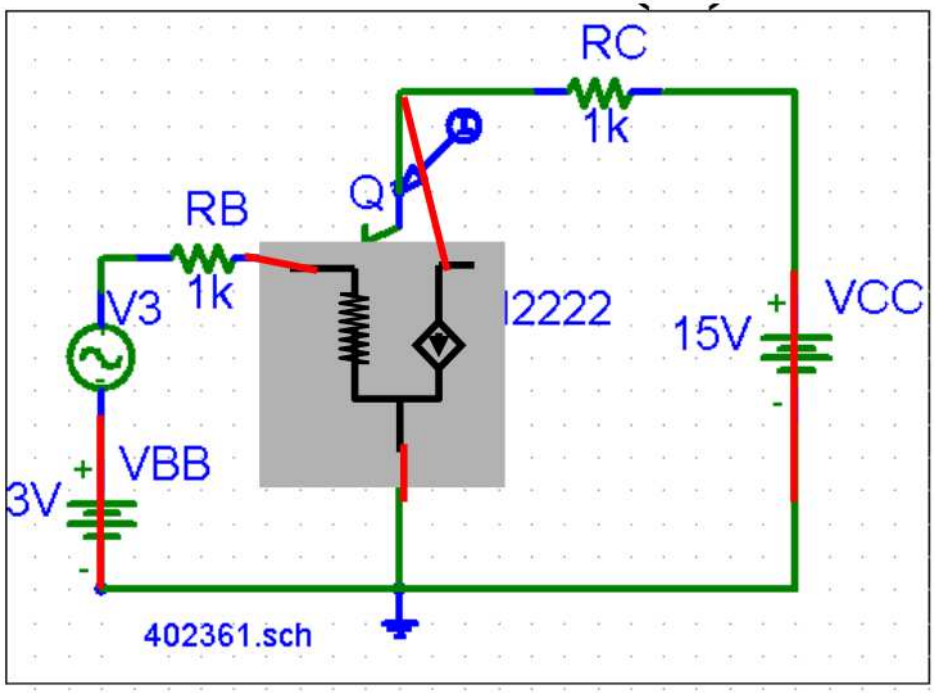
$$V_C^{(DC)} = V_{CE}^{(DC)} = V_{CC} - I_C^{(DC)} R_C$$

$$V_B^{(DC)} = V_{BE}^{(DC)} = V_{BB} - I_B^{(DC)} R_B$$

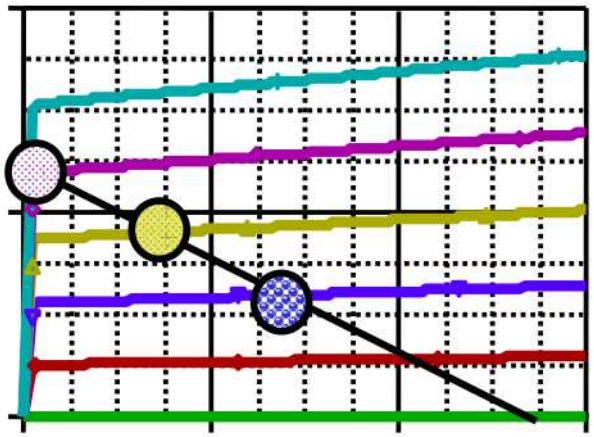
A Look Ahead: AC Model (1)

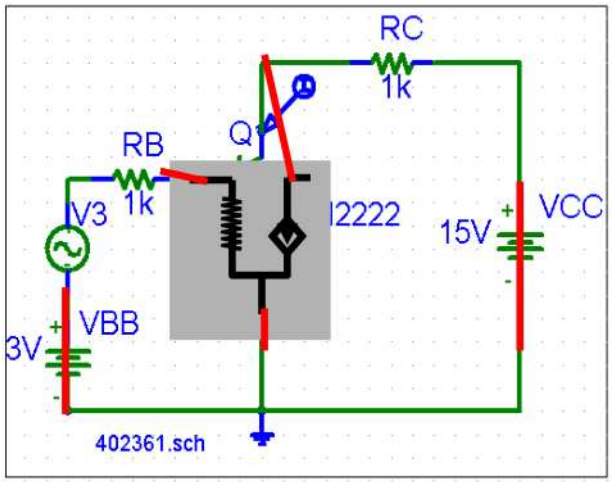






A Look Ahead: AC Model (1)





$$i_c^{(AC)} = \beta i_B^{(AC)}$$

$$i_e^{(AC)} = i_c^{(AC)} / \alpha$$

$$i_b^{(AC)} = \frac{v_3^{(AC)}}{R_B + r_\pi}$$

$$v_c^{(AC)} = - i_c^{(AC)} R_C$$

$$v_b^{(AC)} = i_b^{(AC)} r_\pi$$

April 2001 Chuck DiMarzio, Northeastern University 4023-5-43

Small–Signal (AC) Model

- Calculate Parameters by Taylor Series
- Draw AC Equivalent Circuit
- Parameters Depend on DC Operating Point
  - Transconductance, g
  - Base Parameters, β, r_π
  - Emitter Parameters, α, r_e
  - Voltage Gain

Transconductance (1)

$$v_{BE} = V_{BE}^{(DC)} + v_{be}^{(DC)}$$

$$i_C = I_S e^{v_{BE}/V_T} = I_S e^{\left(V_{BE}^{(DC)} + v_{be}^{(AC)}\right)/V_T}$$

$$i_C = I_S e^{V_{BE}^{(DC)}/V_T} e^{v_{be}^{(AC)}/V_T}$$

$$i_C = I_C^{(DC)} e^{v_{be}^{(AC)}/V_T}$$

$$i_C \approx I_C^{(DC)} \left[1 + \frac{v_{be}^{(AC)}}{V_T} + \frac{1}{2!}\left(\frac{v_{be}^{(AC)}}{V_T} \right)^2 + \cdots \right]$$

Transconductance (2)

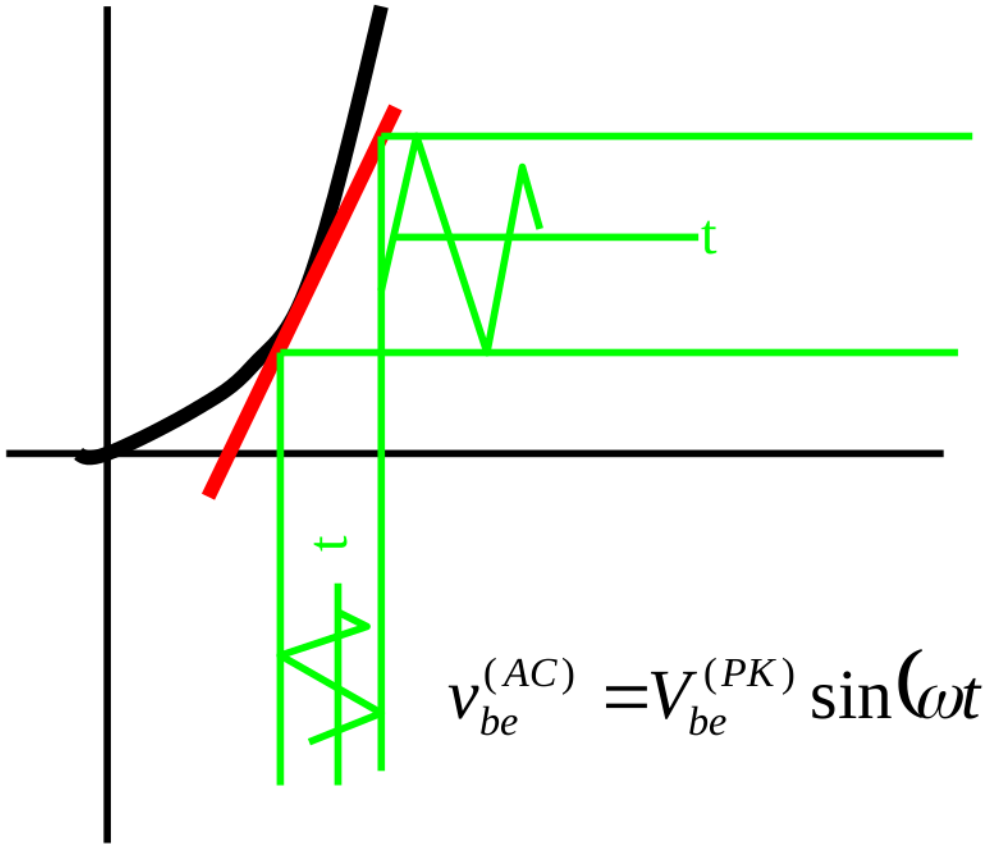
$$i_C \approx I_C^{(DC)}\left[1+\frac{v_{be}^{(AC)}}{V_T}+\frac{1}{2!}\left(\frac{v_{be}^{(AC)}}{V_T}\right)^2+\cdots\right]$$

$$i_C \approx I_C^{(DC)}+I_C^{(DC)}\frac{v_{be}^{(AC)}}{V_T}$$

$$i_c^{(AC)} = I_C^{(DC)}\frac{v_{be}^{(AC)}}{V_T} = \frac{I_C^{(DC)}}{V_T}v_{be}^{(AC)} = g_m v_{be}^{(AC)}$$

$$g_m = \frac{I_C^{(DC)}}{V_T}$$

Transconductance Graphically



$$i_c^{(AC)} = I_c^{(PK)} \sin(\omega t)$$

$$i_c^{(AC)} = g_m v_{be}^{(AC)}$$

$$I_c^{(PK)} = g_m V_{be}^{(PK)}$$

$$v_{be}^{(AC)} = V_{be}^{(PK)} \sin(\omega t)$$

Base Parameters

$$i_B = i_c / \beta$$

$$I_B^{(DC)} = I_C^{(DC)} / \beta \qquad i_b^{(AC)} = i_c^{(AC)} / \beta$$

$$i_c^{(AC)} = g_m v_{be}^{(AC)}$$

$$i_b^{(AC)} = g_m v_{be}^{(AC)} / \beta$$

$$i_b^{(AC)} = (g_m / \beta) v_{be}^{(AC)}$$

$$i_b^{(AC)} = v_{be}^{(AC)} / r_\pi \qquad r_\pi = \beta / g_m$$

Emitter

$$i_E = i_C / \alpha$$

$$I_E^{(DC)} = I_C^{(DC)} / \alpha \qquad i_e^{(AC)} = i_c^{(AC)} / \alpha$$

$$i_e^{(AC)} = \frac{I_C^{(DC)} v_{be}^{(AC)}}{\alpha V_T} = \frac{I_E^{(DC)}}{V_T} v_{be}^{(AC)}$$

$$i_e^{(AC)} = \frac{1}{r_e} v_{be}^{(AC)} \qquad r_e = \frac{V_T}{I_E^{(DC)}}$$

Note That

$$r_\pi = (\beta + 1) r_e$$

Voltage Gain

$$v_c = V_{CC} - i_c R_C$$

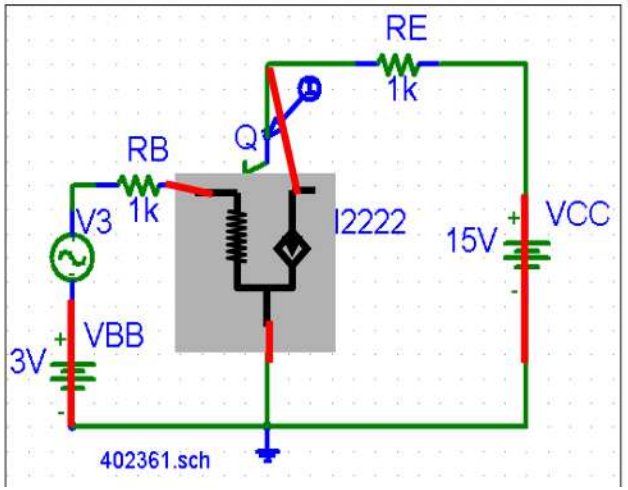
$$V_C^{(DC)} + v_c^{(AC)} = V_{CC} - I_C^{(DC)} R_C - i_c^{(AC)} R_C$$

$$V_C^{(DC)} = V_{CC} - I_C^{(DC)} R_C$$

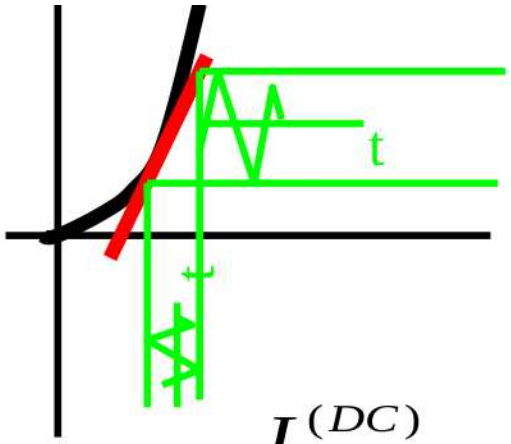
$$v_c^{(AC)} = - i_c^{(AC)} R_C$$

$$v_c^{(AC)} = - g_m R_C v_{be}^{(AC)}$$

$$v_c^{(AC)} = g_v v_{be}^{(AC)} \qquad g_v = - g_m R_C$$



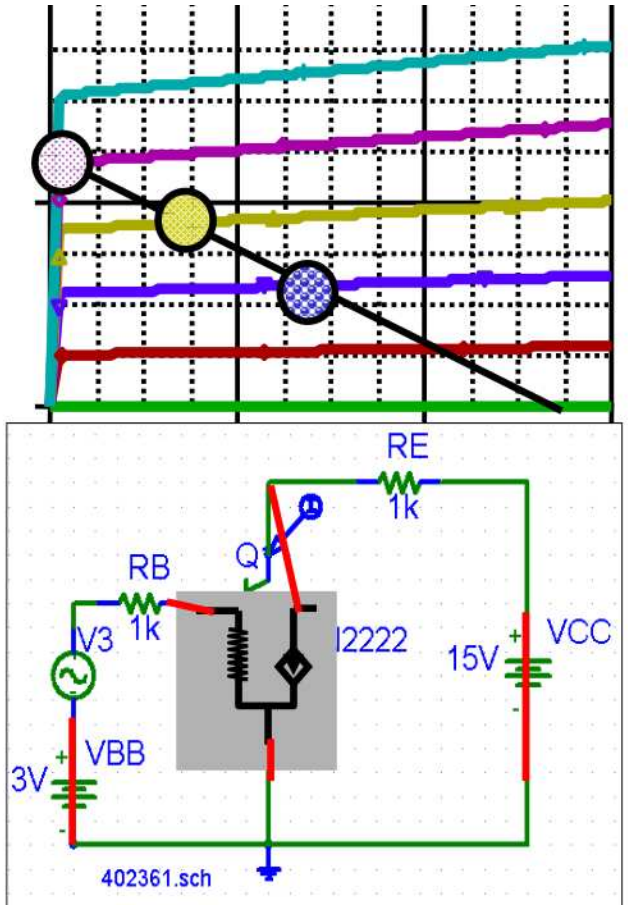
Small–Signal Model Summary



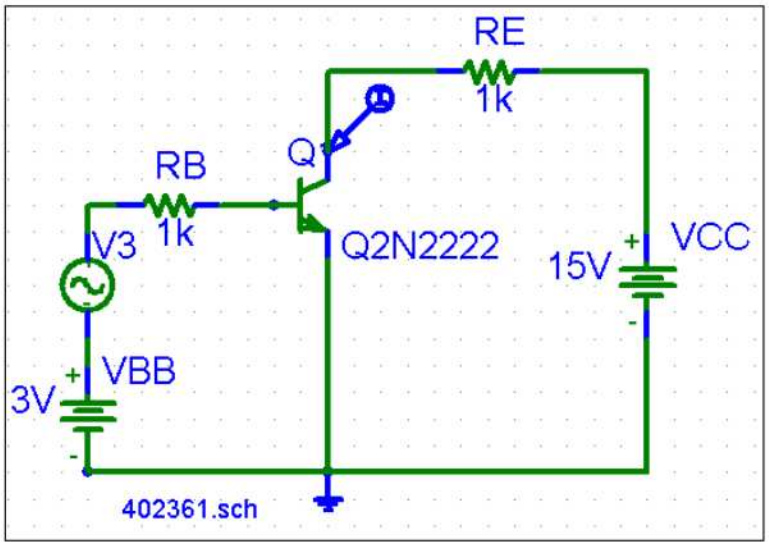
$$g_m = \frac{I_C^{(DC)}}{V_T}$$

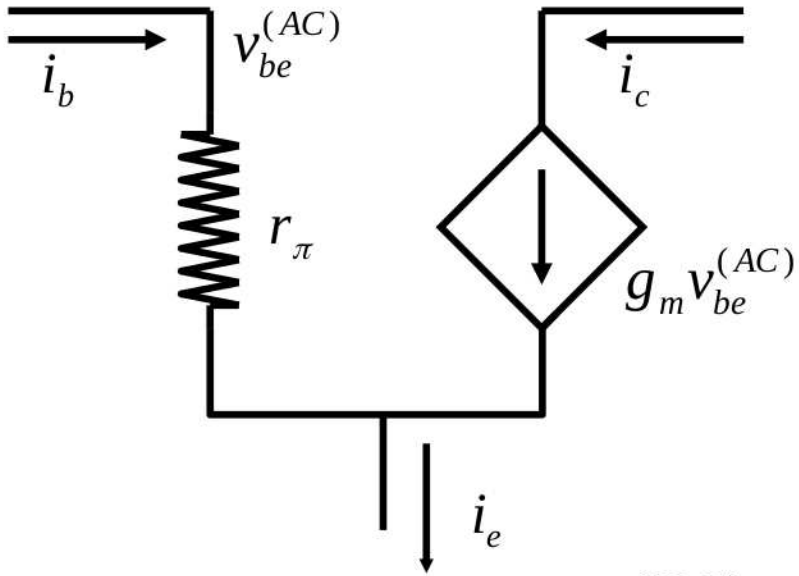
$$r_\pi = \beta / g_m \quad \text{or} \quad r_e = \frac{V_T}{I_E^{(DC)}}$$

$$g_v = - g_m R_C$$



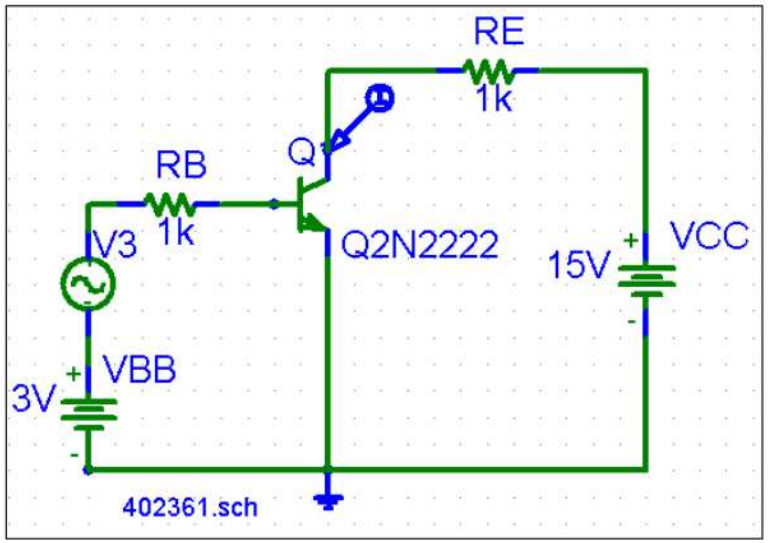
An AC Pi Model

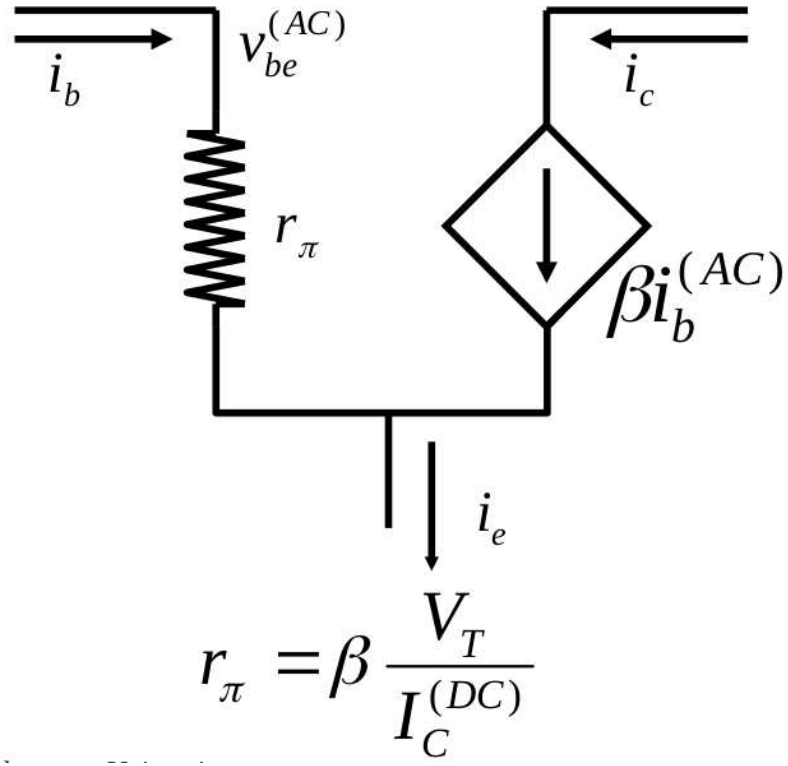




$$r_\pi = \beta / g_m \qquad g_m = \frac{I_C^{(DC)}}{V_T}$$

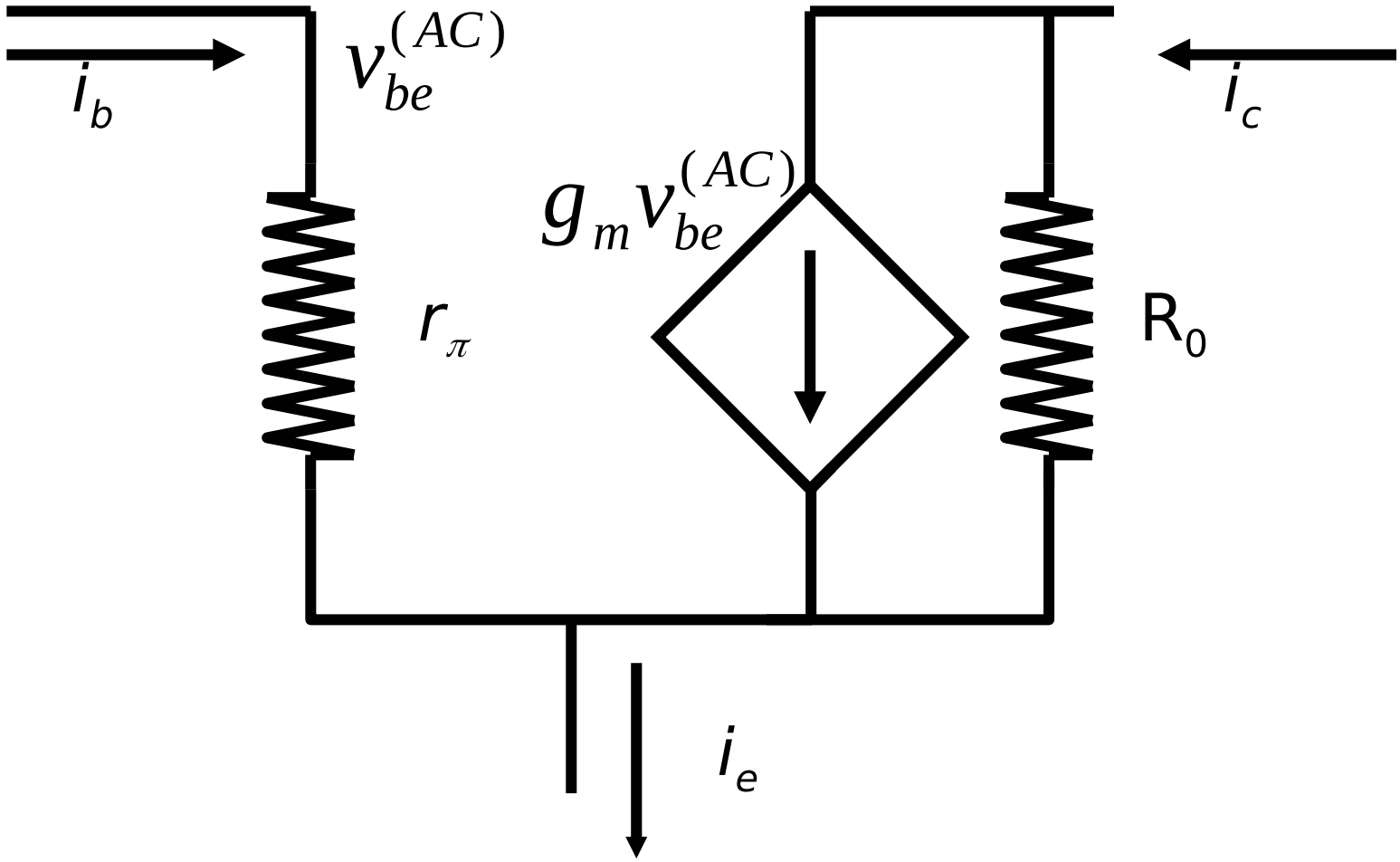
Another AC Pi Model



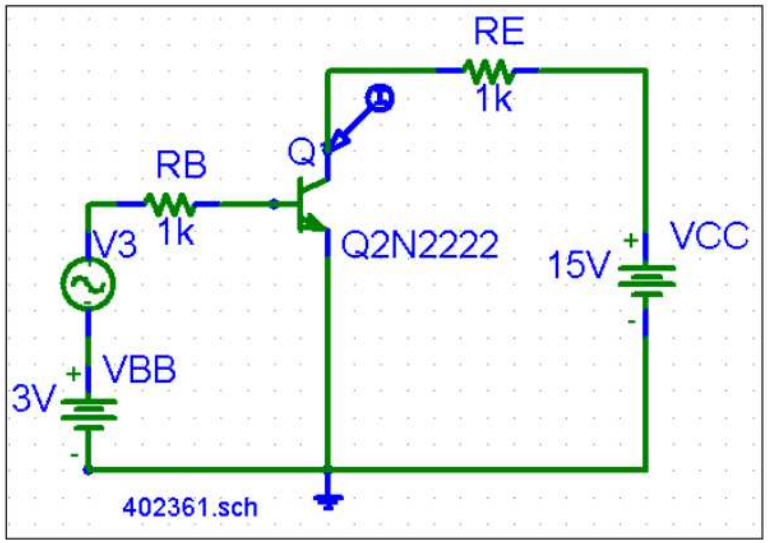


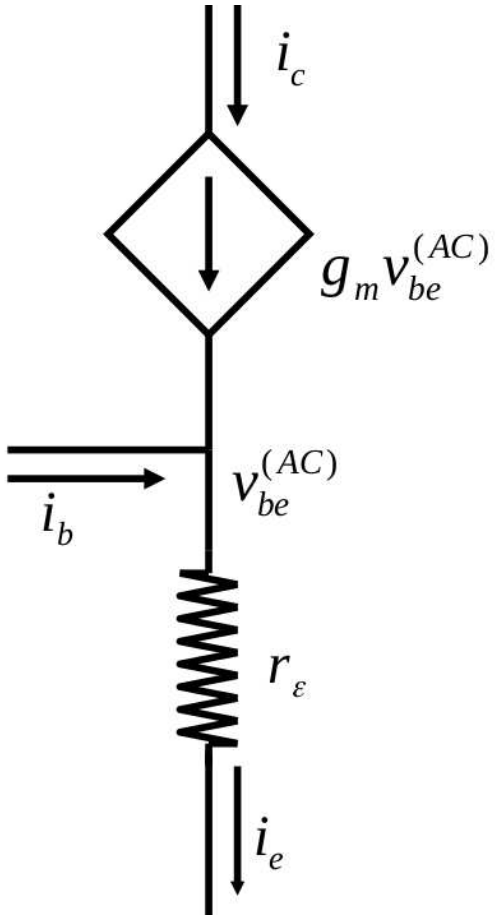
$$r_\pi = \beta \frac{V_T}{I_C^{(DC)}}$$

AC Pi Model with Early Voltage



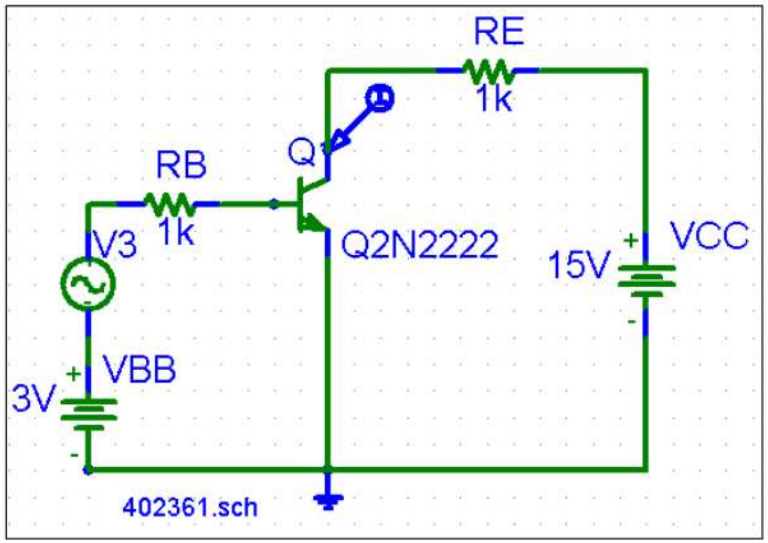
A Tee Model

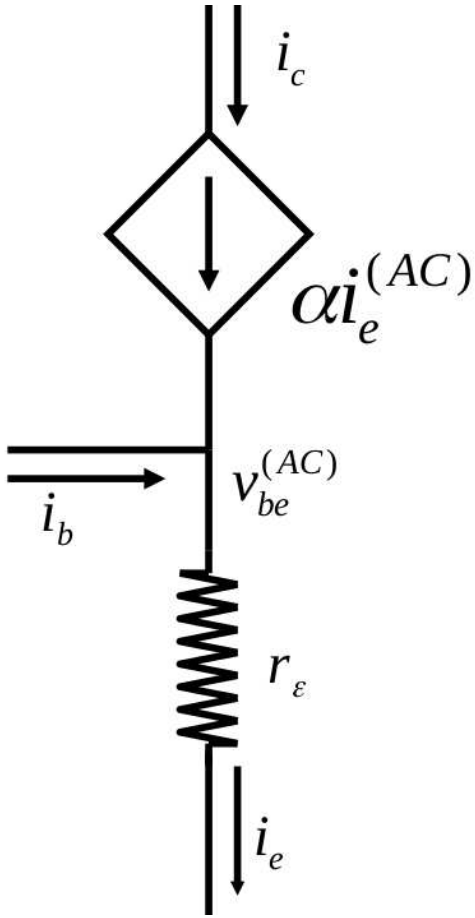




$$g_m = \frac{I_C^{(DC)}}{V_T} \qquad r_e = \frac{V_T}{I_E^{(DC)}}$$

Another Tee Model





$$g_m = \frac{I_C^{(DC)}}{V_T} \qquad r_e = \frac{V_T}{I_E^{(DC)}}$$