EECE5646 Second Exam

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Logistics

Schedule In fairness to all, I must insist on adhering strictly to the schedule. The exam is due electronically on Blackboard before 6:00PM (18:00) Eastern Standard Time (also known as 18:00R, 23:00Z, 23:00GMT), Thursday, 13 December.

Format: Please submit exactly one .pdf file with all the problems answered in sequence. Other formats can lead to font variations, lost artwork, and other unpleasant events.

Filename: Please include some recognizable portion of your name in the filename.

Copy: Please keep a copy yourself, in case any electronic problems occur. I plan to download your work and check that I can open your file shortly after the submission deadline, and I will contact you if there is any problem.

General Instructions

Please do not collaborate with other students or seek help from outside experts. However, you may use any reference book, journal articles, or other readily available resources. Please cite references if you do so.

Please contact me if you are confused about the wording of a problem. I will will clarify the wording of the problems, or correct an error in the questions if someone should find one. Keep an eye on the announcements on the course web site for such updates.

Draw a figure for each of the problems. Usually in my problems, the first step is to generate a layout of the optical system. I give points for figures.

You will want to use a computer for some of the problems. You may use any language you like, but make sure that the equations and graphs are presented in such a way that I don't need to look at your code. When I ask for a plot, I am looking for a correctly labeled one, with correct numerical values. A sketch is not sufficient.

Present your work as clearly as possible. I give partial credit if I can figure out that you know what you are doing. I do not give credit for putting down everything you know and hoping I will find something correct in it. Typesetting or word–processing really helps.

1 Lyot Filter

Here we consider a Lyot Filter, or liquid–crystal tunable filter, often used in hyperspectral imaging. A single "stage" of the Lyot filter consists of a pair of crossed polarizers with a liquid crystal sandwiched between them. A voltage is applied to the liquid crystal layer to vary its birefringence. The axes of the liquid crystal layer are oriented at 45 degrees relative to those of the polarizers. If the birefringence is set so that the layer is a half–wave plate for a wavelength, λ_{set}, then this wavelength is transmitted through the device with the only loss being from absorption and scattering in the materials. At other wavelengths, the stage will transmit less light. By adjusting the voltage, we can tune the filter.

However, there are ambiguities. If the device is a half–wave plate for λ_{set} then it is a 3/2 waveplate for $\lambda_{set}/3$ and so forth. Therefore, multiple stages are needed with different thicknesses. At λ_{set} these stages have birefringence equal to odd half–multiples of the wavelength, $OPD = N\lambda_{set}/2$ where $N = 1, 3, 5 \ldots$.

1.1 Individual Stages

Plot the transmission for unpolarized light as a function of wavelength from 420 to 730 nm with the set wavelength at $\lambda_{set} = 650$ and 480 nm, for stages, $N = 1, 3, 5, 7, 9$. For my solution, I used coherency matrices.

1.2 Performance

Plot the overall transmission of the five–stage device ($N = 1, 3, 5, 7, 9$) on the same figure. What is the linewidth (full–width at half–maximum, FWHM) for each set wavelength?

What is the maximum transmission within the plotted band at a wavelength other than the desired one?

1.3 Improvement

Using up to seven stages (not necessarily consecutive), see how narrow you can make the filter, while keeping the "leakage" of unwanted wavelengths low.

2 Laser Cavity

Let's look at an Argon ion laser designed for operation on the 514 nm line. Now, let's design this cavity so that the minimum beam diameter (Gaussian $1/e^2$ in the cavity is 3 mm, and the beam achieves a focus outside the cavity in front of the laser, with a waist diameter of 500 μm. The cavity must be 30 cm long.

2.1 Cavity Design

Determine

- the distance from the front mirror to the waist,
- the beam diameter at the rear mirror, and
- the radii of curvature of the two mirrors.

Is the cavity stable or unstable? Make a qualitative comment about tolerances for this design, based on stability.

2.2 Multi–Wavelength Operation

Now, suppose that the laser produces output on multiple gain lines of the argon ion. In particular, where is the waist and how large is it, for the 488 nm line?

3 Mode–Locked Pulse

Here we look at a mode–locked laser pulse from a titanium–doped sapphire laser, with a 100 fs pulse length and 80 MHz pulse repetition frequency (Free Spectral Range), and a center wavelength of 730 nm.

In class, we discussed the issue of dispersion through glass and its broadening of the pulse in time.. Suppose that we correct as well as possible for the glass in our microscope so that we deliver a transform–limited pulse to the sample. However, we still have to deliver the light to different depths in the sample. Let's see if this is an important consideration.

3.1 Linewidth

We first direct the laser to a grating spectrometer and measure the distribution of the light across wavelengths. What is the width of the spectrum that we expect to see?

3.2 Dispersion

Here we will assume the sample consists of water and we want to image over a range of 2 mm in depth (about the maximum possible with optical coherence tomography. Look at the website http://refractiveindex.info/ for refractive index information for water. Note that there is a link there to tabular data, which is probably the most useful for this calculation. You probably want to smooth the curve to obtain a single value for $dn/d\lambda$.

Is the peak irradiance reduced by the dispersion? *Hint:* Remember that we can write the field as

$$\sum_{m=-M}^{M} e^{j2\pi(f+m\times FSR)t+j\phi_m}, \tag{1}$$

where $2M \times FSR$ is the linewidth in frequency, M is large enough to account for the full spectrum of the pulse, and we've included a phase for each frequency, ϕ_m. If ϕ varies linearly in frequency, that causes a shift in time (as in increased OPL), but not a broadening. It is a change in the OPL with frequency that changes the irradiance.

4 Radiometry

In this problem, we determine the signals we expect to see with three different cameras, viewing a terrestrial scene. We consider one camera in each of the three important bands: visible light from 400 to 800 nm, the mid–infrared band from 3 to 5 μm and the far–infrared band from 8 to 12 μm. We will consider imaging both with scattered sunlight and thermal emission.

Hint: Calculation of the photon count in each band will require integration. One good approach is to calculate the spectral photon count, N_λ in photons per unit wavelength at each end of the band and then do a two–point trapezoidal integration using

$$N_{band} \approx \frac{N_\lambda(\lambda_1) + N_\lambda(\lambda_2)}{2} \times (\lambda_2 - \lambda_1)\,. \tag{2}$$

4.1 The Cameras

Here we relate the spectral photon count at the camera (in photons per unit wavelength) to the target spectral radiance. In the remaining sections, we will use numbers appropriate to the different wavelengths and targets. For simplicity and comparison, let's assume that all cameras are the same, with square pixels, 20 μm on a side and an f/2 lens. Of course the camera technologies would be quite different, using different detector materials, different lens materials, and more.

Assume that the target has a spectral radiance, $L_\lambda(\lambda)$. Write an equation for the spectral flux on a pixel in Watts per unit area.

Now, assume that the integration time is (1/30) sec, and write an equation for the spectral photon count, $N_\lambda(\lambda)$, in photons per unit wavelength.

4.2 Thermal Emission

Now consider the thermal emission of the target at 300 Kelvins. Compute the number of photons detected in a pixel in each of the three wavelength bands.

4.3 Scattered Sunlight

Repeat the process for scattered sunlight. Assume that the sun is a 5000 K black–body source with 1000 W/m^2 irradiance at the surface of the target. Assume that the surface of the target reflects ten percent of the light in a Lambertian pattern.

4.4 Summary

Present the results in some reasonable way, and discuss them. Consider which situations produce usable signals, which bands are more appropriate to thermal or reflected–light imaging, and which would work at night.