

EECE5646 Midterm Exam

Prof. Charles A. DiMarzio
Department of Electrical and Computer Engineering
Northeastern University

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Logistics

Schedule In fairness to all, I must insist on adhering strictly to the schedule. The exam is due electronically on Blackboard before 12:00 noon, Monday 22 October.

Format: Please submit exactly one .pdf file with all the problems answered in sequence. Other formats can lead to font variations, lost artwork, and other unpleasant events.

Filename: Please include some recognizable portion of your name in the filename.

Copy: Please keep a copy yourself, in case any electronic problems occur. I plan to download your work and check that I can open your file shortly after the submission deadline, and I will contact you if there is any problem.

General Instructions

Please do not collaborate with other students or seek help from outside experts. However, you may use any reference book, journal articles, or other readily available resources. Please cite references if you do so.

Please contact me if you are confused about the wording of a problem. I will clarify the wording of the problems, or correct an error in the questions if someone should find one. Keep an eye on the announcements on the course web site for such updates.

Draw a figure for each of the problems. Usually in my problems, the first

step is to generate a layout of the optical system. I give points for figures.

You will want to use a computer for some of the problems. You may use any language you like, but make sure that the equations and graphs are presented in such a way that I don't need to look at your code. When I ask for a plot, I am looking for a correctly labeled one, with correct numerical values. A sketch is not sufficient.

Present your work as clearly as possible. I give partial credit if I can figure out that you know what you are doing. I do not give credit for putting down everything you know and hoping I will find something correct in it.

1 Snell's Window

The goal here is to construct a synthetic image of the view from a swimming pool, looking up, to illustrate Snell's Window. At angles below the critical angle, we will assume that no reflection occurs, and thus solve only the refraction problem. After Chapter 6, we could modify the result to account for the actual amount of light reflected and refracted at each angle, but we will not do so here.

The pool is square, with a width of 12 meters and a depth of 3 meters. The room in which it is located is also square, with walls separated by 18 meters and a ceiling 4 meters above the water surface.

1.1 Simple Camera

Design a camera (Specify the focal length of the lens and the distance to the image plane) such that the entrance window will be a 4-meter square on the surface when the camera is placed in the center of the pool at a depth of 1.5 meters. The design of this camera is not critical to the problem, so you may assume water on both sides of the lens. In my solution, I assumed 401 pixels in each direction, to keep the computation time short.

For your choice of focal length, what would be the focal length of this lens in air?

1.2 Reflection

We have discussed the refraction equation in vector form. Derive an equation that can be used similarly for reflection.

1.3 Synthetic Image

In this case, it's hard to trace rays forward and predict which pixel (if any) they will hit. It is easier to trace backward and “close” the ray to find where it originated. Trace rays from the camera pupil to each pixel in the entrance window (on the surface), and then consider refraction or reflection as appropriate, and compute the intersection with the surfaces that comprise the sides and bottom of the pool, the sides of the room, and the ceiling. Put some pattern on the wall, so that you can interpret the synthetic image. I simply truncated each of the coordinates (x,y,z) of the final point to the nearest meter, and determined if the product was odd or even to generate a 1 for a bright panel or 0 for a dark one. The calculation was relatively easy, and the result was easy to interpret.

Hint: Start with the pool bottom and ceiling. Once those are working, start work on one side, because there is one additional complication on the sides that does not occur on the horizontal surfaces. *Another hint:* Calculate the unit vector for refraction before trying reflection. If the refraction calculation fails in some way, it tells you that you are beyond the critical angle. *Yet another hint:* You will have multiple solutions to “close” each ray. Give some thought to the easiest way to pick the correct solution.

2 Zoom Lens

We can make a zoom lens (one that has a variable focal length) for a camera, starting with a camera lens of fixed focal length, f_4 , by placing an afocal telescope before the lens. If the afocal telescope has an adjustable magnification, m , the focal length of the combination will change. We would like to do this in such a way that we don't have to move the image plane (focus the camera) when we adjust the focus (zoom).

2.1 Zoom Concept

The most general matrix for an afocal telescope in air is

$$\mathcal{A} = \begin{pmatrix} m & q \\ 0 & 1/m \end{pmatrix},$$

where m is the magnification, and q can have any value, depending on the implementation of the telescope.

If such a telescope is placed in front of a camera lens having a focal length, f_4 , what is the focal length of the combination?

Hint: Matrix optics is probably the easiest approach here.

Assuming thin lenses and a telescope with a total length, z_t , where are the principal planes relative to the microscope lens? Write your answer as a function of f_4 , m , q , and z_t .

For an object at infinity, where is the image?

Explain why this answer is so simple. *Hint:* If it wasn't simple, you might want to go back and look at your equations again. How would the result be more complicated if the object distance were finite?

2.2 Example

Consider a telescope consisting of three lenses, f_1 , f_2 , f_3 mounted in front of a camera lens, f_4 ;

$$f_1 = f_3 = 70 \text{ mm} \quad f_2 = -22.5 \text{ mm} \quad f_4 = 100 \text{ mm}.$$

The distance between the last telescope lens and the camera lens is

$$z_{34} = 10 \text{ mm}.$$

Write the matrices, and find the principal planes, focal length, and focal points for

$$z_{12} = 25 \text{ mm} \quad z_{23} = 25 \text{ mm},$$

$$z_{12} = 10 \text{ mm} \quad z_{23} = 34 \text{ mm},$$

and

$$z_{12} = 34 \text{ mm} \quad z_{23} = 10 \text{ mm}.$$

Write the matrix for the telescope alone. Use the condition that the telescope is afocal to determine z_{23} and the telescope length, $z_{23} + z_{12}$ as functions of z_{12} . Compare your results to the three fixed points above, and comment on the useful range of focal lengths with the zoom lens in this problem.

3 Stops

Here we discuss the finite lens diameters in the zoom lens from Problem 2. We'll first find the location of all the apertures in image space, and the amount by which they are magnified. Next we'll choose to place the aperture stop at the first lens. To make sure it functions as the aperture stop, we will determine the minimum diameter of the other lenses so that they do not become aperture stops. Then we want the diagonal of the camera to be the exit window, so we will decide on the minimum lens sizes required to make sure none of them limits the field of view more than that diagonal. The diameters of the lenses must be large enough to satisfy both conditions.

3.1 Image Space

For the zoom lens in Problem 2, locate the images of all the lenses in image space. Use the mid-point configuration, where $z_{12} = 25$ mm. Identify each as real or virtual. Continue to assume thin lenses.

3.2 Aperture Stop

We want Lens 1 to be the aperture stop, so we want its image in image space to be the exit pupil. Find the diameter of the exit pupil to make the lens $f/4$, and then find the actual diameter of the aperture stop (Lens 1). Find the minimum diameters of the other lenses to make sure they do not become aperture stops.

3.3 Field Stop

We want the diagonal of the imaging chip, 11 mm to limit the field of view. Every circular aperture should subtend an angle from the exit pupil that is larger than the angle subtended by the chip diagonal. Compute the minimum diameters of the lenses. The final design must have diameters that satisfy this condition and the one you obtained in Part 3.2.

4 Thick Lenses

Finally, let's put this zoom lens together using thick lenses. Assume that you have a collection of lenses with the desired focal lengths that are bi-convex, bi-concave, plano-convex, and plano-concave. Pick the best choice for each lens, and compute the vertex-to-vertex distance. Assume the lenses have vertex thicknesses of 5 mm.