Optics for Engineers

Chapter 11

Charles A. DiMarzio
Northeastern University

Nov. 2012

Fourier Optics Terminology

	Field Plane	Fourier Plane
C	Field Amplitude, $E(x,y)$	$\tilde{E}(f_x,f_y)$
	Amplitude Point–Spread Function, $h(x,y)$ Coherent Point–Spread Function Point–Spread Function PSF, APSF, CPSF	Amplitude Transfer Function, $\tilde{h}(x,y)$ Coherent Transfer Function Transfer Function ATF, CTF
	$E_{image} = E_{object} \otimes h$	$\tilde{E}_{image} = \tilde{E}_{object} \times \tilde{h}$
I	Irradiance, $I(x,y)$	$\tilde{I}(f_x,f_y)$
	Incoherent Point–Spread Function, $H(x,y)$ Point–Spread Function PSF, IPSF	Optical Transfer Function, $\tilde{H}(f_x,f_y)$ OTF Modulation Transfer Function, $\left\|\tilde{H}\right\|$ MTF Phase Transfer Function, $\angle\tilde{H}$ PTF
	$I_{image} = I_{object} \otimes H$	$\tilde{I}_{image} = \tilde{I}_{object} \times \tilde{H}$

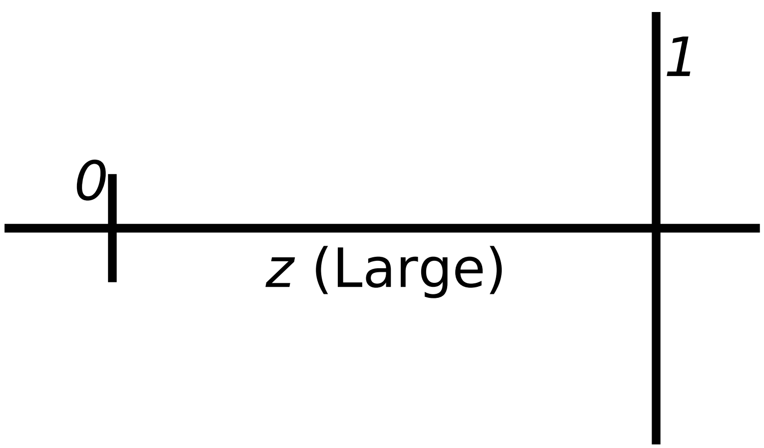
Three Configurations for Fourier Optics

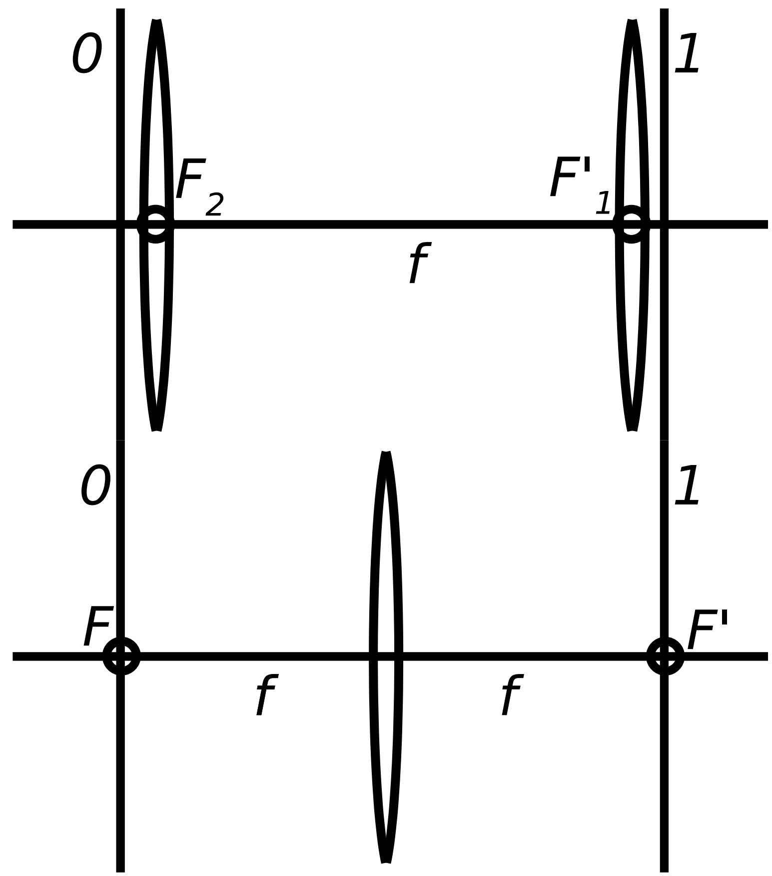
See Chapter 8 for Fresnel–Kirchoff Integral Equation

Use One of These Configurations to Remove Curvature

$$E\left(x_1,y_1,z_1\right)=\frac{jke^{jkz_1}}{2\pi z_1}e^{jk\frac{\left(x_1^2+y_1^2\right)}{2z_1}}\times$$

$$\int\int E\left(x,y,0\right)e^{jk\frac{\left(x^2+y^2\right)}{2z_1}}e^{-jk\frac{\left(xx_1+yy_1\right)}{z_1}}dxdy$$





Fourier Optics Equations (1)

- Fresnel–Kirchoff Integral

$$E\left(x_1,y_1,z_1\right)=\frac{jke^{jkz_1}}{2\pi z_1}e^{jk\frac{\left(x_1^2+y_1^2\right)}{2z_1}}\times$$

$$\int\int E\left(x,y,0\right)e^{jk\frac{\left(x^2+y^2\right)}{2z_1}}e^{-jk\frac{\left(xx_1+yy_1\right)}{z_1}}dxdy$$

- In Spatial Frequency with Source Curvature Removed

$$E\left(f_x,f_y,z_1\right)=\frac{j2\pi z_1e^{jkz_1}}{k}e^{jk\frac{\left(x_1^2+y_1^2\right)}{z_1}}\int\int E\left(x,y,0\right)e^{-j2\pi\left(f_xx+f_yy\right)}dxdy$$

- Both Curvatures Removed

$$E\left(f_x,f_y,z_1\right)=\frac{j2\pi z_1e^{jkz_1}}{k}\int\int E\left(x,y,0\right)e^{-j2\pi\left(f_xx+f_yy\right)}dxdy$$

Fourier Optics Equations (2)

- Define Frequency–Domain Field

$$\tilde{E}\left(f_x, f_y\right) = j\ z_1 \lambda e^{jkz_1} E\left(f_x, f_y, z_1\right),$$

- Fourier Transform

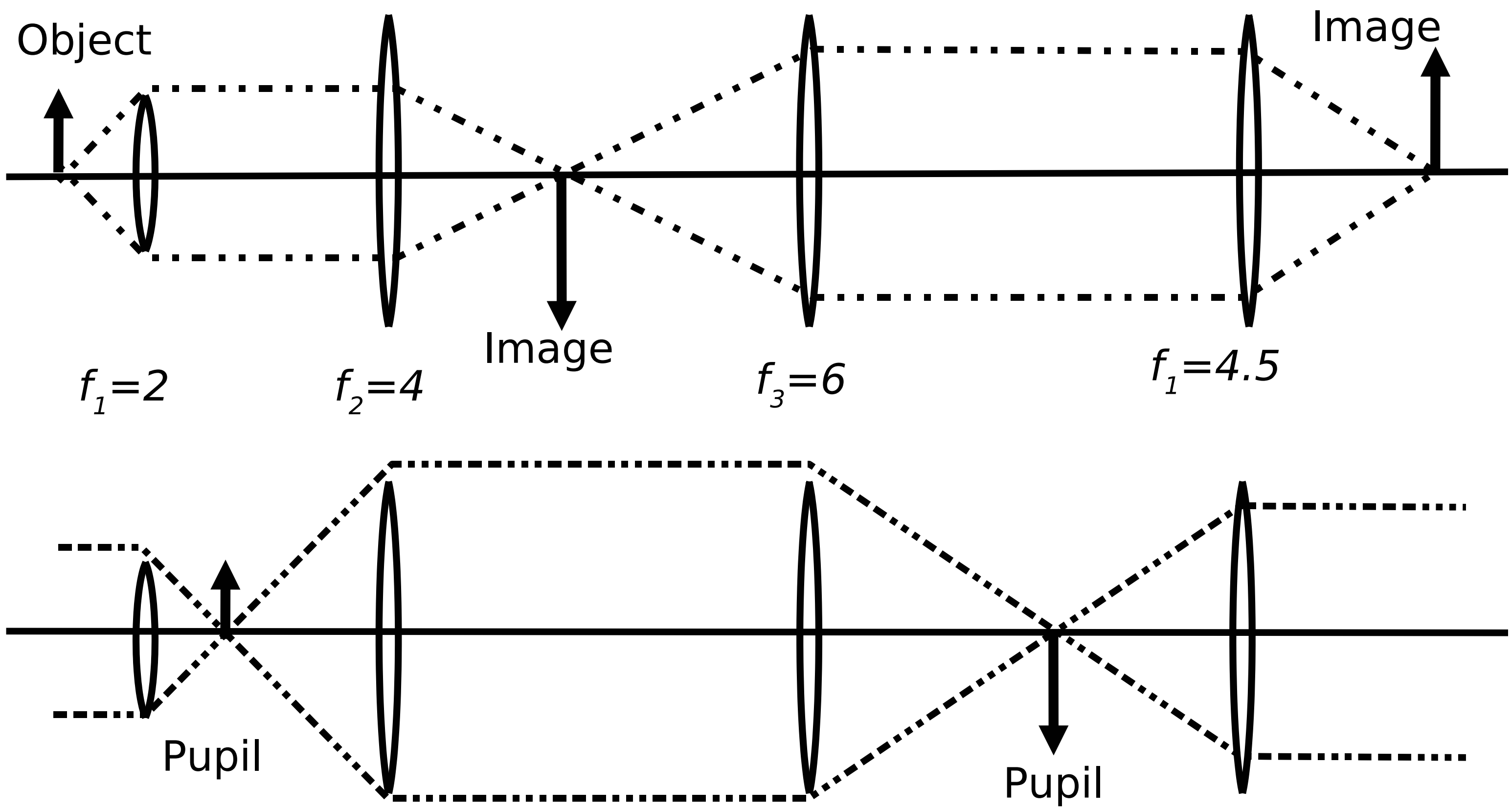
$$\tilde{E}\left(f_x, f_y\right) = \int\int E\left(x, y, 0\right) e^{-j2\pi\left(f_x x + f_y y\right)} dx dy,$$

- Inverse Fourier Transform

$$E\left(x, y\right) = \int\int \tilde{E}\left(f_x, f_y\right) e^{-j2\pi\left(f_x x + f_y y\right)} df_x df_y$$

Optical Fourier Transform

$$f_x = \frac{k}{2\pi z} x_1 = \frac{x_1}{\lambda z} \qquad f_y = \frac{k}{2\pi z} y_1 = \frac{y_1}{\lambda z}$$



Fourier Analysis: FT and IFT

- Pupil to Field (x_1, y_1) to (x, y): Fourier Transform

$$E(x_1, y_1, z_1) = \frac{jke^{jkz_1}}{2\pi z_1} \int\int E(x, y, 0)\, e^{-jk\frac{(xx_1+yy_1)}{z_1}} dxdy$$

  – Field to Pupil: (x, y) to (x_2, y_2): Fourier Transform...

$$E(x_2, y_2, z_2) \stackrel{?}{=} \frac{jke^{jkz_2}}{2\pi z_2} \int\int E(x, y, 0)\, e^{-jk\frac{(xx_2+yy_2)}{z_2}} dxdy$$

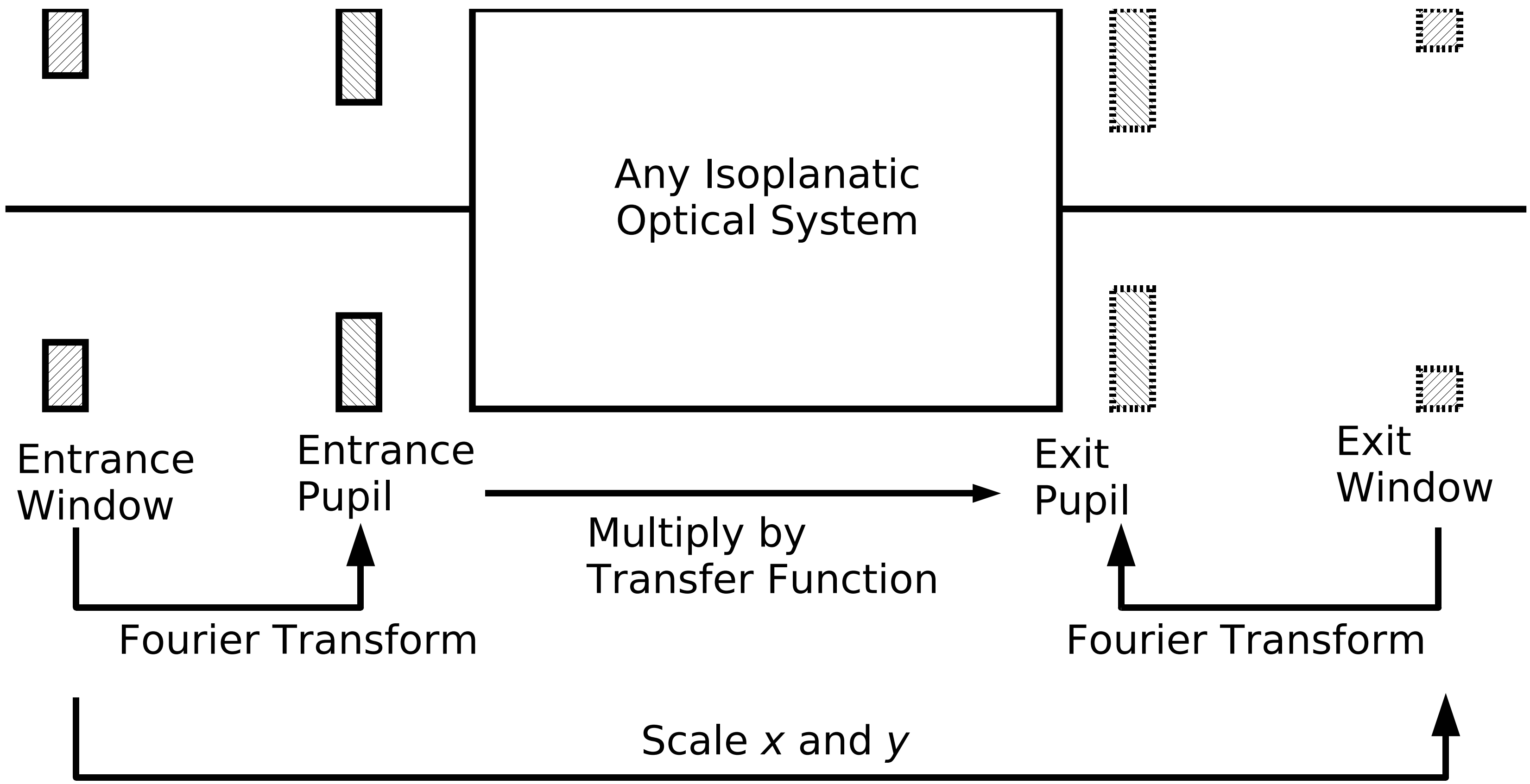
  – ...or Inverse Fourier Transform?

$$E(x_2, y_2, z_2) \stackrel{?}{=} \frac{jke^{-jkz_1}}{2\pi z_1} \int\int E(x, y, 0)\, e^{jk\frac{(xx_2+yy_2)}{z_1}} dxdy$$

  – Negative Signs and Scaling (z_1 *vs.* z_2)

$$x_2 = -\frac{f_2}{f_1}x_1 \qquad y_2 = -\frac{f_2}{f_1}y_1$$

Computation: The Amplitude Transfer Function



- Field Plane: Convolve with Point–Spread Function
- Pupil Plane: Multiply by Amplitude Transfer Function

Computation: Steps

1. Multiply the object by a binary mask to account for the entrance window, and by any other functions needed to account for non–uniform illumination, transmission effects in field planes, *etc.*,

2. Fourier transform.

3. Multiply by the ATF, which normally includes a binary mask to account for the pupil, and any other functions that multiply the field amplitude in the pupil planes.

4. Inverse Fourier transform.

5. Scale by the magnification of the system.

Isoplanatic Systems

- Analogy to Temporal Signal Processing
  - Linear Time–Shift–Invariant Systems
  - Convolution with Impulse Response in Time Domain
  - Multiplication with Transfer Function in Frequency Domain
  - Fourier Optics Assumption
    * Linear Space–Shift–Invariant Systems
    * Convolution with Point–Spread Function in Image
    * Multiplication with 2–D Transfer Functions in Pupil

Non–Isoplanatic Systems

- Some Aberrations Depend on Field Location
  - Coma
  - Astigmatism and Field Curvature
  - Distortion
  - Somewhat Isoplanatic over Small Regions
- Twisted Fiber Bundle
  - Random Re–location of light from pixels
  - Not at All Shift–Invariant

Anti–Aliasing Filter

- Spatial Frequency in the Pupil Plane

$$f_x = \frac{u}{\lambda}$$

- Cutoff Frequency

$$f_{cutoff} = \frac{NA}{\lambda}$$

- Example: $\lambda = 500$nm and $NA = 0.5$

$$f_{cutoff} = 1\text{cycle}/\mu\text{m}$$

  – Nyquist Sampling in the Object Plane

$$f_{sample} = 2\text{cycles}/\mu\text{m}$$

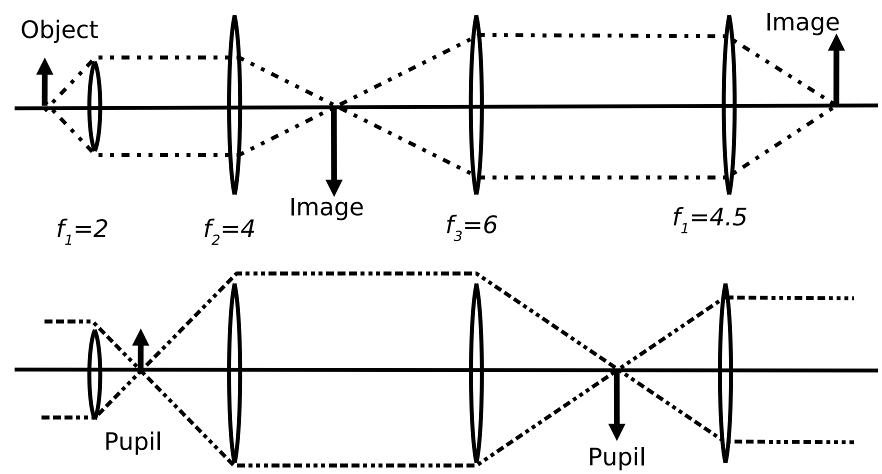
- In Image Plane

$$m = \frac{4}{2} \times \frac{4.5}{6} = 1.5$$

  – Pixel pitch

$$0.5\mu\text{m} \times m = 1.33\mu\text{m}$$

  – Smaller than Practical
  – Need More Magnification



Anti–Aliasing: Practical Examples

- Microscope, $\lambda = 500$nm
  - $100X$, Oil Immersion $NA = 1.4$
  - Object Plane

$$f_{cutoff} = \frac{NA}{\lambda} =$$

2.8Cycles/μm

  - Image Plane

$$f_{cutoff} = \frac{NA}{\lambda}/m =$$

$$f_{sample} = 2f_{cutoff} =$$

0.056pixels/μm

  - pixel spacing $\leq 4.5\mu$m

- Camera (Small m)
  - Small NA, Large F

$$NA_{image} = n'\frac{1}{|m-1|\,2F} \approx \frac{1}{2F}$$

$$f_{sample} = 2f_{cutoff} =$$

$$2 \times \frac{NA}{\lambda} = \frac{1}{\lambda F_{min}}$$

- F–Number

$$F_{min} = \frac{1}{\lambda f_{sample}} =$$

$$\frac{x_{pixel}}{\lambda} = \frac{5\mu\text{m}}{500\text{nm}} = 10$$

Fourier Optics with Coherent Light

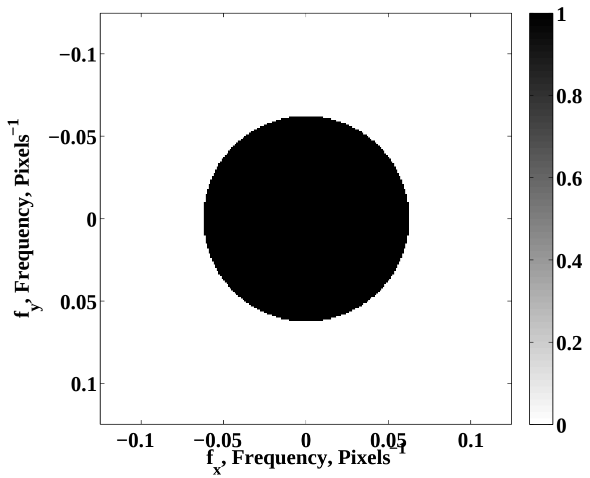
PSF $$h\left(x,y\right)=\int\int\tilde{h}\left(f_x,f_y,0\right)e^{-j2\pi\left(f_xx+f_yy\right)}df_xdf_y$$

$$E_{image}\left(x,y\right)=E_{object}\left(x,y\right)\otimes h\left(x,y\right)$$

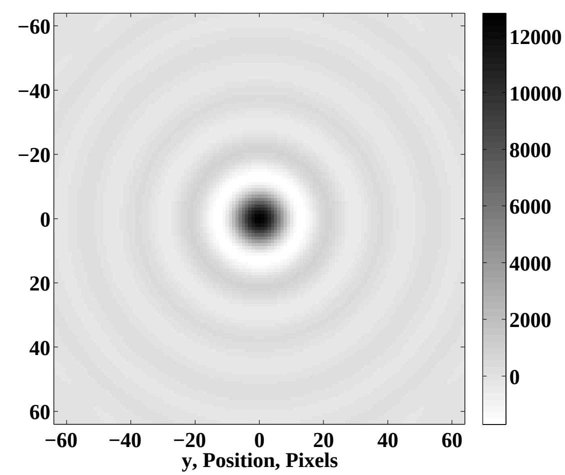
ATF $$\tilde{h}\left(f_x,f_y\right)=\int\int h\left(x,y,0\right)e^{j2\pi\left(f_xx+f_yy\right)}dxdy$$

$$\tilde{E}_{image}\left(f_x,f_y\right)=\tilde{E}_{object}\left(f_x,f_y\right)\tilde{h}\left(f_x,f_y\right)$$

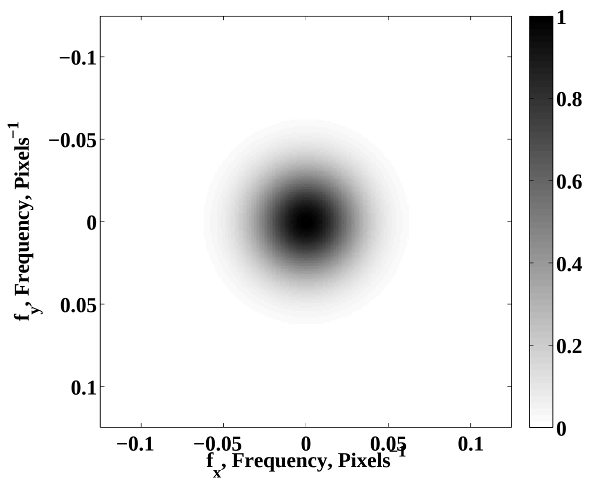
Gaussian Apodization Degrades Resolution, Reduces Sidelobes



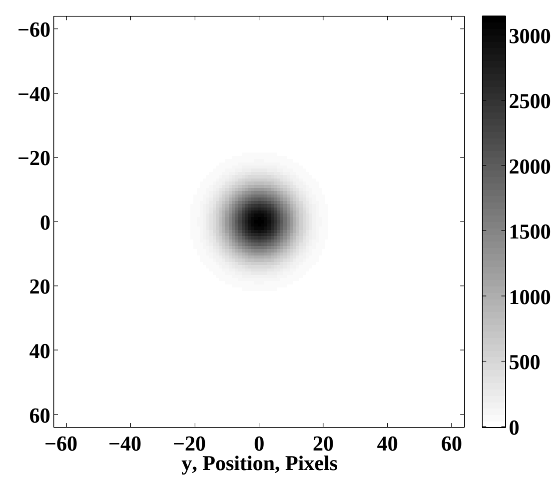
A. Aperture



B. Airy Function PSF

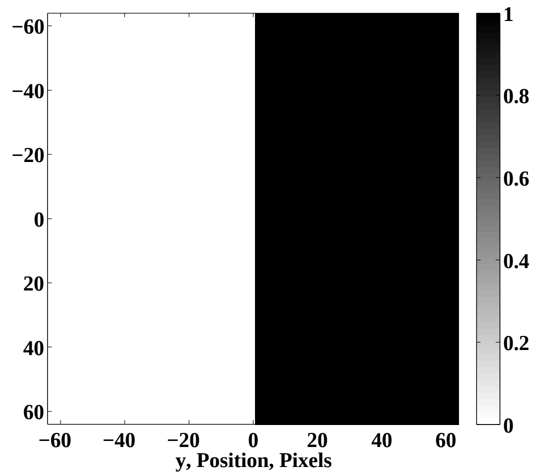


C. Gaussian Apodization

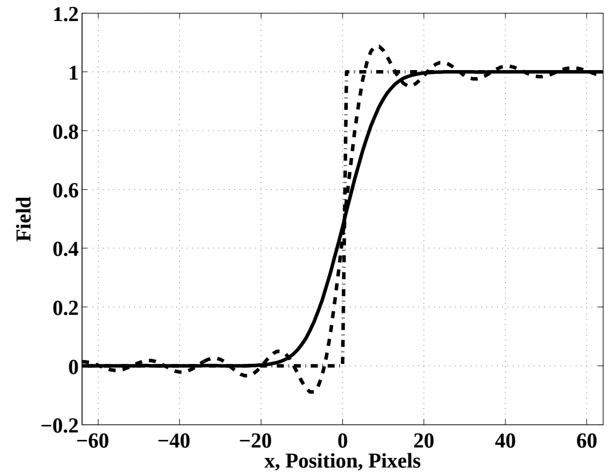


D. Gaussian PSF

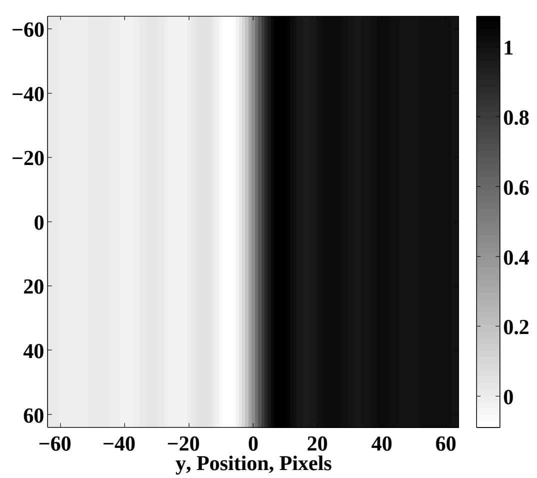
Improved (?) Imaging with Gaussian Apodization



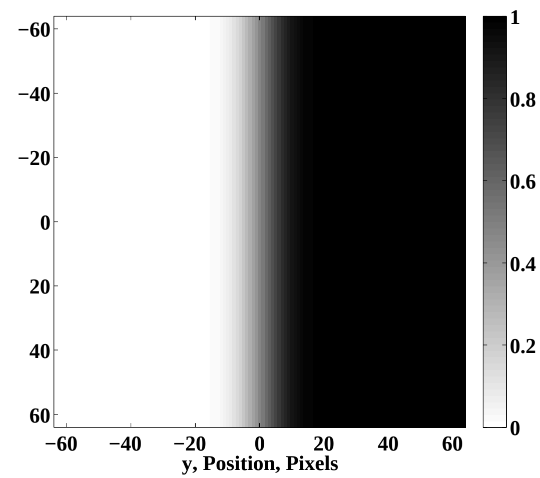
A. Knife Edge Object



B. Image Slices



C. Image with Aperture



D. Image with Gaussian

Coherent Fourier Optics Summary

- Isoplanatic imaging system; pairs of planes such that field in one is scaled Fourier transform of that in the other. Several Configurations.
- It is often useful to place the pupil at one of these planes and the image at another.
- Then the aperture stop acts as a low–pass filter on the Fourier transform of the image. This filter can be used a an anti–aliasing filter for a subsequent sampling process.
- Other issues in an optical system can be addressed in this plane, all combined to produce the transfer function.
- The point–spread function is a scaled version of the inverse Fourier transform of the transfer function.
- The transfer function is the Fourier transform of the point–spread function.
- The image can be viewed as a convolution of the object with the point–spread function.

Fourier Optics for Incoherent Imaging (1)

- Even an LED Source Usually Results in an Incoherent Image

$$\delta\lambda = \lambda/30 \qquad \tau_c \approx \frac{30}{\nu} = 30\frac{\lambda}{c} \approx 60\text{fs} \qquad \langle h(x,y)\rangle = 0 \qquad T \gg \tau_c$$

- Incoherent Point–Spread Function

$$H(x,y) = \langle h(x,y)\, h^*(x,y)\rangle$$

$$I_{image}(x,y) = \left[h(x,y) \otimes E_{object}(x,y)\right]\left[h^*(x,y) \otimes E^*_{object}(x,y)\right]$$

- Cross Terms to Zero: Linear Equation

$$I_{image}(x,y) = \left[h(x,y)\, h^*(x,y)\right] \otimes \left[E_{object}(x,y)\, E^*_{object}(x,y)\right]$$

Fourier Optics for Incoherent Imaging (2)

- Incoherent Image as Convolution

$$I_{image}(x,y) = H(x,y) \otimes I_{object}(x,y)$$

$$\tilde{H}(f_x, f_y) = \tilde{h}(f_x, f_y) \otimes \tilde{h}^*(f_x, f_y)$$

$$\tilde{h}^*(f_x, f_y) = \tilde{h}(-f_x, -f_y)$$

- OTF (Incoherent) is autocorrelation of ATF (Coherent)

Optical Transfer Function

- Optical Transfer Function, OTF (Previous Page)

$$\tilde{H}\left(f_x, f_y\right) = \tilde{h}\left(f_x, f_y\right) \otimes \tilde{h}^*\left(f_x, f_y\right)$$

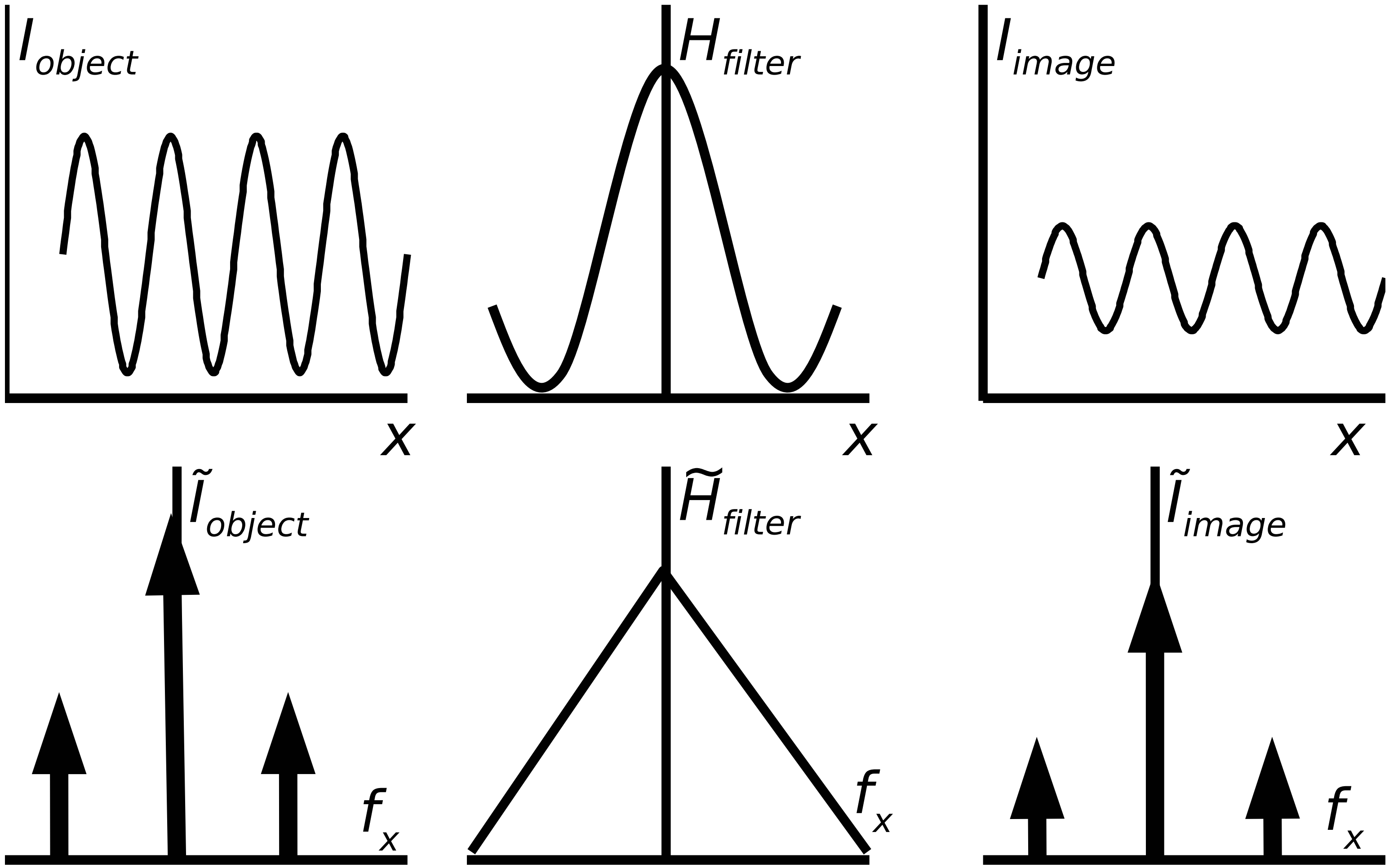
- Modulation Transfer Function, MTF

$$\left|\tilde{H}\left(f_x, f_y\right)\right|$$

- Phase Transfer Function, PTF

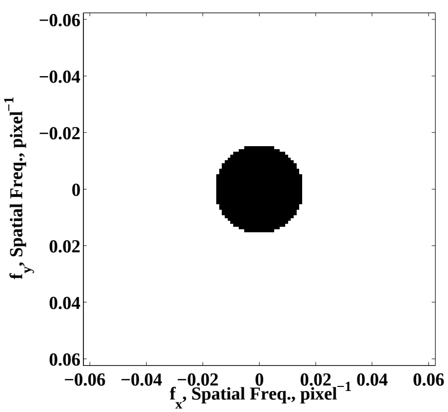
$$\angle\left[\tilde{H}\left(f_x, f_y\right)\right]$$

Fourier Optics Example: Square Aperture ATF

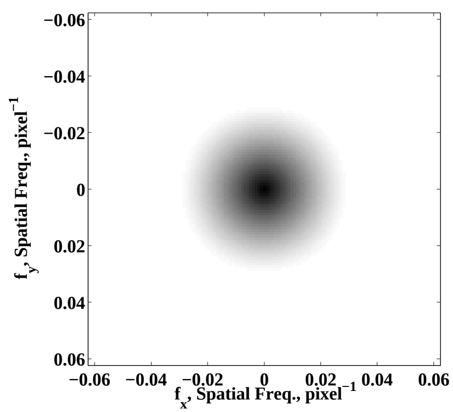


"AC" Amplitude Reduced more than "DC:" Contrast Degraded

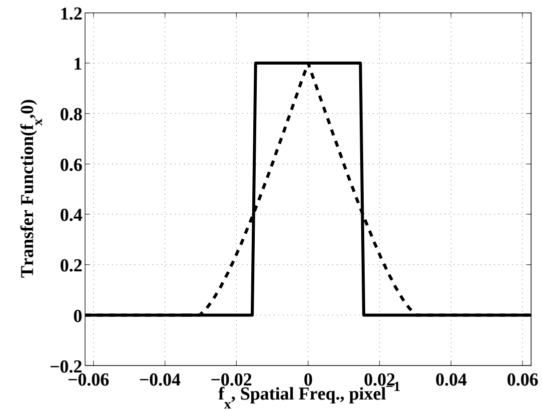
Fourier Optics Example: Coherent and Incoherent



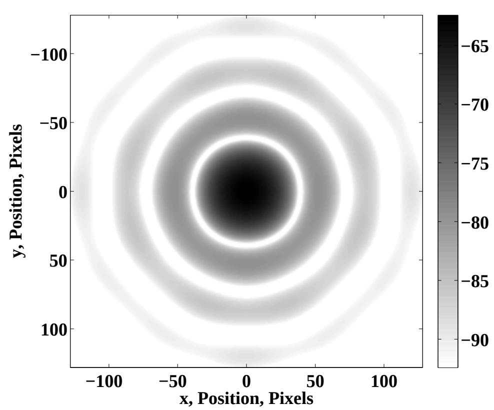
Coherent ATF



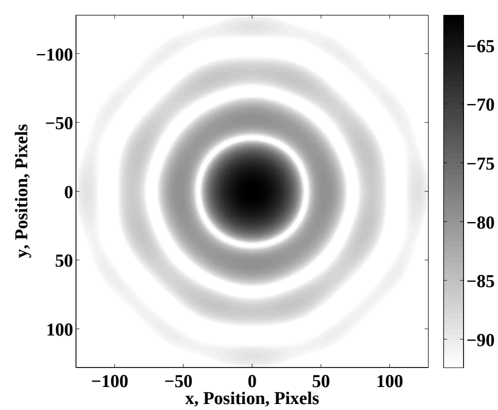
Incoherent OTF



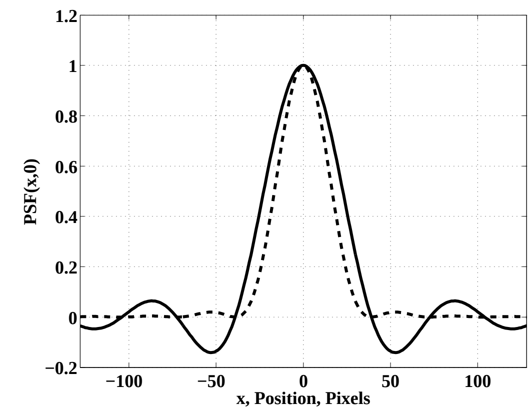
Transfer Function



Coherent PSF (dB)



Incoherent PSF (dB)

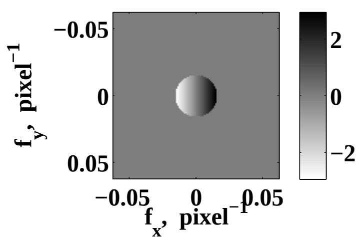


PSF (— Coh) (- - Inc)

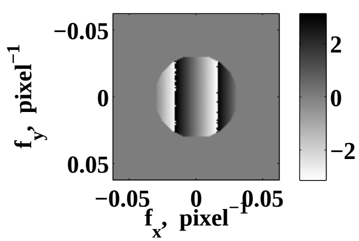
Image Shift (Prism in Pupil)

- Amplitude Transfer Function: Phase Ramp

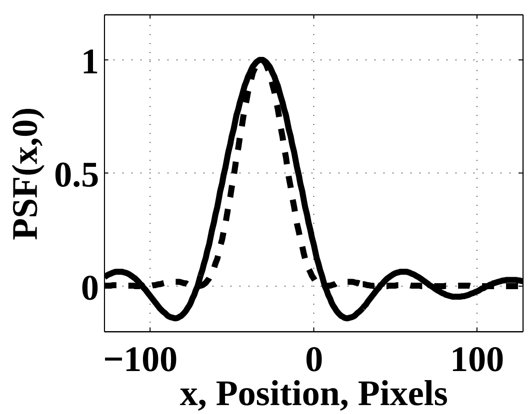
$$g\tilde{h}\left(f_x,f_y\right) = \begin{cases} \exp\left[i2\pi\frac{f_x/4}{1/128}\right] & \text{for } f_x^2+f_y^2 \le f_{max}^2 \\ 0 & \text{Otherwise} \end{cases}$$



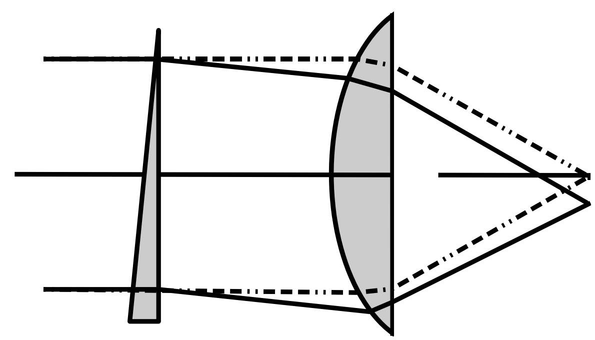
Coh. PTF



Inc. PTF



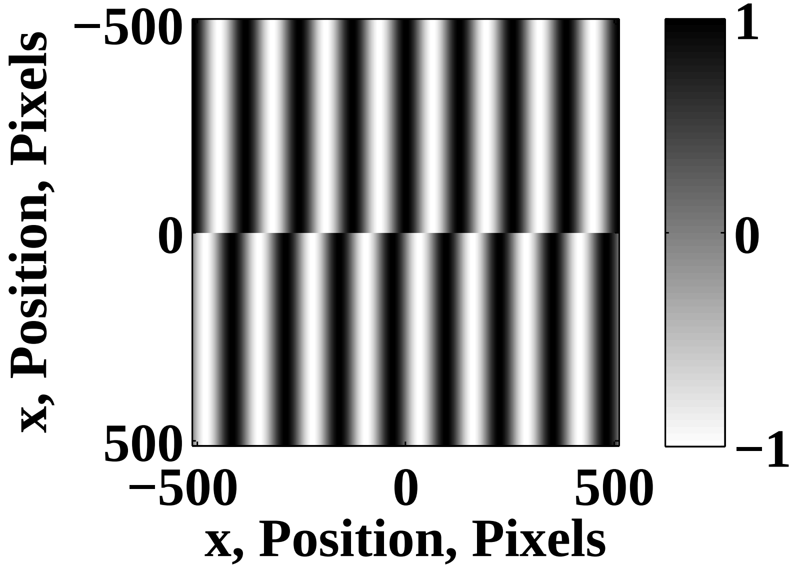
Slices of PSFs



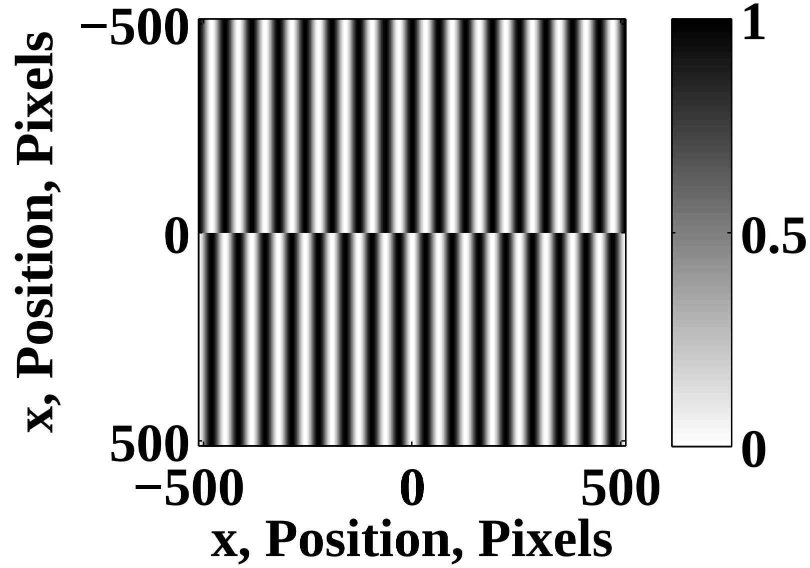
Physical Configuration

Image through Image Shifter

Object Above, Image Below Coherent Object: $\sin(2\pi f_x x)$



(A) Coherent Image



(B) Incoherent Image

Incoherent Object: $[\sin(2\pi f_x x)]^2 = \frac{1}{2} - \frac{1}{2}\cos(2\pi \times 2 f_x x)$

Incoherent Imaging with Camera

- Pixel Current

$$i_{mn} = \int\int_{pixel} \rho_i \left(x - m\delta x, y - n\delta y\right) E(x,y)E^*(x,y)\ dx\ dy$$

- Signal as Convolution with Pixel

$$i_{mn} = \left\{\left[E(x,y)E^*(x,y)\right] \otimes \rho_i\right\} \times \delta\left(x - x_m\right)\delta\left(y - y_n\right)$$

- Complete Transfer Function

$$\tilde{i} = \tilde{I}\tilde{H}\tilde{\rho}_i$$

Summary of Incoherent Imaging

- The incoherent point–spread function is the squared magnitude of the coherent one.
- The optical transfer function (incoherent) is the autocorrelation of the amplitude transfer function (coherent).
- The OTF is the Fourier transform of the IPSF.
- The DC term in the OTF measures transmission.
- The OTF at higher frequencies is usually reduced both by transmission and by the width of the point–spread function, leading to less contrast in the image than the object.
- The MTF is the magnitude of the OTF. It is often normalized to unity at DC. The PTF is the phase of the OTF. It describes a displacement of a sinusoidal pattern.

Characterizing an Optical System

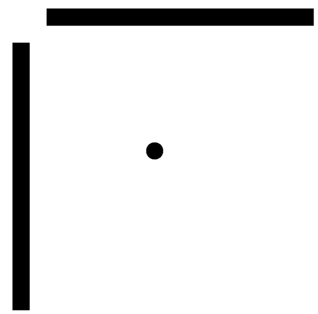
- Overall light transmission. *e.g.* OTF at Dc, or equivalently the integral under the incoherent PSF.
- The 3–dB (or other) bandwidth or the maximum frequency at which the transmission exceeds half (or other fraction) of that at the peak.
- The maximum frequency that where the MTF is above some very low value which can be considered zero. (Think Nyquist)
- Height, phase, and location of sidelobes.
- The number and location of zeros in the spectrum. (Missing spatial frequencies)
- The spatial distribution of the answers to any of these questions in the case that the system is not isoplanatic.

System Metrics

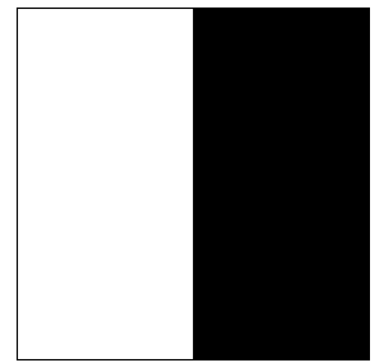
- What is the diffraction–limited system performance, given the available aperture?
- What is the predicted performance of the system as designed?
- What are the tolerances on system parameters to stay within specified performance limits?
- What is the actual performance of a specific one of the systems as built?

Test Objects

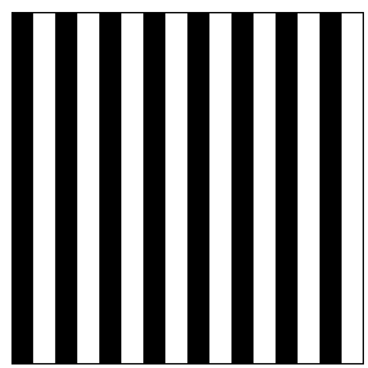
- PSF
- LSF: Line Spread Function
- ESF: Edge Spread Function
- MTF and . . .
- PTF (partial)



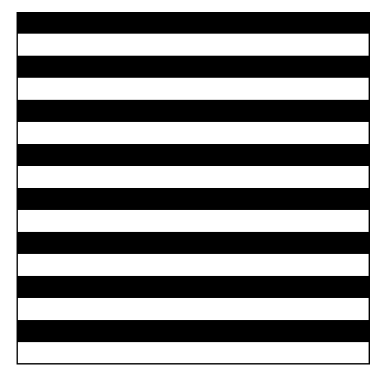
A. Point and Lines



B. Knife Edge

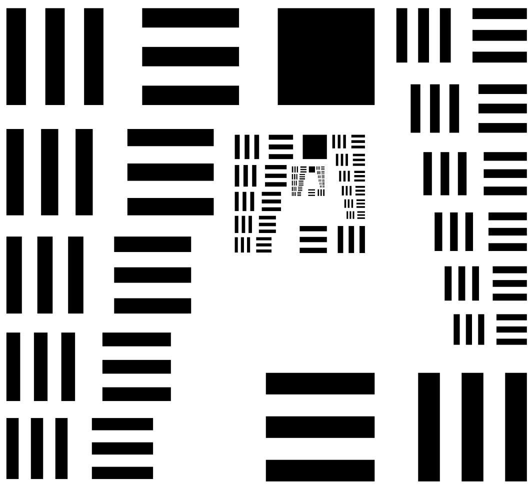


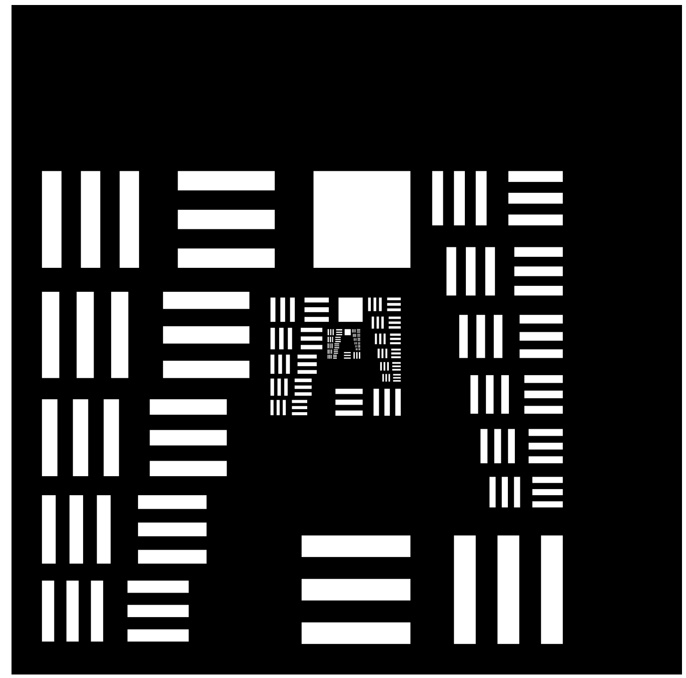
C. X Bar Chart



D. Y Bar Chart

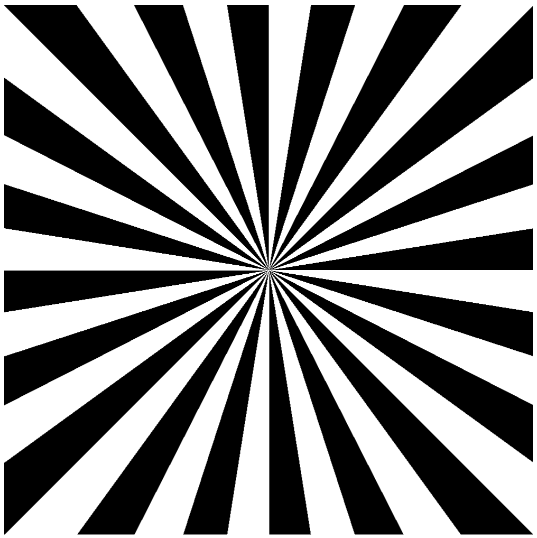
The Air–Force Resolution Chart

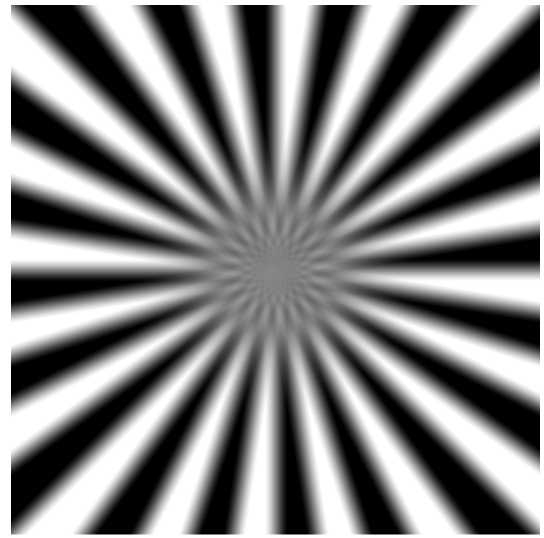




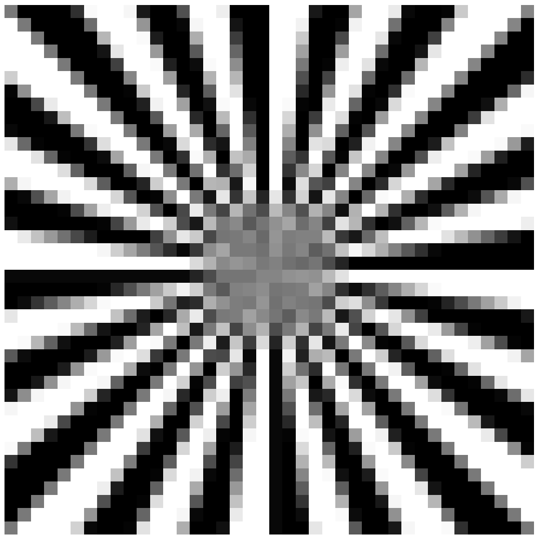
$$x = \frac{1\text{mm}}{2^{G+1+(E-1)/6}}$$

Radial Bar Chart

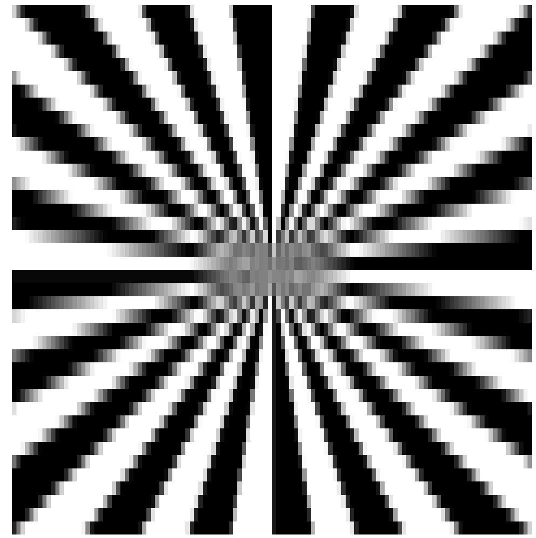




Effect of Pixels



(A) 30X30



(B) 30X10

Summary of System Characterization

- Some systems may be characterized by measuring the PSF directly.
- Often there is insufficient light to do this.
- Alternatives include measurement of LSF or ESF.
- The OTF can be measured directly with a sinusoidal chart.
- Often it is too tedious to use the number of experiments required to characterize a system this way.
- A variety of resolution charts exist to characterize a system. All of them provide limited information.