Optics for Engineers
Chapter 2

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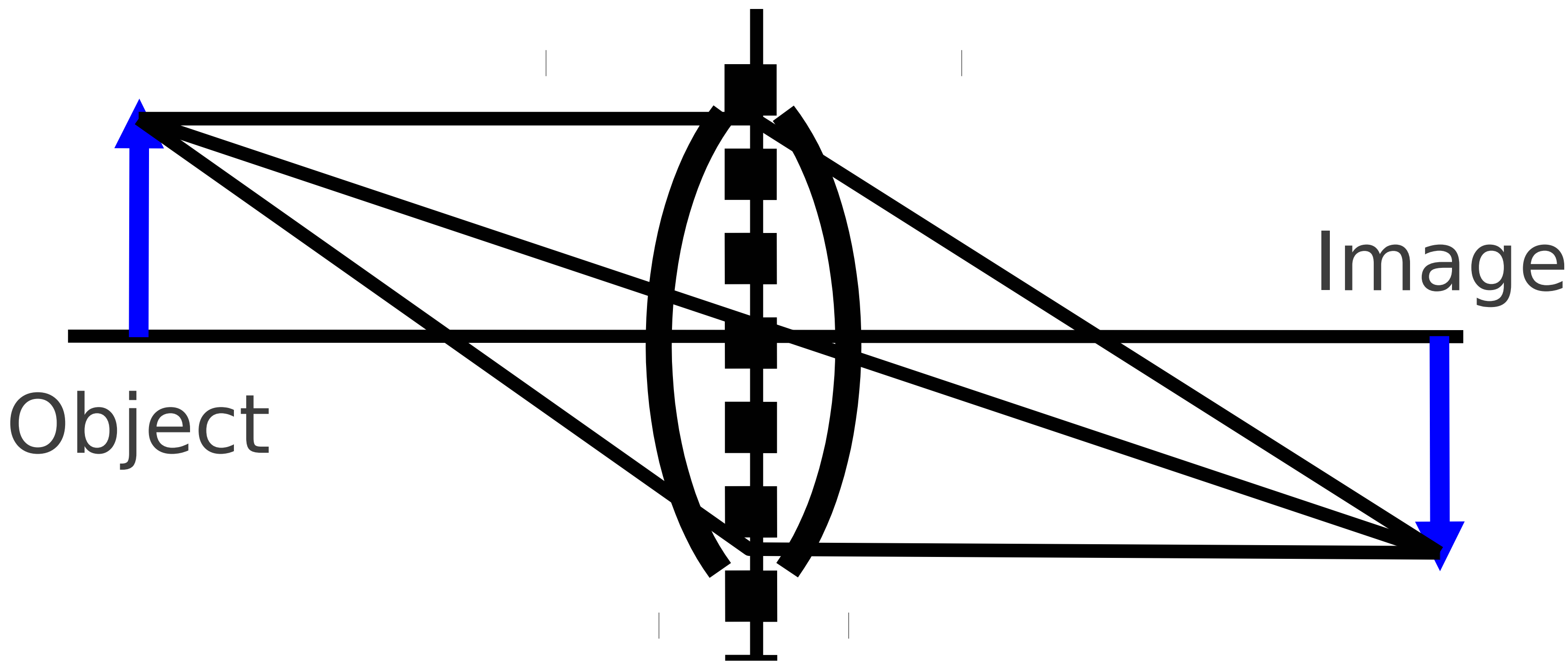
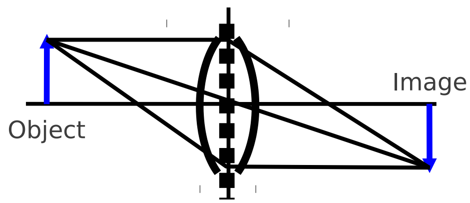
Outline of Geometric Optics

- Chapter 2
  - Snell's Law from Fermat's Principle
  - Mirrors and Refractive Surfaces
  - Multiple Surfaces: Simple Lenses: The Thin Lens
  - Image Location, Orientation, Magnification
- Chapter 3: Matrix Optics: Principal Planes
- Chapter 4: Stops Limit Light Gathering and FOV
- Chapter 5: Aberrations Limit Resolution
- Later: Wave Optics: Diffraction–Limited Resolution in Ch. 8

“High–School Optics”

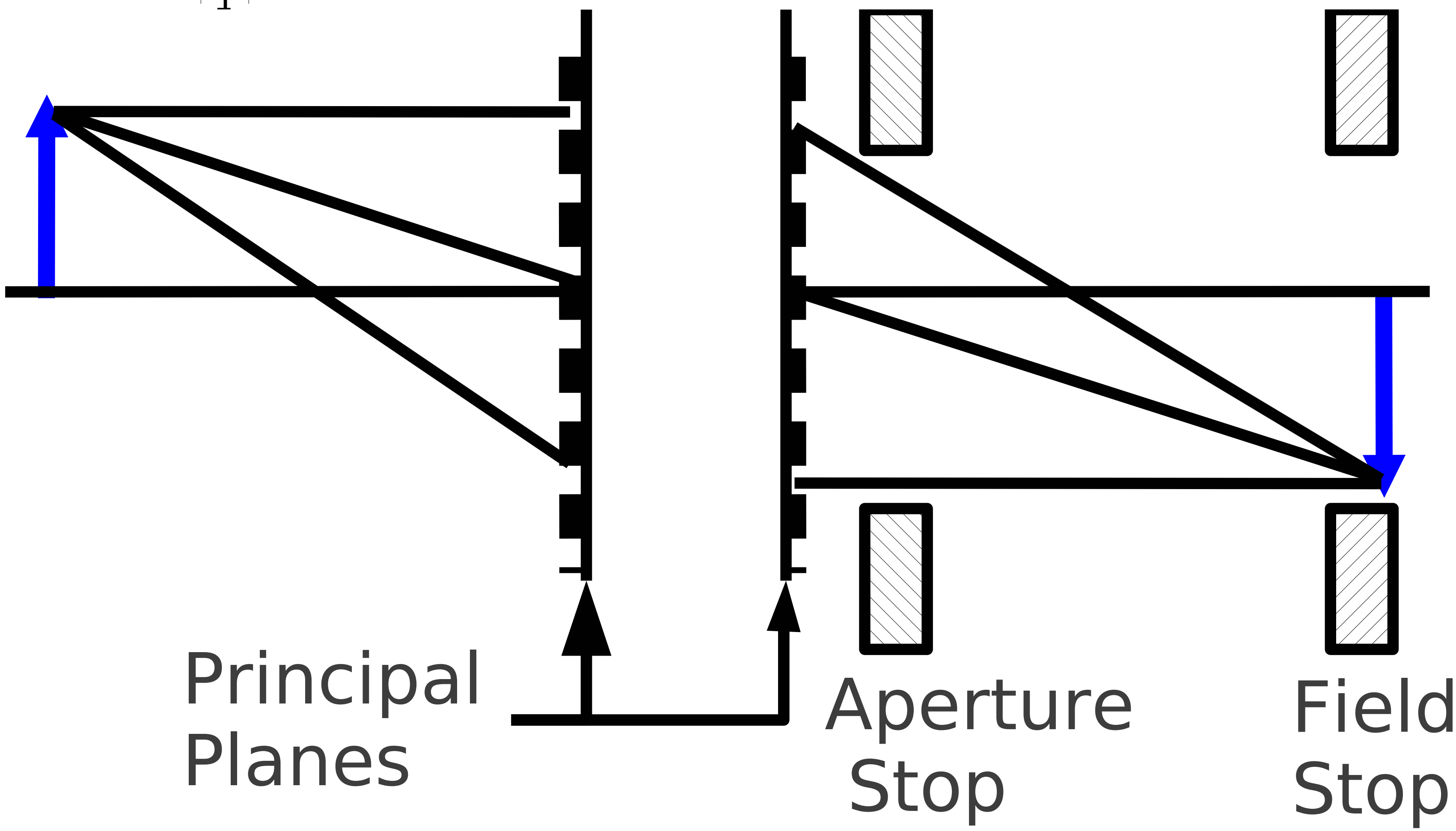


“The AP Version”



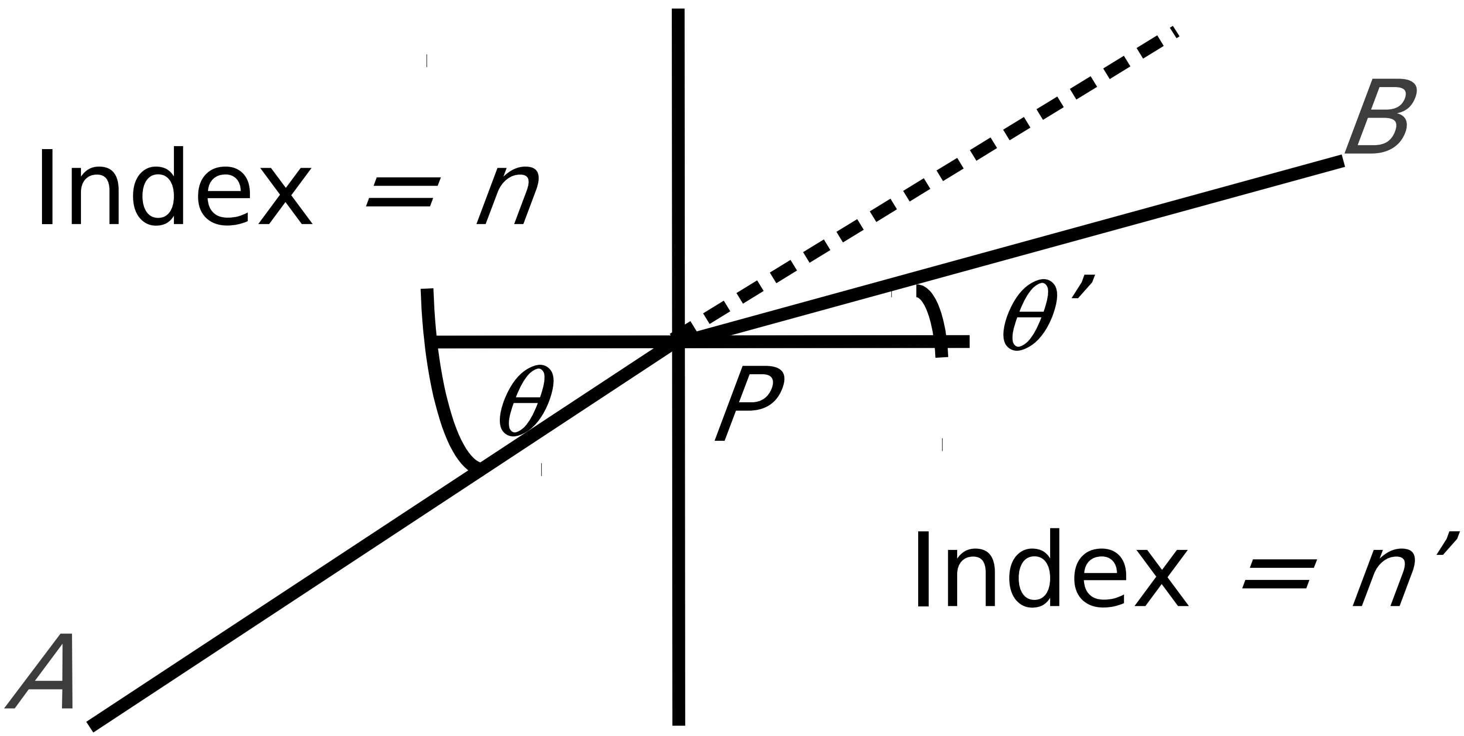
Principal Planes: Ch. 3

Stops: Ch. 4

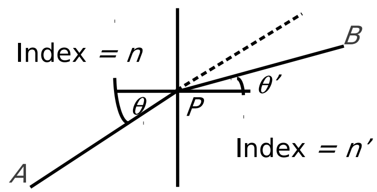


Concepts for Refraction

- Plane of Incidence Contains Incident (and Exiting) Ray and Normal (and is the plane of the 2–D drawing)
- Angle of Incidence Is Defined Relative to Normal



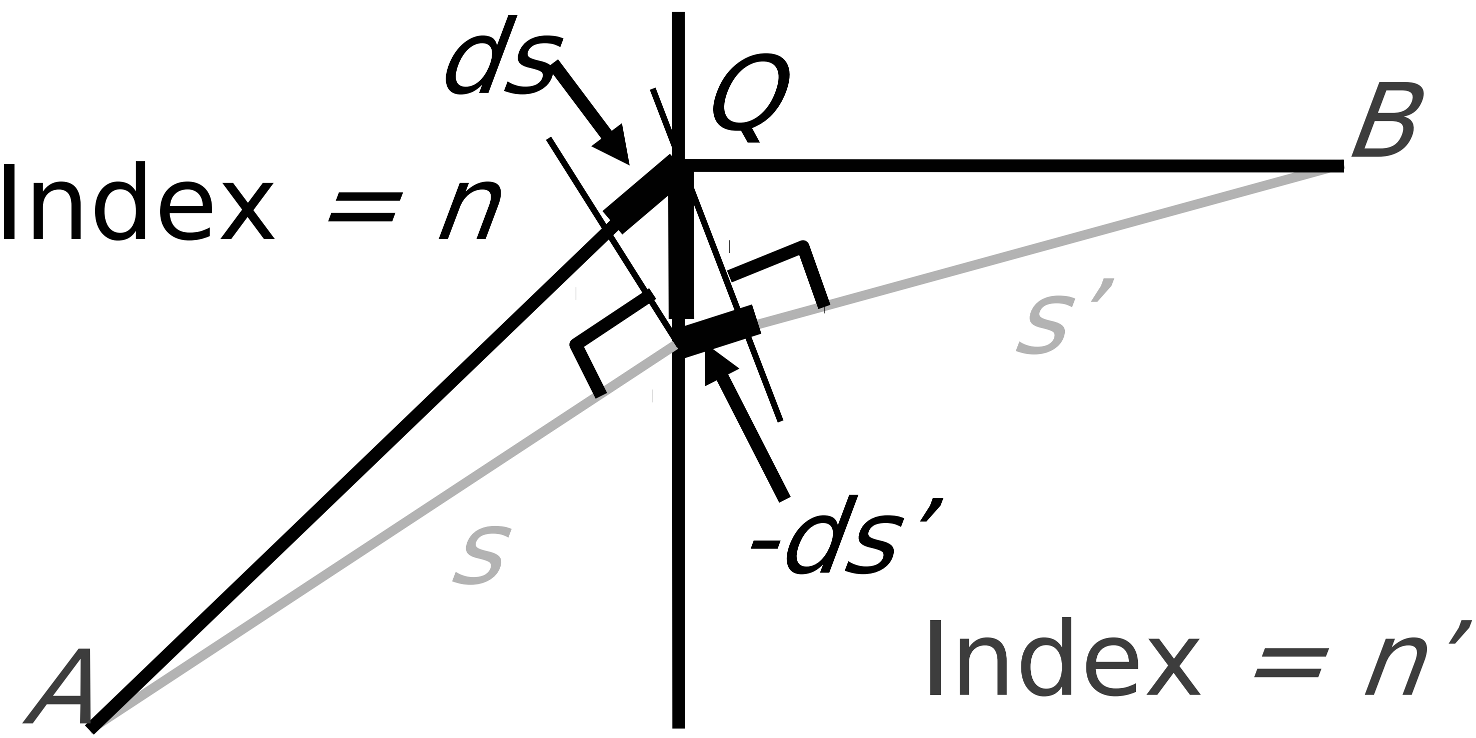
Snell's Law



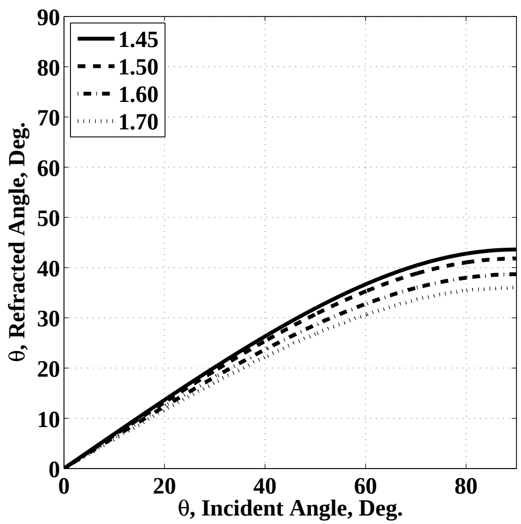
Variational Approach from Minimal Path, AB

$$nds = n'ds'$$

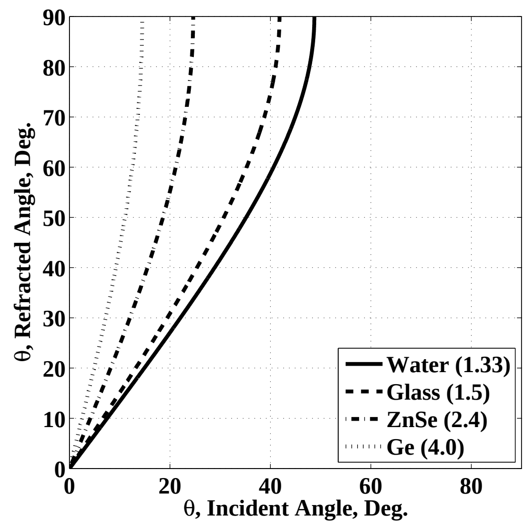
$$\boxed{n \sin\theta = n' \sin\theta'.}$$



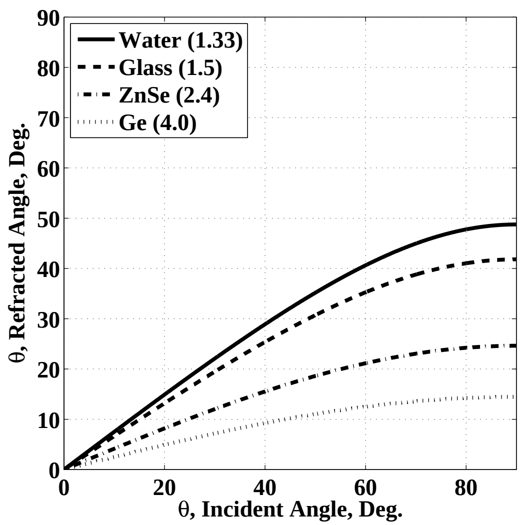
Snell's Law: Examples



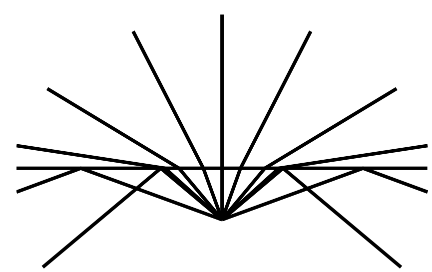
Air to Glass



Air to Material

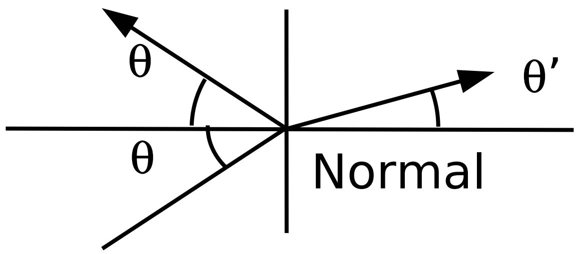


Material to Air



Total Internal Reflection

Reflection and Refraction



Reflection:

$$\boxed{\theta_r = \theta.}$$

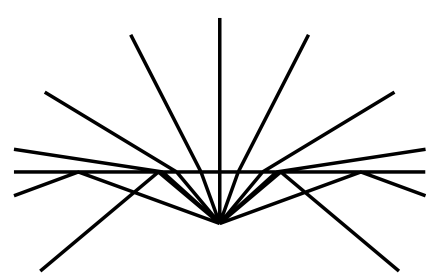
Refraction

Total Internal Reflection

- Critical Angle (No Solution for θ')

$$n \sin \theta_c = 1$$

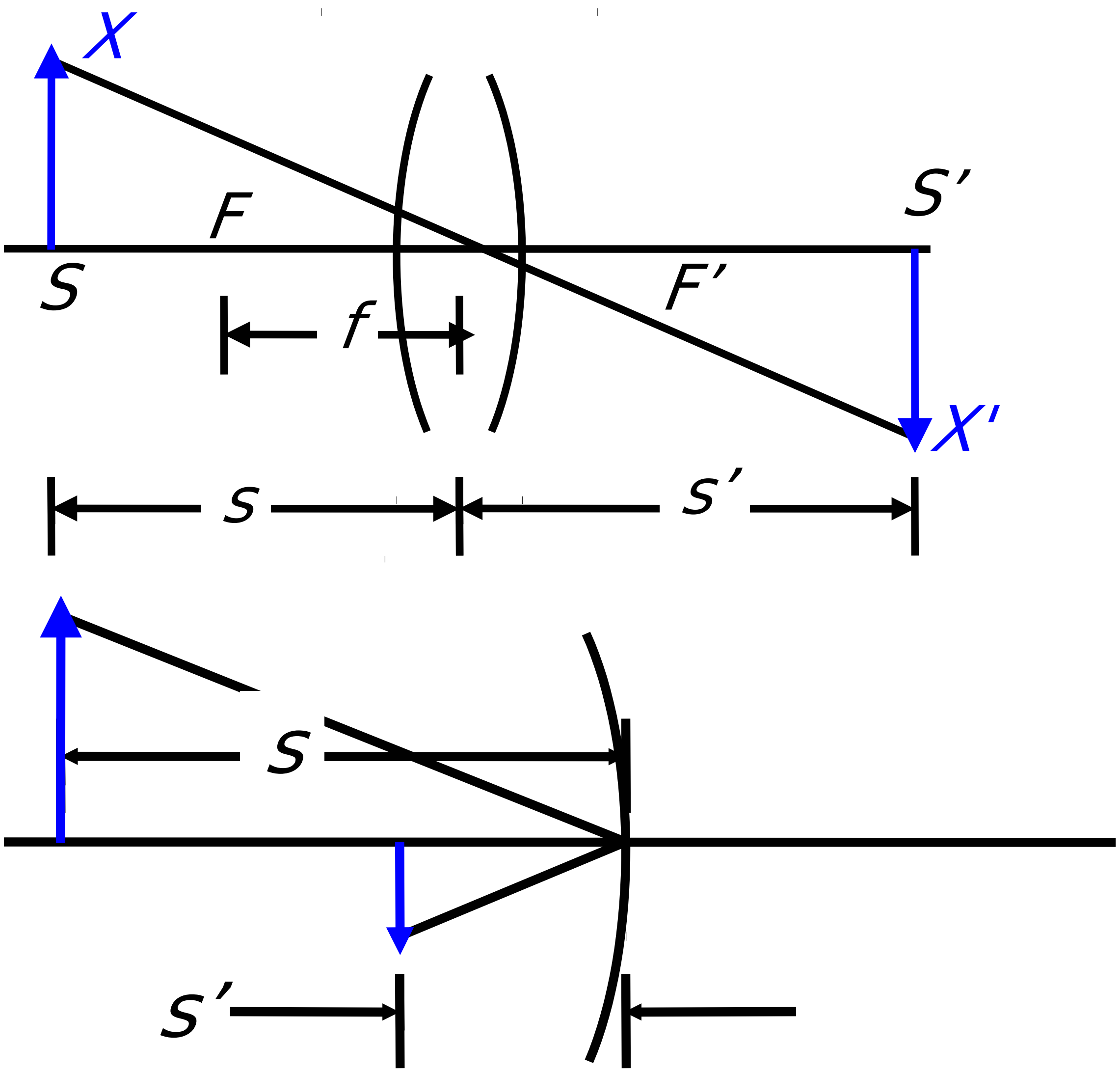
- For $\theta < \theta_c$ Reflection and Refraction
- For $\theta > \theta_c$ 100% Reflection



Snell's Window

Carol Grant

Imaging Sign Conventions



- Lens
  - $s > 0$ to Left
  - $s' > 0$ to Right
  - $f > 0$ for Converging
- Mirror
  - $s > 0$ to Left
  - $s' > 0$ to Left
  - $f > 0$ for Concave

Upper Case for points, Lower for lengths — $x, x' > 0$ Upright

Imaging Terms

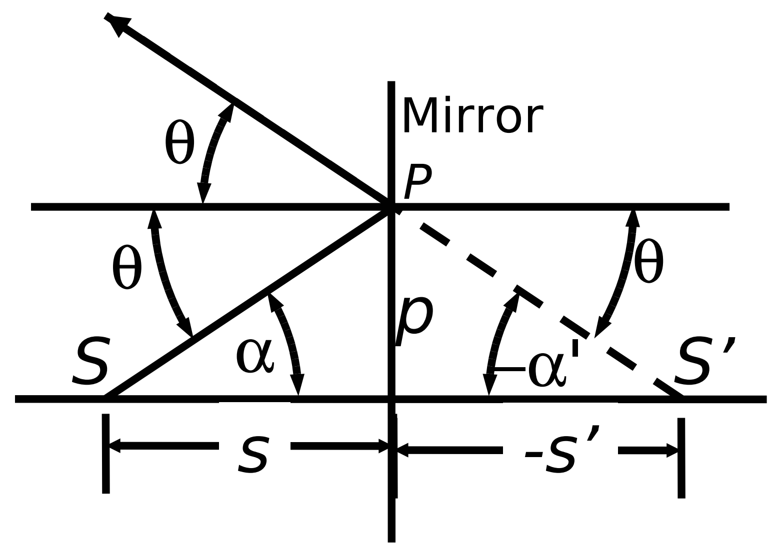
We will discuss these in detail later.
The important issues now are the definitions.

Quantity	**Definition**	**Equation**	**Notes**
Object distance	s		Positive to the left
Image distance	s'	$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$	Positive to the right for interface or lens. Positive to the left for mirror.
Magnification	$m=\frac{x'}{x}$	$m=-\frac{x'}{x}$	
Angular magnification	$m_\alpha=\frac{\partial\alpha'}{\partial\alpha}$	$\lvert m_\alpha\rvert=\frac{1}{\lvert m\rvert}$	
Axial Magnification	$m_z=\frac{\partial s'}{\partial s}$	$\lvert m_z\rvert=\lvert m\rvert^2$	

Reflection at a Plane Mirror (1)

- Narcissus
- "...the looking glasses of the women..." Exodus 38:8

Image Location
Similar Triangles
$s' = -s$ (Planar reflector)
Virtual Image as Shown



- Question: Could we have a virtual object? How?

Reflection at a Plane Mirror (2)

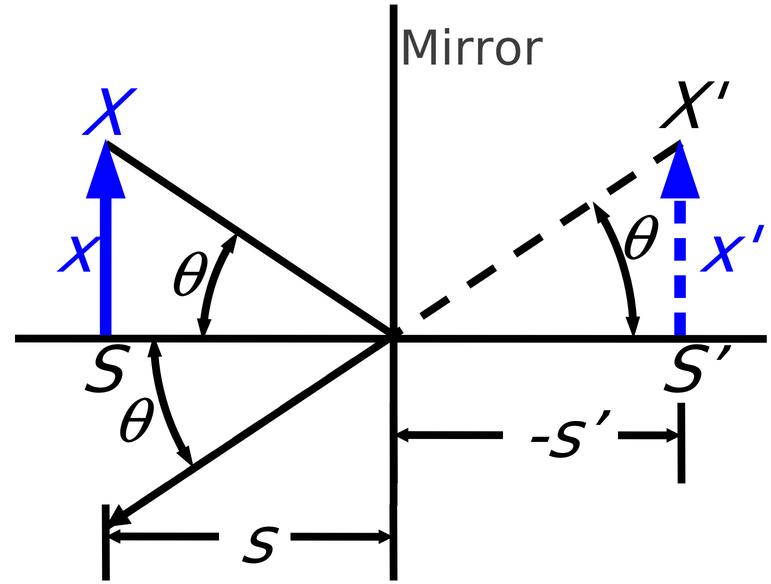
Magnification (Transverse)

More Similar Triangles

$x' = x \qquad\qquad m = 1$

$$m = \frac{x'}{x} = \frac{-s'}{s} = 1$$

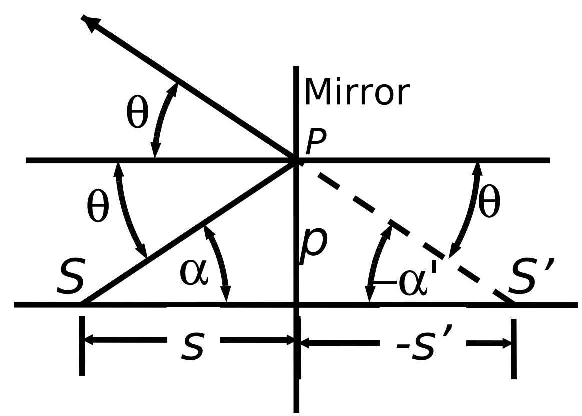
(Planar reflector)



Upright ($m > 0$) & Virtual (Dotted Lines)

Angular Magnification

$$m_\alpha = \frac{d\alpha'}{d\alpha} = -1 \qquad \text{(Planar reflector)}$$



Reflection at a Plane Mirror (3)

Axial Magnification

$$m_z = \frac{ds'}{ds} = \frac{s'}{s} = -1 \qquad \text{(Planar reflector)}$$

Summary

$$s = -s' \qquad m = 1 \qquad m_z = -1$$

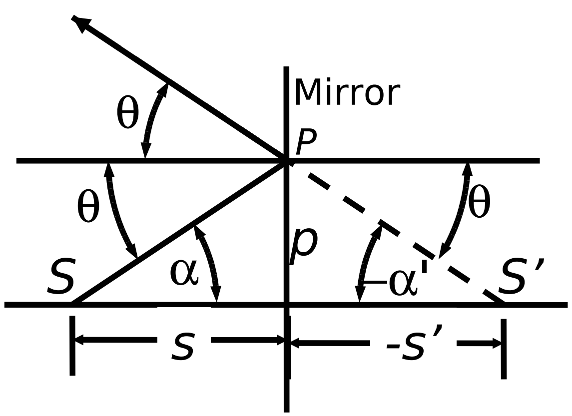
Upright, Virtual, Perverted*, but Not Distorted**
*Right–Handed Coordinate System Imaged to Left–Handed
**Distorted Means $m_z \neq m$.

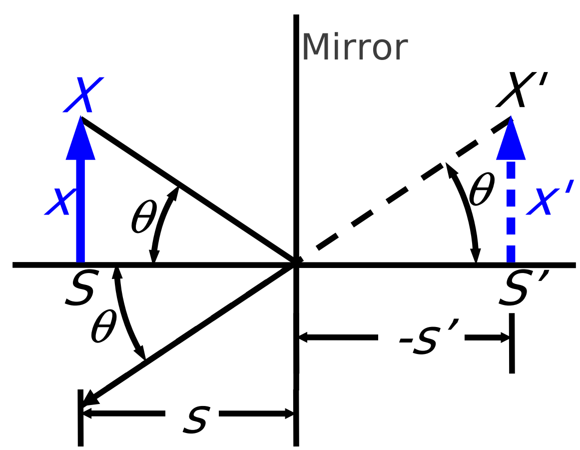
Misconception: Mirror Does Not Reverse Left and Right
Left is Left, Right is Right, but Front is Back

Imaging Equations

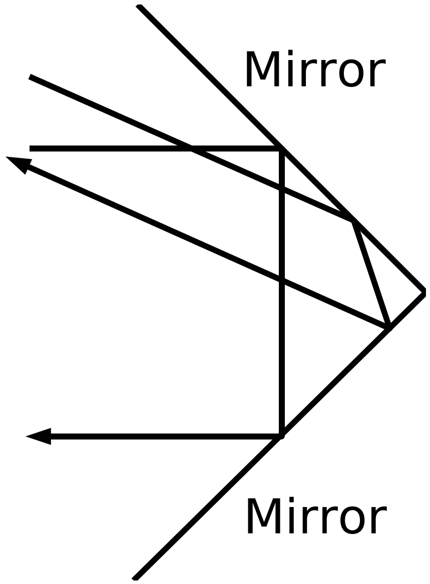
Surface	s'	m	m_α	m_z	**Image****	**O***	**D***	**P***
Planar Mirror***	$s' = -s$	1	-1	-1	Virtual	Upright	No	Yes
Concave Mirror $s > f$	$\frac{1}{s'} + \frac{1}{s} = \frac{1}{f}$	$-s'/s$	$-m^2$	$-1/m$	Real	Inverted	Yes	No
Convex Mirror	$\frac{1}{s'} + \frac{1}{s} = \frac{1}{f}$	$-s'/s$	$-m^2$	$-1/m$	Virtual	Upright	Yes	Yes
Planar Refractor	$\frac{s}{n} = \frac{s'}{n'}$	1	$\frac{n}{n'}$	$\frac{n'}{n}$	Virtual	Upright	Yes	No
Curved Refractor $s > f$	$\frac{n}{s} + \frac{n'}{s'} = \frac{n'-n}{r}$	$-\frac{ns'}{n's}$	-1	$-\frac{n}{n'}m^2$	Real	Inverted	Yes	Yes

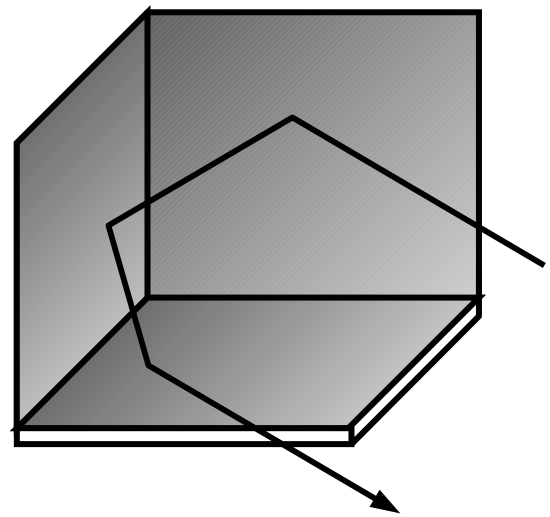
* "O" mean Orientation, "D" means Distortion, and "P" means Perversion of coordinates.
** The Image is Defined as Real or Virtual for a Real Object
*** Complete Analysis in green text

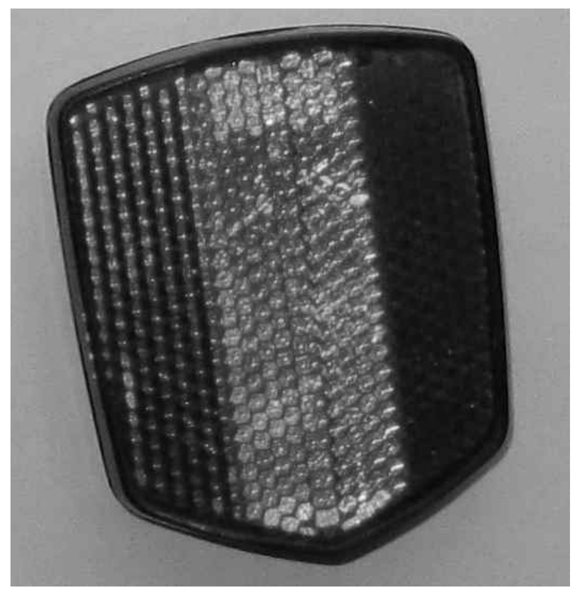




The Retroreflector

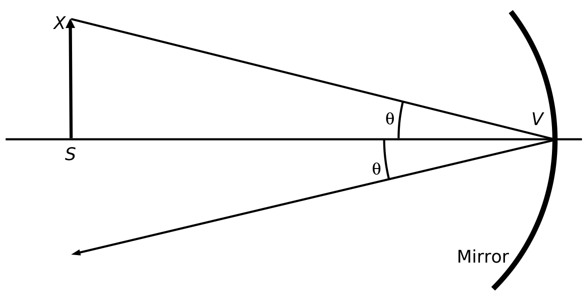




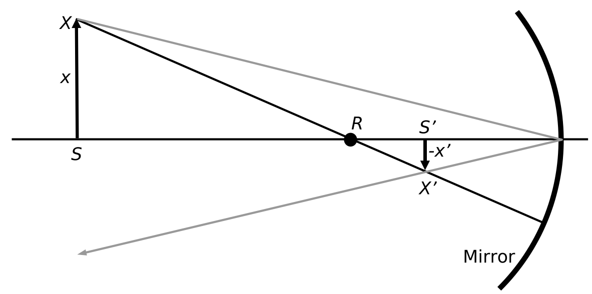


Curved (Spherical) Mirror (1)

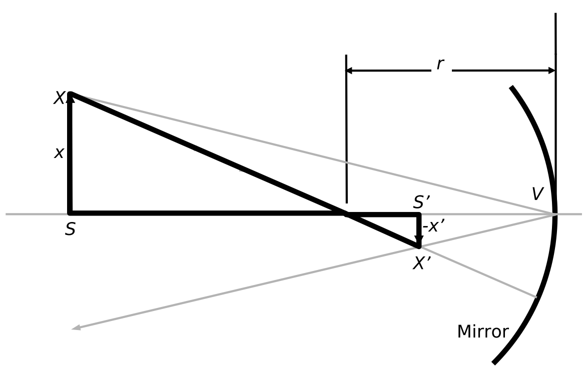
All Rays from the Object Go Through the Image (No Aberrations).
Work with the Easy Ones.



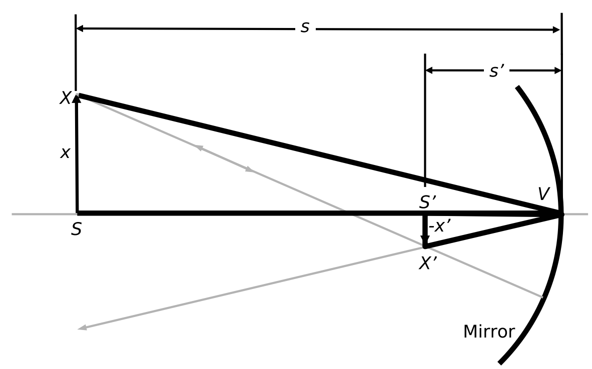
A. Vertex Ray



B. Radial Ray



C. Ray Intersection



D. Similar Triangles

$$\frac{x}{s-r} = \frac{-x'}{r-s'} \qquad (D)$$

$$\frac{x}{s} = \frac{-x'}{s'} \qquad (C)$$

Image Location

$$\frac{1}{s'} + \frac{1}{s} = \frac{2}{r}$$

Magnification

$$m = \frac{x'}{x} = -\frac{s'}{s}$$

Curved (Spherical) Mirror (2)

- Focal Length Defined in General

$$\boxed{\frac{1}{s'}+\frac{1}{s}=\frac{1}{f}}$$

- Specific Result for Spherical Mirror

$$\boxed{f=\frac{r}{2}\qquad \text{(Spherical reflector)}}$$

- Physical Signficance

$$s'\rightarrow f \qquad s\rightarrow\infty \qquad \text{or} \qquad s\rightarrow f \qquad s'\rightarrow\infty.$$

Curved (Spherical) Mirror (3)

- Angular Magnification

$$m_\alpha = \frac{s}{s'} \qquad |m_\alpha| = |1/m| \qquad \text{(Spherical reflector)}$$

- Axial Magnification

$$\frac{1}{s'} + \frac{1}{s} = \frac{2}{r}$$

$$-\frac{ds}{s^2} - \frac{ds'}{(s')^2} = 0$$

$$m_z = \frac{ds'}{ds} = -\left(\frac{s'}{s}\right)^2 \qquad m_z = -m^2 \qquad |m_z| = |m|^2$$

Curved (Spherical) Mirror (4)

- Imaging Equation

$$\frac{1}{s'} + \frac{1}{s} = \frac{1}{f} \qquad f = \frac{r}{2}$$

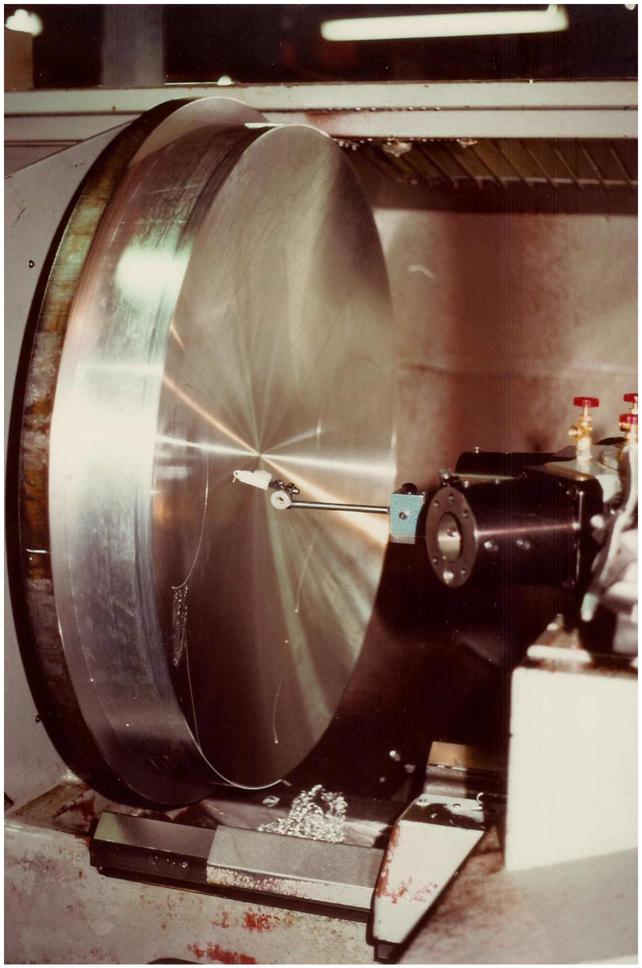
- Magnification

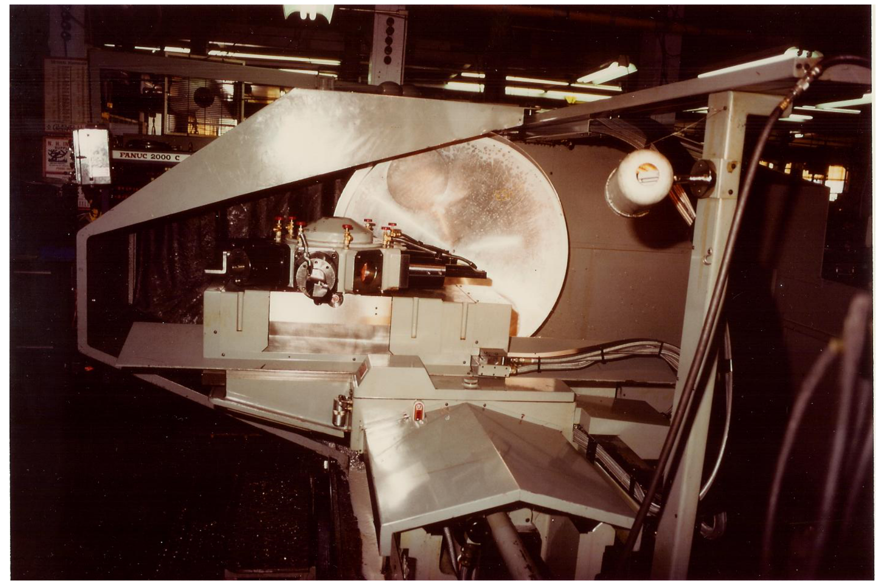
$$m = -\frac{s'}{s} \qquad m_\alpha = \frac{1}{m} \qquad m_z = -m^2$$

- Summary: The Image in this Case is...
  - Real
  - Inverted
  - Distorted (Unless $s = s'$)
  - Handedness–Preserved

- Question: Can a Concave Mirror Ever Produce a Virtual Image of a Real Object? (Hint: What if $s' = 0$?)

Large Reflective Optics

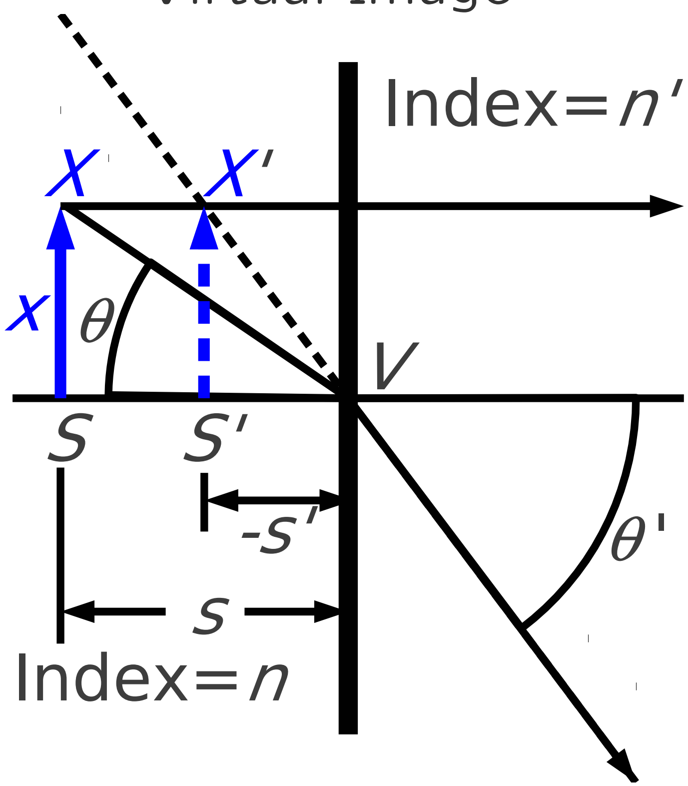




"Every Material that Transmits 10μm Light is Expensive."
Not Completely True, but Close.

The Fishtank Problem (1)

- Fishtank Setup
  - Object Inside
  - Viewer Outside
  - Virtual Image



- Geometry

$$\tan\theta = \frac{x}{s} \qquad \tan\theta' = \frac{x'}{s'} = \frac{x}{s'}$$

- Snell's Law (Small Angles)

$$n\sin\theta \approx n\frac{x}{s} \qquad n'\sin\theta' \approx n'\frac{x}{s'}$$

- Refraction at a planar Interface

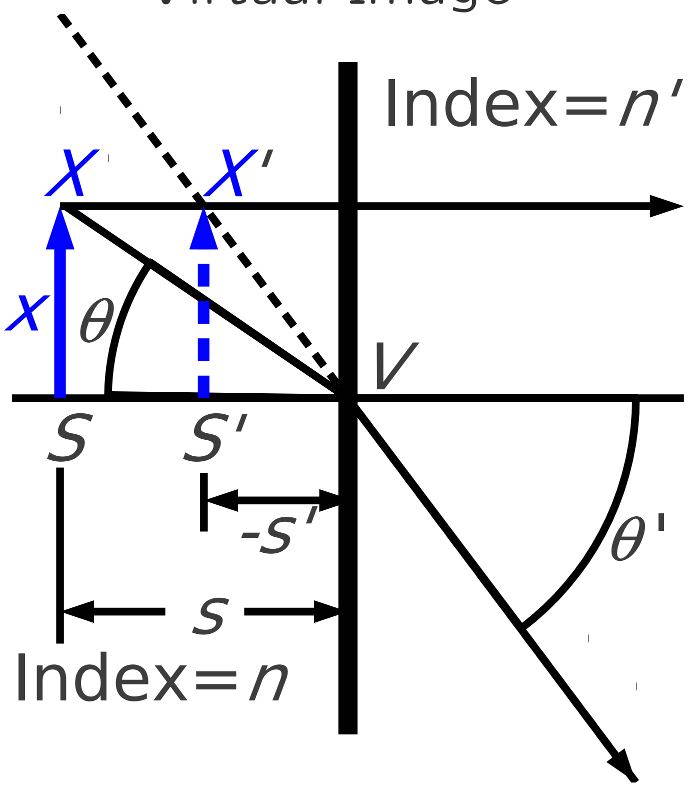
$$\boxed{\frac{n}{s} = \frac{n'}{s'}}$$

- Fishtank from Outside

$$n = 1.33 \qquad n' = 1 \qquad s' = \frac{1}{1.33}s$$

The Fishtank Problem (2)

- Fishtank Setup
  - Object Inside
  - Viewer Outside
  - Virtual Image



- Fishtank Paradox
  - Physical Thickness $= z$
  - Geometric Thickness

$$\ell_g = \frac{z}{n}$$

  - Optical Pathlength

$$OPL = zn$$

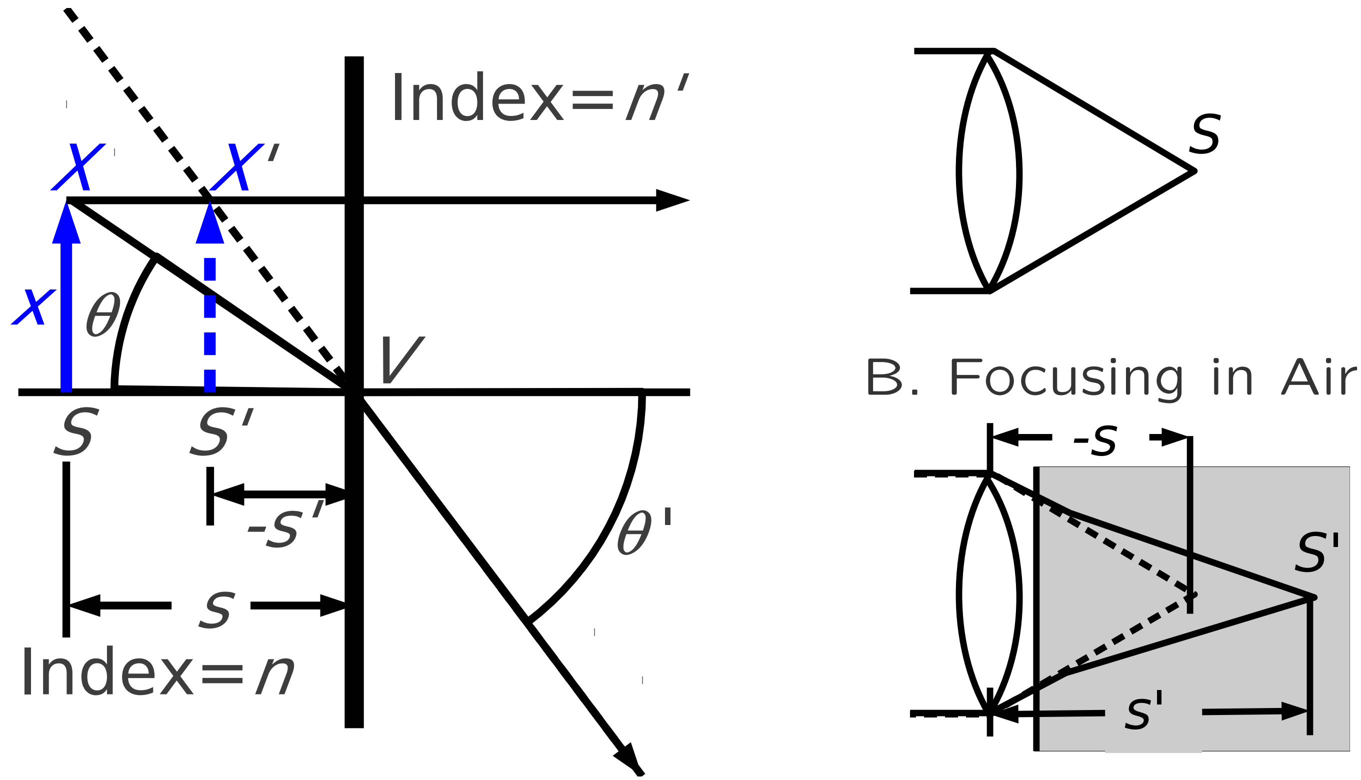
- Magnifications

$$m = \frac{x'}{x} = 1 \qquad m_\alpha = \frac{n}{n'}$$

$$m_z = \frac{ds'}{ds} = \frac{n'}{n}$$

- Virtual, Upright, Distorted

Practical Example



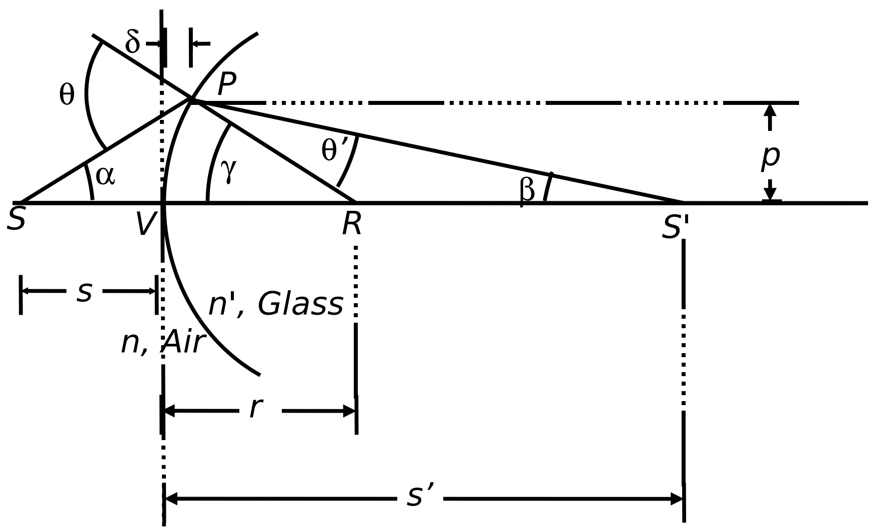
A. Planar Interface

B. Focusing in Air

C. Focusing in Skin

Focusing Depth Decreases, but OPL Increases. *e.g.* Focus to 100μm and image 75μm. Time Gate at 133μm (Optical Coherence Tomography) Together, measure index and depth?

Refraction: Curved Interface (1)



$\theta = \alpha + \gamma$ from $\triangle\ S, P, R,$

and

$\gamma = \theta' + \beta$ from $\triangle\ S', P, R$

$$\tan\alpha = \frac{p}{s+\delta} \qquad \tan\beta = \frac{p}{s'-\delta} \qquad \tan\gamma = \frac{p}{r-\delta}$$

For Small Angles tan ? = sin ? =? and $\delta \to 0$

$$\alpha = \frac{p}{s} \qquad \beta = \frac{p}{s'} \qquad \gamma = \frac{p}{r}$$

$$\theta = \frac{p}{s} + \frac{p}{r} \qquad \theta' = \frac{p}{r} - \frac{p}{s'}$$

Refraction: Curved Interface (2)

- Previous Page...

$$\theta = \frac{p}{s} + \frac{p}{r} \qquad\qquad \theta' = \frac{p}{r} - \frac{p}{s'}$$

- Snell's Law (Small Angles sin ? =?)

$$n\theta = n'\theta'$$

$$\frac{np}{s} + \frac{np}{r} = \frac{n'p}{r} - \frac{n'p}{s'}$$

$$\boxed{\frac{n}{s} + \frac{n'}{s'} = \frac{n' - n}{r} \qquad \text{(Refraction at a curved surface)}}$$

Refraction: Curved Interface (3)

- Focal Lengths (More Complicated Now)
  - Back Focal Length (Refraction at a curved surface)

$$s \to \infty \qquad BFL = f' = s' \frac{n'r}{n' - n}$$

  - Front Focal Length

$$s' \to \infty \qquad FFL = f = s = \frac{nr}{n' - n}$$

- Ratio (Calculated for this Example, but Much More General)

$$\frac{f'}{f} = \frac{n'}{n}$$

Refracting Power

- Definition

$$P = \frac{f}{n} = \frac{f'}{n'}$$

- Units

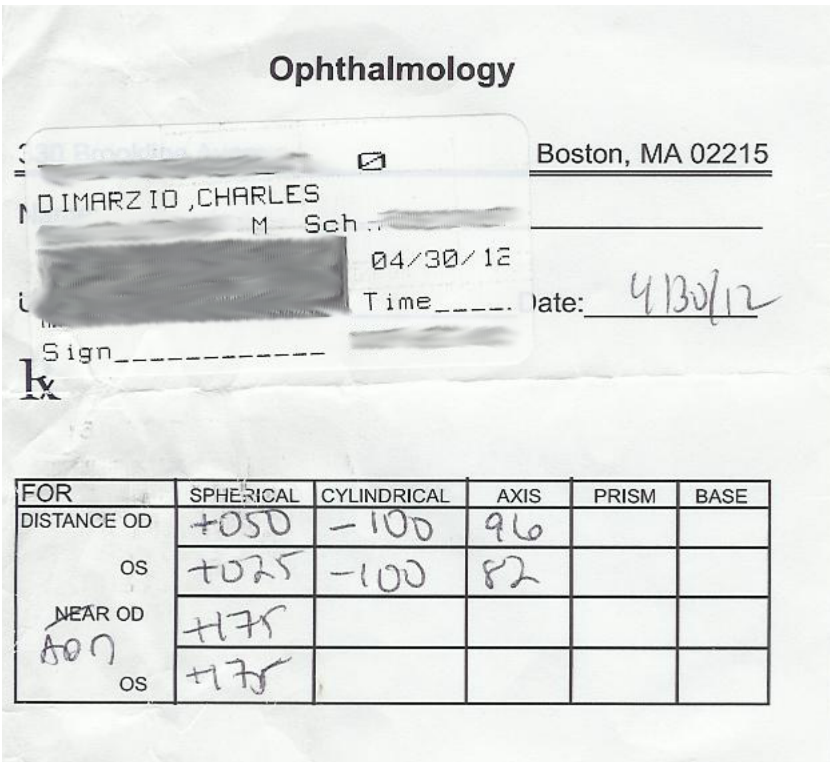
$$\text{Diopter} = \text{m}^{-1}$$

- Refraction at a Curved Interface

$$\boxed{P = \frac{n' - n}{r}}$$

Q: What combinations of n, n', and r yield positive (or negative) refracting power?

Eyeglass Prescription



Ophthalmology

[redacted] Boston, MA 02215

DIMARZIO, CHARLES

M Sch [redacted]

[redacted]

04/30/12

Time____ Date: 4/30/12

[redacted]

Sign____________

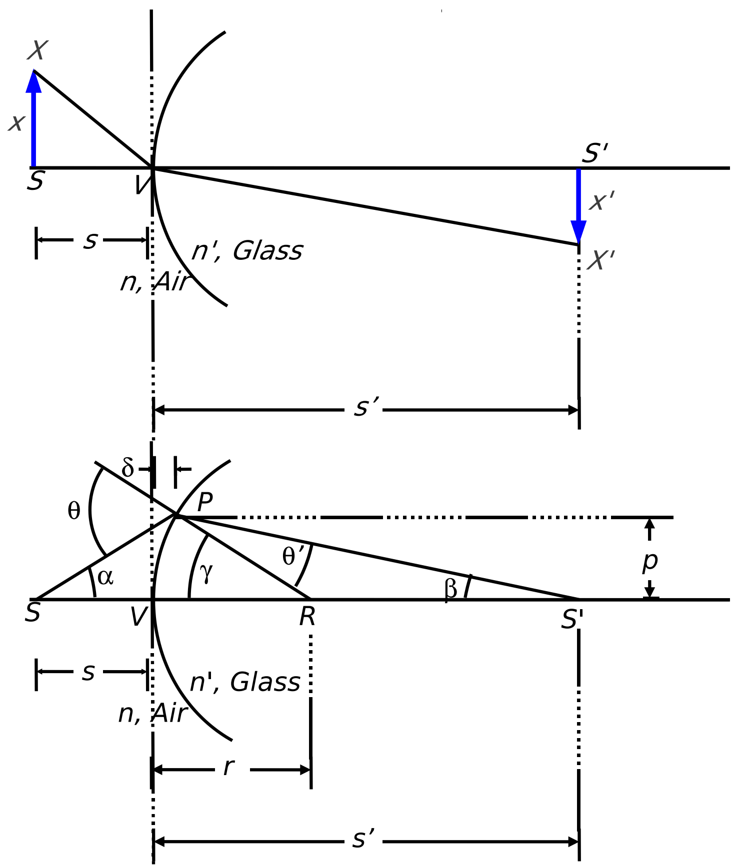
Rx

FOR	SPHERICAL	CYLINDRICAL	AXIS	PRISM	BASE
DISTANCE OD	+050	−100	96		
OS	+025	−100	82		
NEAR OD ADD	+175				
OS	+175				

- In Hundreths of Diopters
- Near Values Add to Far

- Left Eye (Oculus Sinister) Far:
  - 0.50 diopter 4° from Horizontal
  - -0.50 diopter 96°
- Left Eye Near (Add 1.75):
  - 2.25 diopter 4°
  - -1.25 diopter 96°
- Right Eye (Oculus Dexter) Far:
  - 0.25 diopter -8°
  - -0.75 diopter 82°
- Right Eye Near (Add 1.75):
  - 2.00 diopter -8°
  - -1.00 diopter 82°

Magnifications



Snell's Law at the Vertex

$$m = -\frac{ns'}{n's}$$

$$m_\alpha = \frac{-d\beta}{d\alpha} = -\frac{s'}{s} = -\frac{n}{n'}\frac{1}{m}$$

$$\frac{ds'}{ds} = -\frac{n}{n'}\left(\frac{s'}{s^2}\right)^2$$

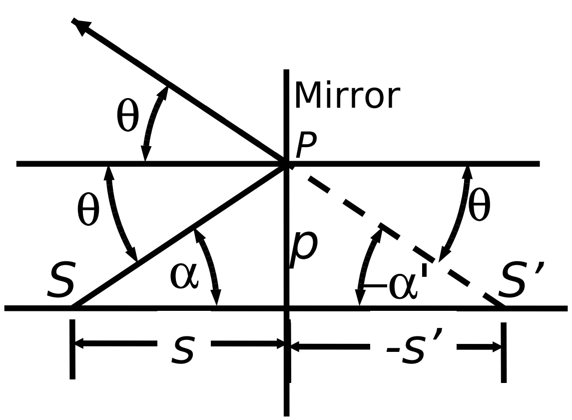
$$\frac{n}{s} + \frac{n'}{s'} = \frac{n'-n}{r} \qquad -\frac{n\ ds}{s^2} - \frac{n'\ ds'}{(s')^2} = 0$$

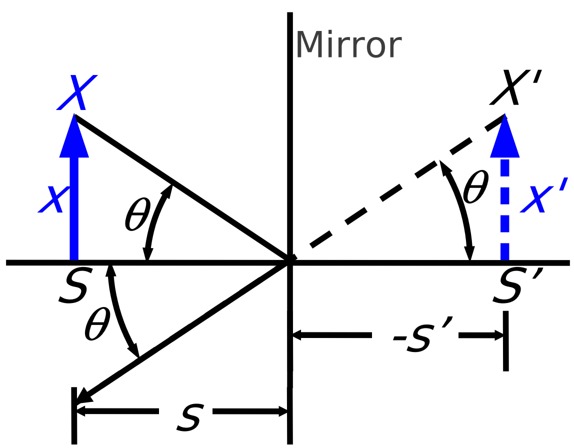
$$m_z = -\frac{n}{n'}m^2$$

Imaging Equations

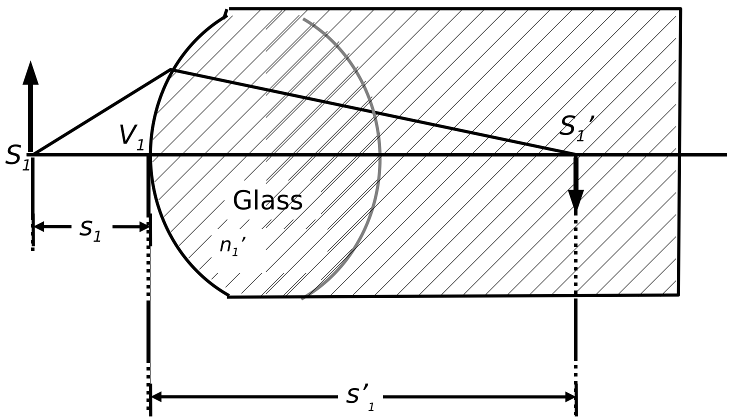
Surface	s'	m	m_α	m_z	Image**	O*	D*	P*
Planar Mirror	$s' = -s$	1	-1	-1	Virtual	Upright	No	Yes
Concave Mirror $s > f$	$\frac{1}{s'} + \frac{1}{s} = \frac{1}{f}$	$-s'/s$	$-m^2$	$-1/m$	Real	Inverted	Yes	No
Convex Mirror	$\frac{1}{s'} + \frac{1}{s} = \frac{1}{f}$	$-s'/s$	$-m^2$	$-1/m$	Virtual	Upright	Yes	Yes
Planar Refractor	$\frac{s}{n} = \frac{s'}{n'}$	1	$\frac{n}{n'}$	$\frac{n'}{n}$	Virtual	Upright	Yes	No
Curved Refractor $s > f$	$\frac{n}{s} + \frac{n'}{s'} = \frac{n'-n}{r}$	$-\frac{ns'}{n's}$	-1	$-\frac{n}{n'}m^2$	Real	Inverted	Yes	Yes

* "O" mean Orientation, "D" means Distortion, and "P" means Perversion of coordinates.
** The Image is Defined as Real or Virtual for a Real Object

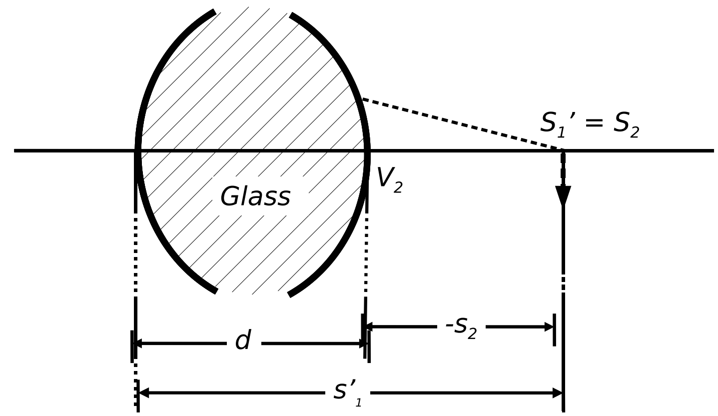




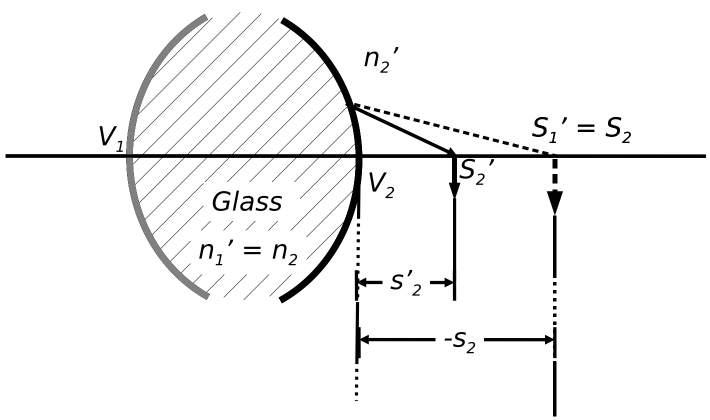
The Simple Lens



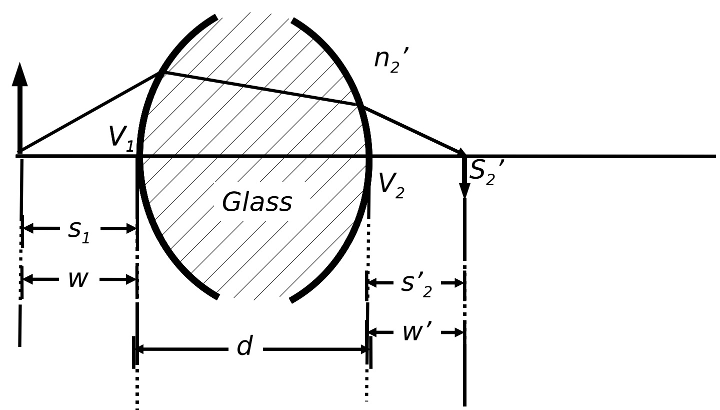
First Surface Object and Image



Second Surface Object

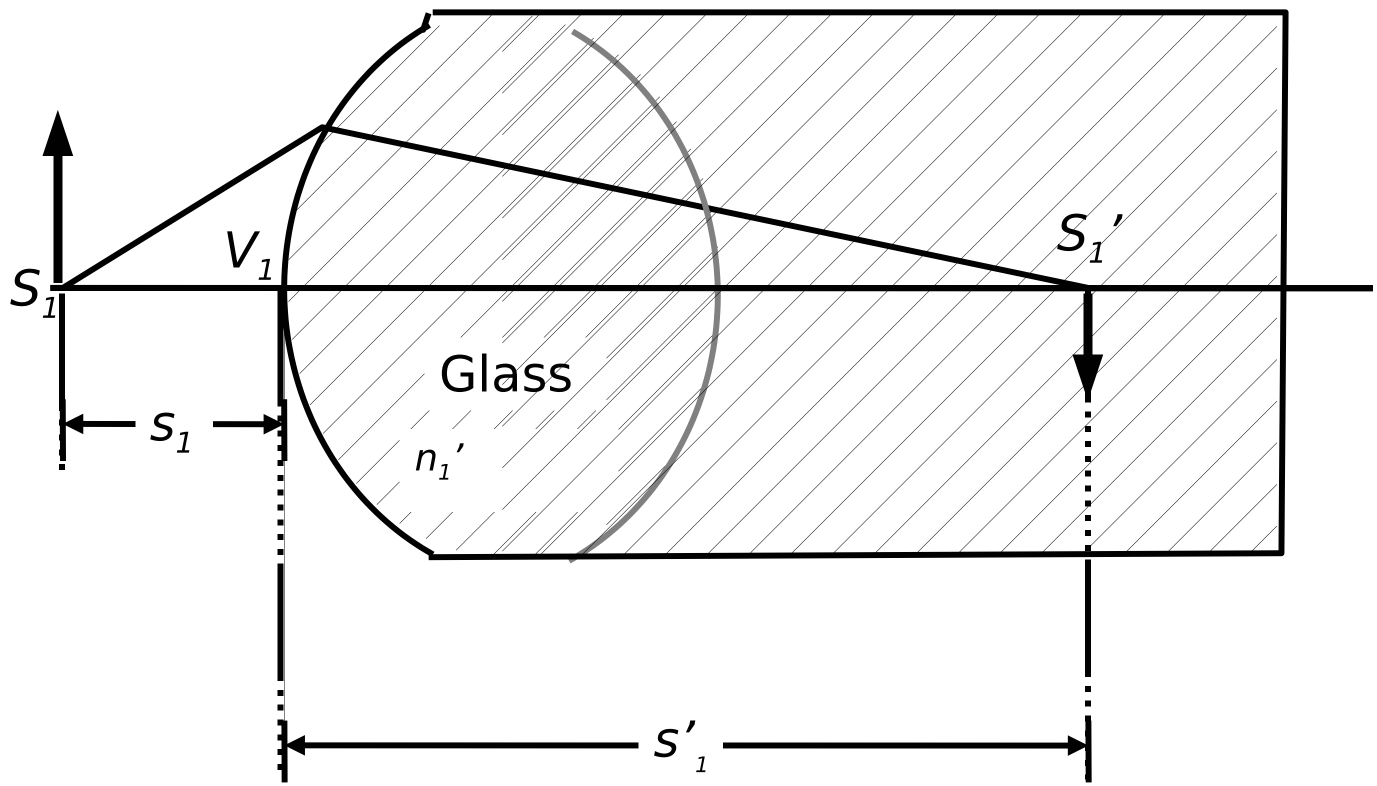


Second Surface Image



Complete Lens

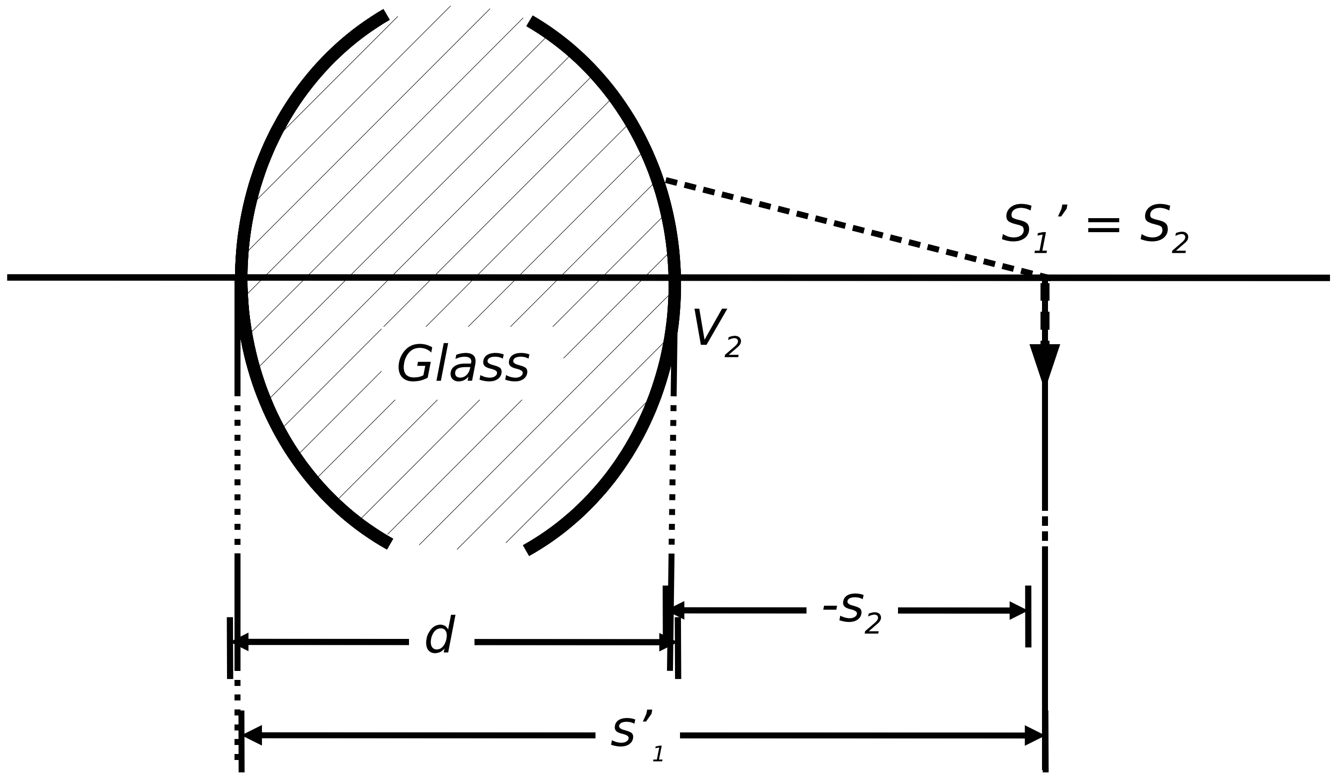
First Surface Solution



$$\frac{n_1}{s_1} + \frac{n_1'}{s_1'} = \frac{n_1' - n_1}{r_1}$$

Note Subscript $_1$

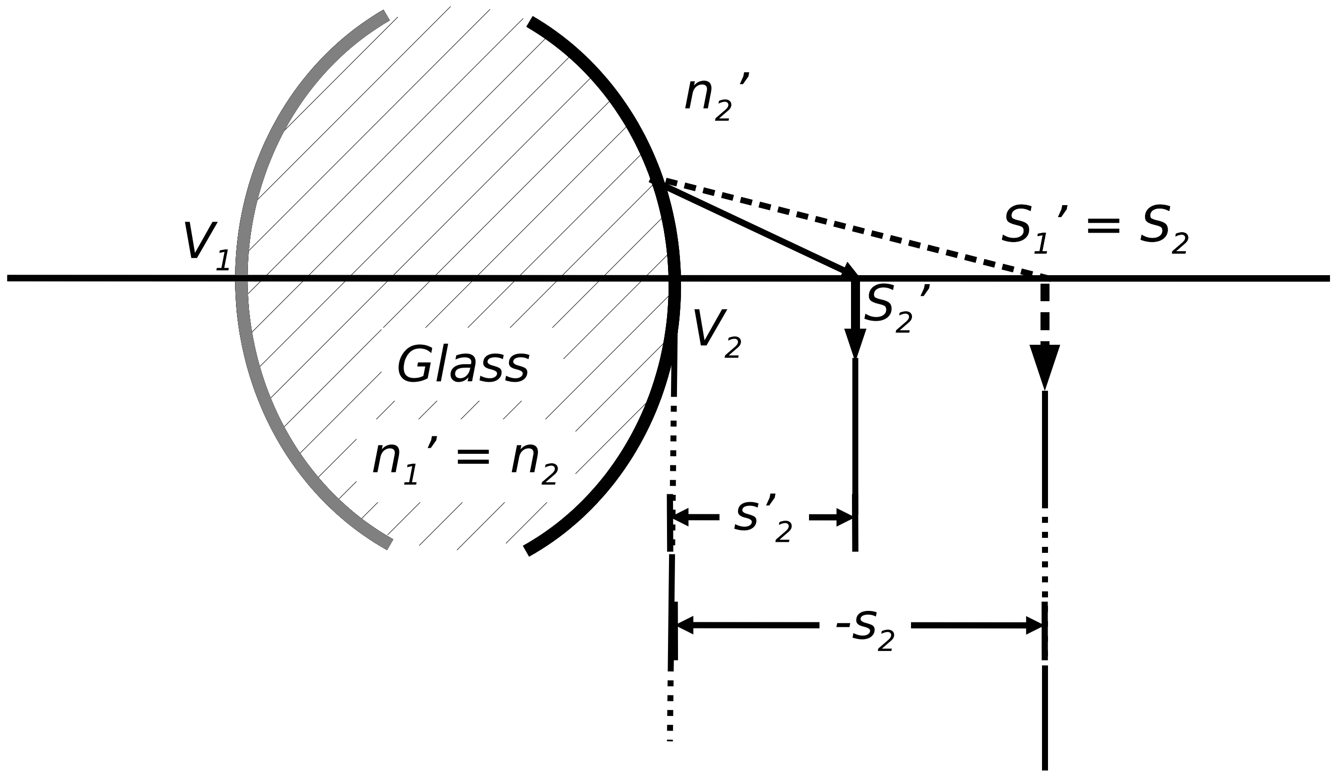
Second Surface Object



$$s_2 = -\left(s_1' - d\right) \qquad n_2 = n_1'$$

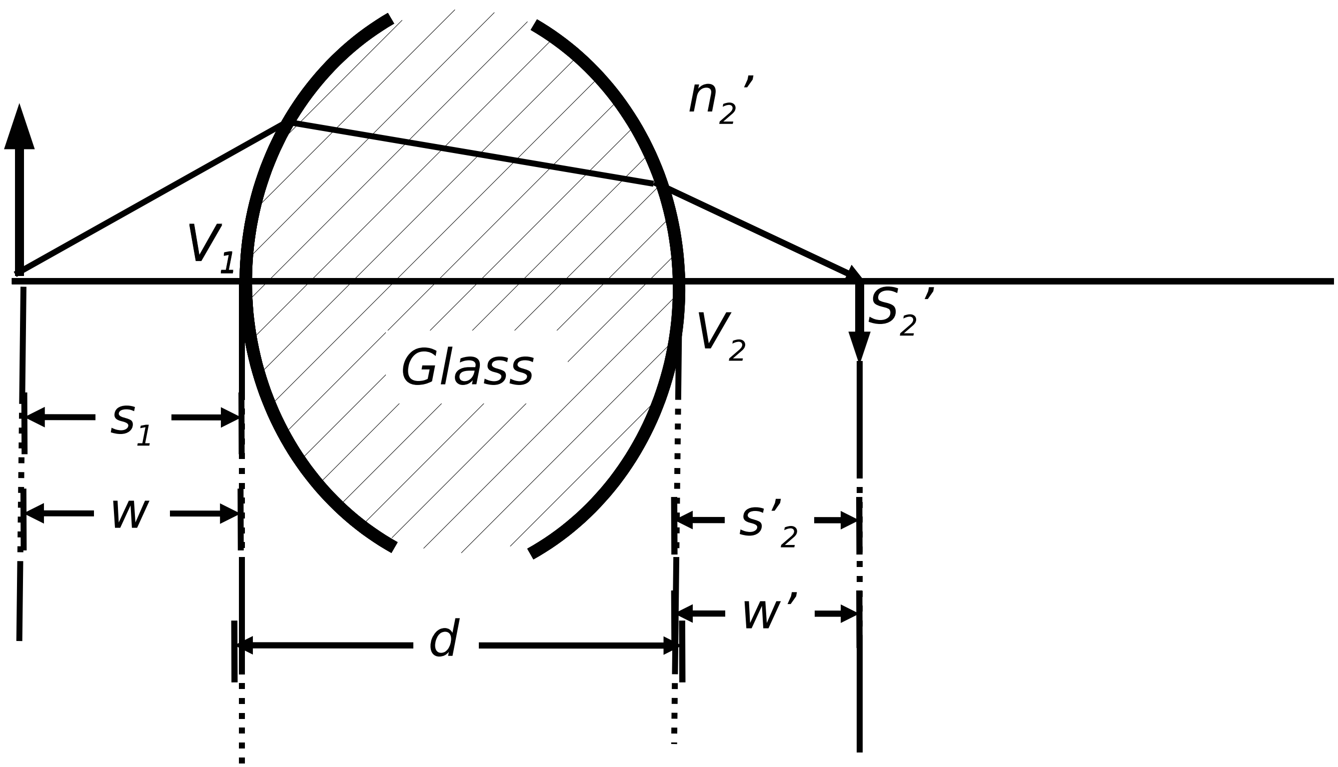
Virtual Object in this Case (Often but not Always)

Second Surface Solution



$$\frac{n_2}{s_2}+\frac{n_2'}{s_2'}=\frac{n_2'-n_2}{r_2} \qquad \frac{n_1'}{d-s_1'}+\frac{n_2'}{s_2'}=\frac{n_2'-n_1'}{r_2}$$

Complete Simple Lens (1)



$$\frac{n_2'}{s_2'} = \frac{n_2' - n_1'}{r_2} - n_1' \frac{n_1' - n_1 - r_1 \frac{n_1}{s_1}}{d\left(n_1' - n_1\right) - n_1' r_1 - d n_1' r_1 \frac{n_1}{s_1}}$$

Complete Simple Lens (2)

$$\frac{n_2'}{s_2'} = \frac{n_2' - n_1'}{r_2} - n_1' \frac{n_1' - n_1 - r_1 \frac{n_1}{s_1}}{d\left(n_1' - n_1\right) - n_1' r_1 - d n_1' r_1 \frac{n_1}{s_1}}$$

That's Ugly! Let's Define Some New Notation:

$$w = s_1 \qquad w' = s_2'$$

$$n = n_1 \qquad n' = n_2' \qquad n_\ell = n_1' = n_2$$

$$\frac{n'}{w'} = \frac{n' - n_\ell}{r_2} - n_\ell \frac{n_\ell - n - r_1 \frac{n}{w}}{d\left(n_\ell - n\right) - n_\ell r_1 - d n_\ell r_1 \frac{n}{w}}$$

Complete Simple Lens (3)

$$\frac{n'}{w'} = \frac{n' - n_\ell}{r_2} - n_\ell \frac{n_\ell - n - r_1 \frac{n}{w}}{d\left(n_\ell - n\right) - n_\ell r_1 - d n_\ell r_1 \frac{n}{w}}$$

That's Still Ugly. Set $n = n' = 1$. Not General, but Useful.

$$\frac{1}{w'} = \frac{1 - n_\ell}{r_2} - n_\ell \frac{n_\ell - 1 - r_1 \frac{1}{w}}{d\left(n_\ell - 1\right) - n_\ell r_1 - d n_\ell \frac{r_1}{w}}$$

Or Even Simpler, Set $d = 0$.

$$\frac{n}{w} + \frac{n'}{w'} = \frac{n' - n_\ell}{r_2} + \frac{n_\ell - n}{r_1}$$

The Thin Lens (1)

$$\frac{n}{w}+\frac{n'}{w'}=\frac{n'-n_\ell}{r_2}+\frac{n_\ell-n}{r_1}$$

Now The s *vs.* w Distinction Doesn't Matter.

$$\frac{n}{s}+\frac{n'}{s'}=\frac{n'-n_\ell}{r_2}+\frac{n_\ell-n}{r_1}$$

$$\frac{n}{s}+\frac{n'}{s'}=P_1+P_2=P$$

Back and Front Focal Lengths

$$BFL=f'=\frac{n'}{P_1+P_2} \qquad FFL=f=\frac{n}{P_1+P_2}$$

where $\quad P_1=\dfrac{n_\ell-n}{r_1} \qquad P_2=\dfrac{n'-n_\ell}{r_2}$

The Thin Lens (2)

$$BFL = f' = \frac{n'}{P_1 + P_2} \qquad FFL = f = \frac{n}{P_1 + P_2}$$

Focal–Length Relationship (Generally True)

$$\frac{f'}{f} = \frac{n'}{n}$$

Specifically

$$f = f' \qquad \text{if} \qquad n = n'$$

And In Air (Probably the Most–Used Equation in Optics)

$$\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}}$$

The Thin Lens (3)

- The Lensmaker's Equation

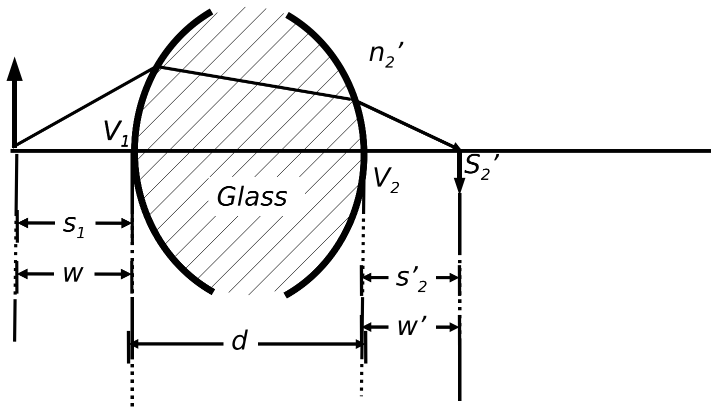
$$\boxed{\frac{1}{f} = \frac{1}{f'} = P_1 + P_2 = (n_\ell - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

- Be Careful About Signs (Biconvex Means $r_1 > 0$ and $r_2 < 0$)

$$P_1 = \frac{n_\ell - 1}{r_1} \qquad P_2 = \frac{n_\ell - 1}{-r_2}$$

- Powers Add for Thin Lenses

The Thin Lens Magnification



For $n = n'$ and $d = 0$

$$\frac{x'}{x} = \frac{-s'}{s}$$

$$m = \frac{-s'}{s} \qquad \text{(Lens in Air)}$$

$$m = \frac{-ns'}{n's} \qquad \text{(General)}$$

Axial Magnification

$$m_z = \frac{ds'}{ds} = \frac{n}{n'}\left(\frac{s'}{s}\right)^2 = \frac{n'}{n}m^2$$

Thin Lens in Air: Summary

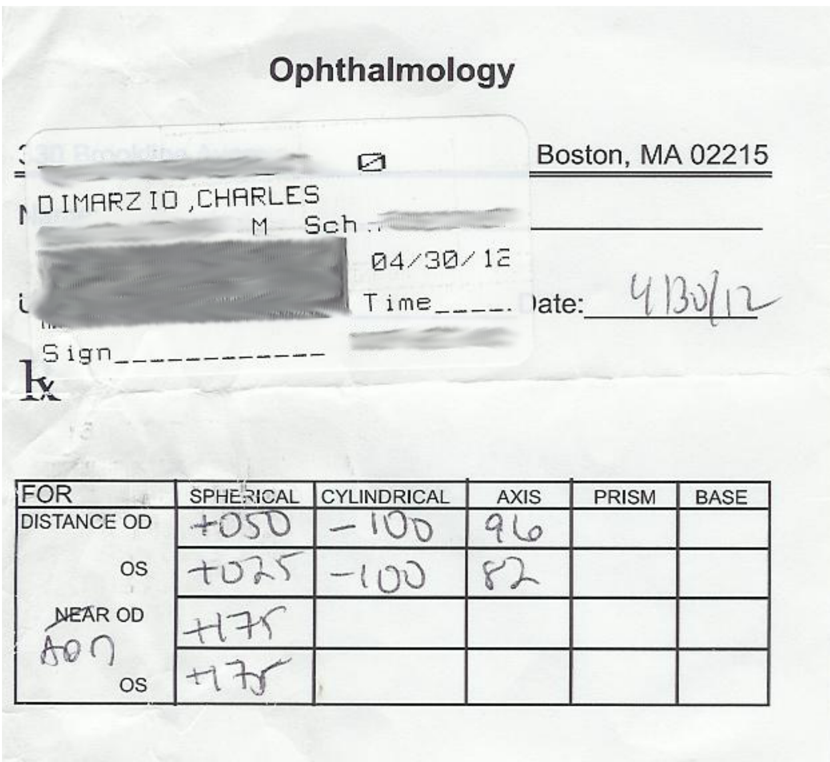
- Making The Lens (We Still Have Some Choices)

$$\boxed{\frac{1}{f} = \frac{1}{f'} = P_1 + P_2 = (n_\ell - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

- Using the Lens

$$\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad\qquad m = -\frac{s'}{s}}$$

Eyeglass Prescription Revisited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[redacted] 04/30/12

Time____ Date: 4/30/12

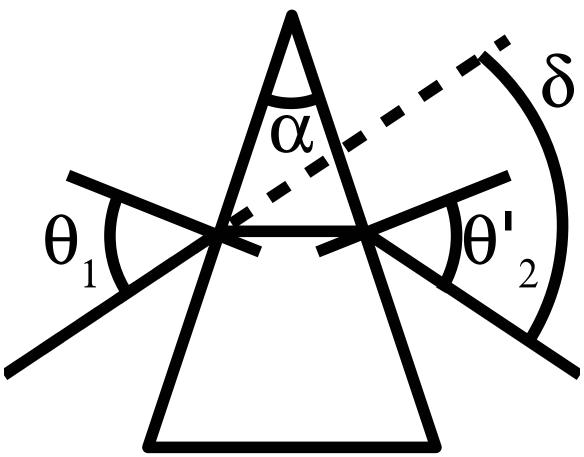
Sign___________ [redacted]

Rx

FOR	SPHERICAL	CYLINDRICAL	AXIS	PRISM	BASE
DISTANCE OD	+050	-100	96		
OS	+025	-100	82		
NEAR OD	+175				
Add OS	+175				

- Adding Powers
- Convex Front
- Concave Back
- Cylinder
- Many Options

Prisms (1)



$$\theta_1' = \frac{\theta_1}{n}$$

$$\left(90^\circ - \theta_2\right) + \left(90^\circ - \theta_1'\right) + \alpha = 180^\circ$$

$$\theta_2 + \theta_1' = \alpha.$$

Applying Snell's law,

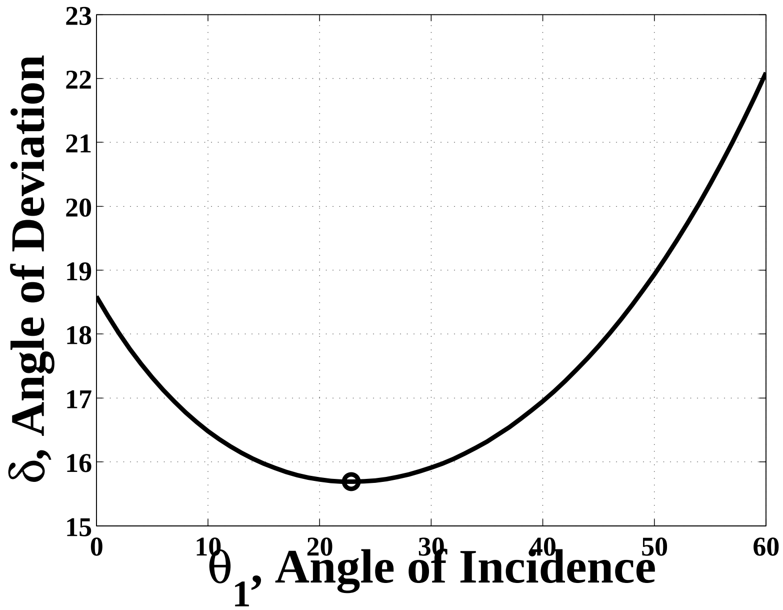
$$\sin\theta_2' = n\sin\theta_2 = n\sin\alpha - \theta_1'$$

$$\sin\theta_2' = n\left(\cos\theta_1'\sin\alpha - n\sin\theta_1'\cos\alpha\right)$$

$$\sin\theta_2' = \sqrt{n^2 - \sin^2\theta_1}\sin\alpha - \sin\theta_1'\cos\alpha$$

$$\delta = \theta_1 + \theta_2' - \alpha$$

Prisms (2)



$\alpha = 30^\circ$ and $n = 1.5$

- Deviation

$$\delta = \theta_1 + \theta_2' - \alpha$$

- Minimum Deviation

$$\delta_{min} = 2 \sin^{-1}\left(n \sin\frac{\alpha}{2}\right) - \alpha$$

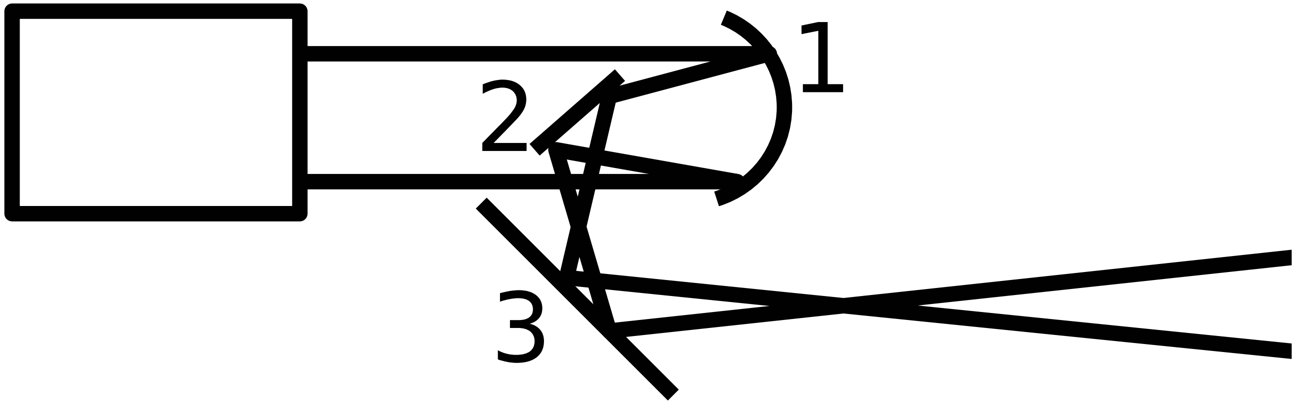
at $$\theta_1 = \sin^{-1}\left(n \sin\frac{\alpha}{2}\right)$$

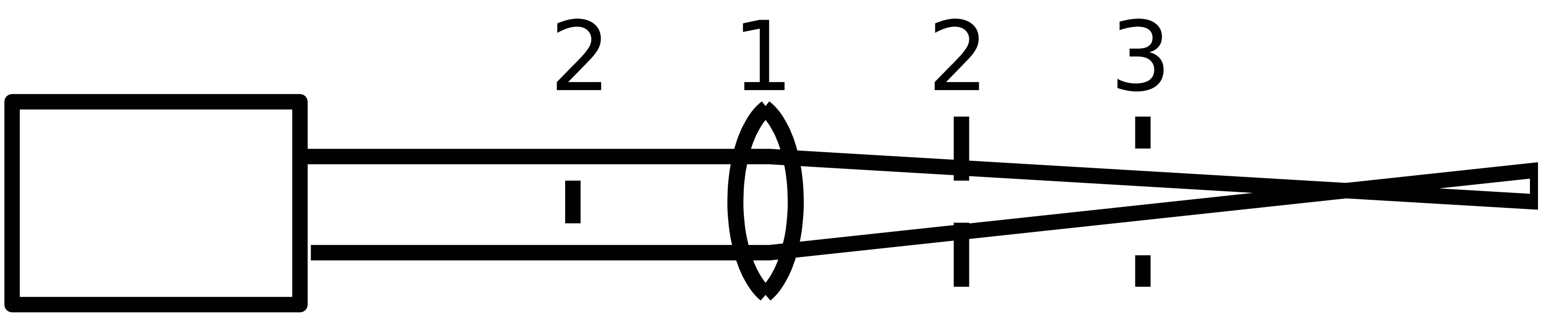
- Small Prism Angles

$$\delta_{min} \approx (n-1)\,\alpha$$

at $$\theta_1 = \frac{n\alpha}{2}$$

"Unfolding" Reflective Systems





Top Shows Actual System. Bottom Shows it Unfolded for Analysis