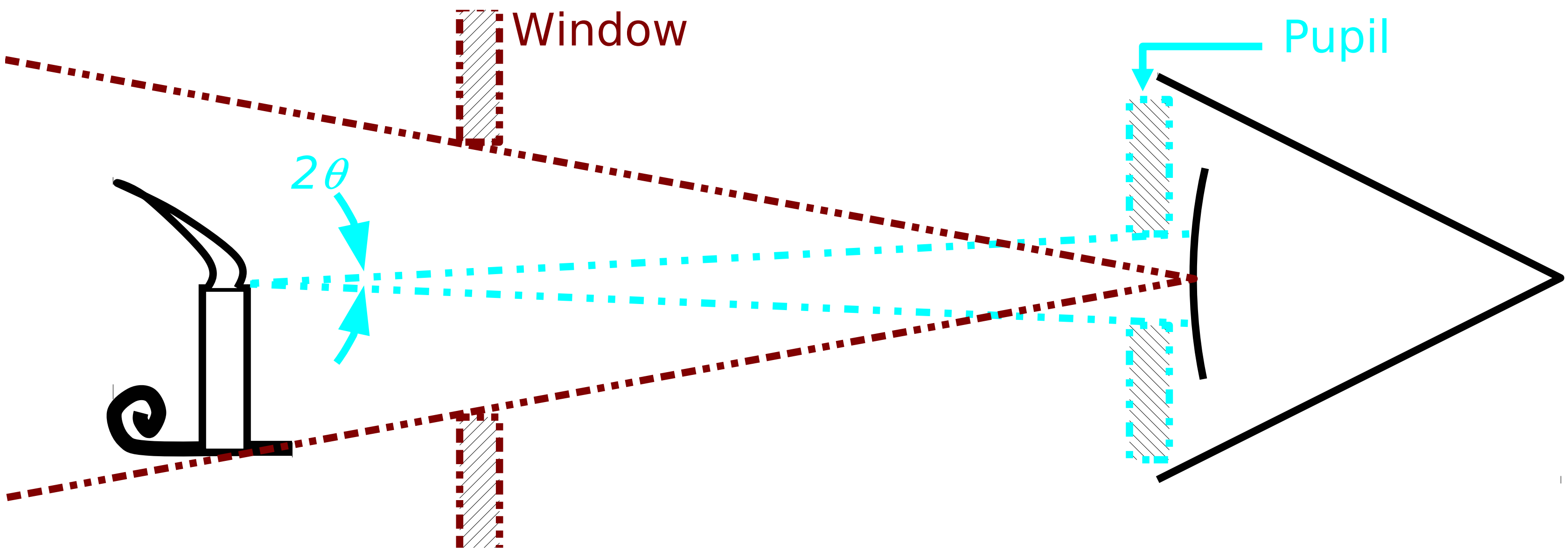
Optics for Engineers
Chapter 4

Charles A. DiMarzio
Northeastern University

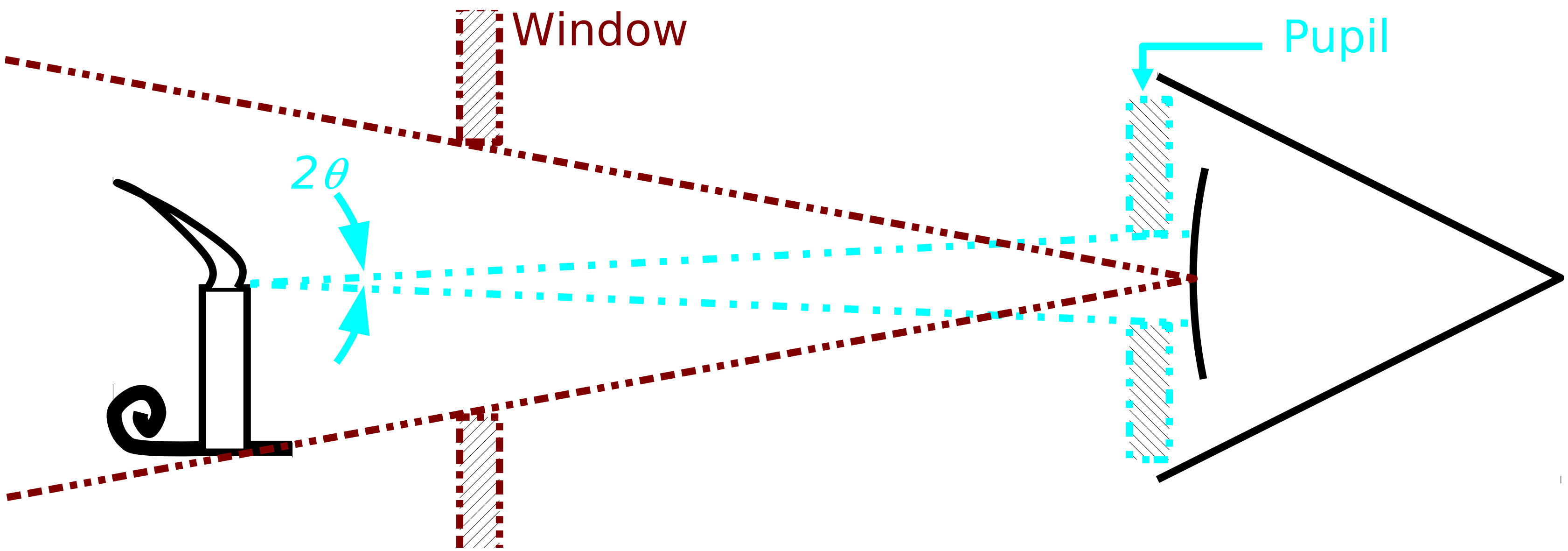
July 2012

Stops



- Pupil Diameter, D, Limits Light Gathering Ability
  - Usually Defined by f–number or Numerical Aperture
- Window Limits Field of View
  - Usually Defined by Angle(s) or Linear Dimension(s)

Numerical Aperture



$$NA_{object} = n \sin\theta = n \frac{D/2}{\sqrt{s^2 + (D/2)^2}}$$

$$NA_{image} = n' \sin\theta' = n' \frac{D/2}{\sqrt{(s')^2 + (D/2)^2}}$$

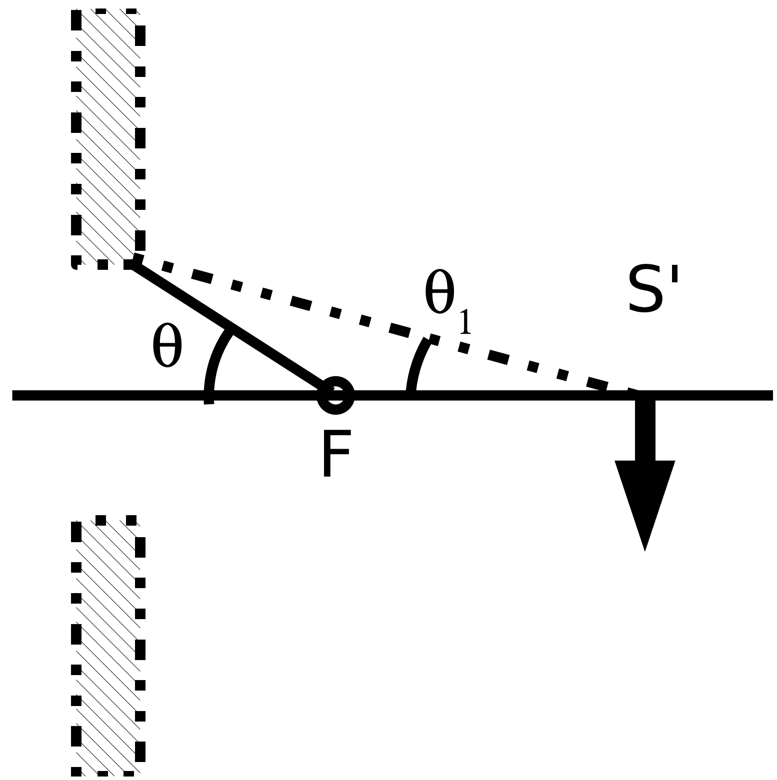
F–Number and NA (1)

$$F = \frac{f}{D}$$

$$NA = n \sin\theta$$

Differences
Summarized

	F	NA
Angle Vertex	Focal Point	Object or Image
Trig.	tan	sin
Dep.	Inv.	Lin.
"Fast" Lens	↓	↑
Aperture	Dia.	Rad.



F–Number and NA (2)

$$F = \frac{f}{D} \qquad NA = n\sin\theta$$

$$\frac{1}{f} = \frac{-m}{s'} + \frac{1}{s'} \qquad s' = (1-m)f$$

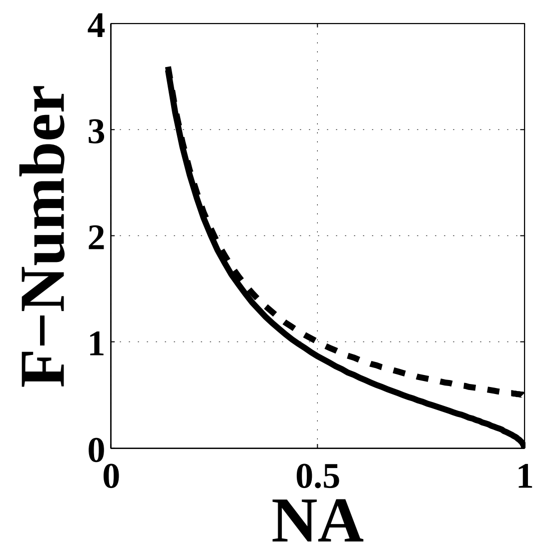
$$NA_{image} = n' \frac{1}{\sqrt{\left(|m-1| \times 2F\right)^2 + 1}}$$

$$NA_{object} = n \frac{1}{\sqrt{\left(\left|\frac{1}{m} - 1\right| \times 2F\right)^2 + 1}}$$

Small NA, Large F

$$NA_{image} =$$

$$n' \frac{1}{|m-1| \times 2F}$$



1:1 Relay ($m = -1$)

$$NA = \frac{1}{\sqrt{4F^2 + 1}}$$

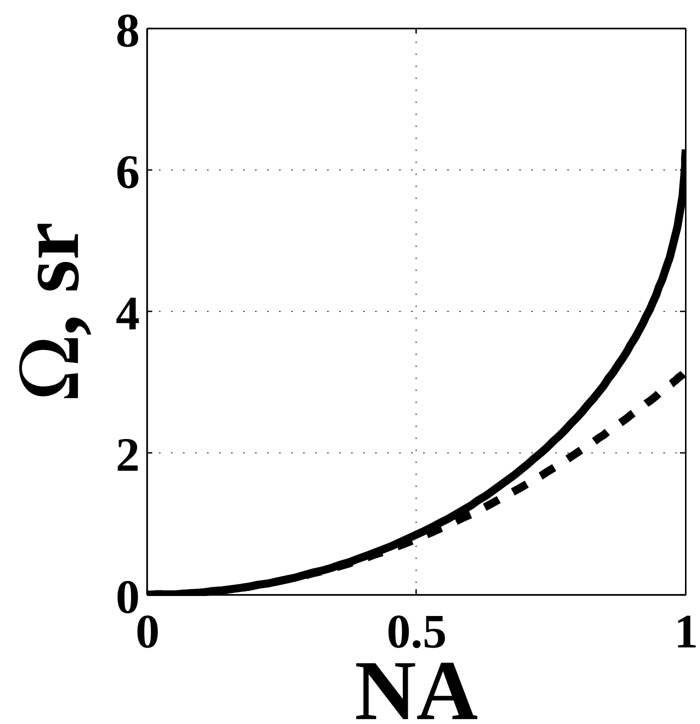
Light–Gathering Ability

$$dP_{aperture} = dI \times \Omega$$

$$NA_{object} = n \sin\theta = n \frac{D/2}{\sqrt{s^2 + (D/2)^2}}.$$

$$\Omega = 2\pi \left(1 - \sqrt{1 - \left(\frac{NA}{n}\right)^2}\right)$$

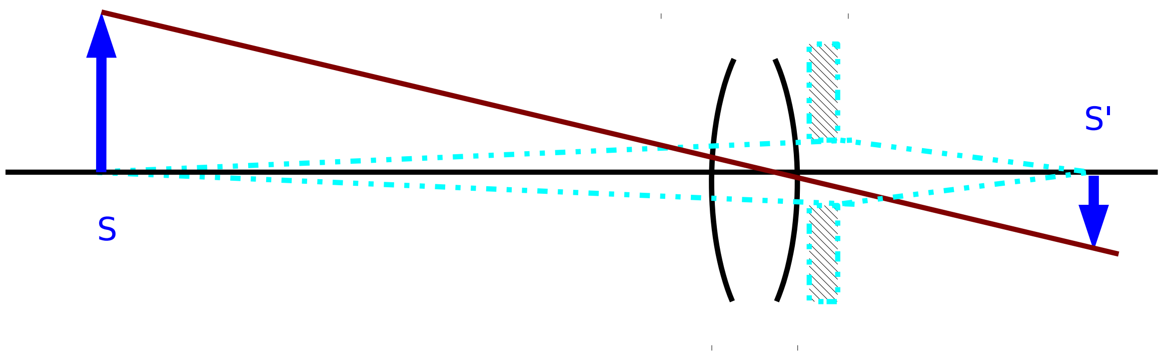
$$\Omega = \frac{\pi}{4}\left(\frac{D}{s}\right)^2 \qquad \text{(Small NA)}$$



Example: Camera (1)

Object: $dI = LdA$ (W/sr)
(Radiance, L in Ch. 12)
Change x_{pixel} to x'_{pixel} in Text

Object Distance	s	1000m
Camera Pixel	x'_{pixel}	7.4μm
Lens	f D	9mm $f/2$



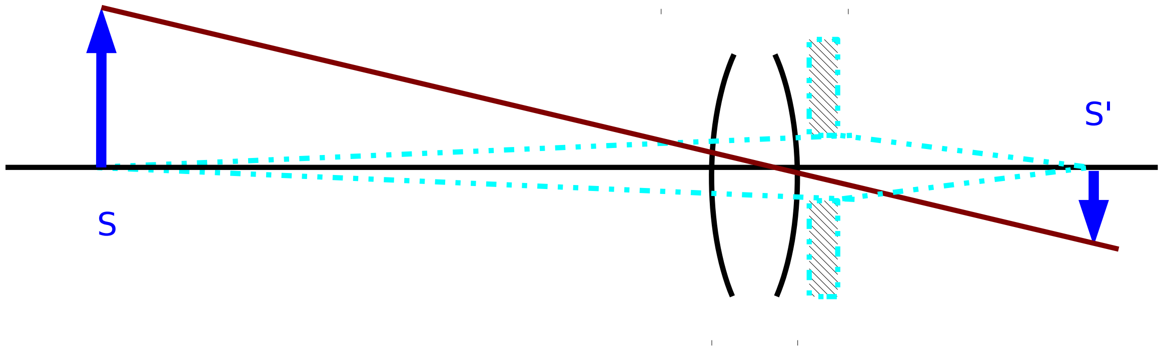
$$x_{pixel} = x'_{pixel}/m = \left(7.4 \times 10^{-6}\text{m}\right) / \left(9 \times 10^{-6}\right)$$

Area $=0.68\text{m}^2$

Intensity of Scattered Sunlight (See Ch. 12)

$$\frac{1000\text{W/m}^2}{\pi} \times 0.25 \approx 50\text{W/sr}$$

Example: Camera (2)



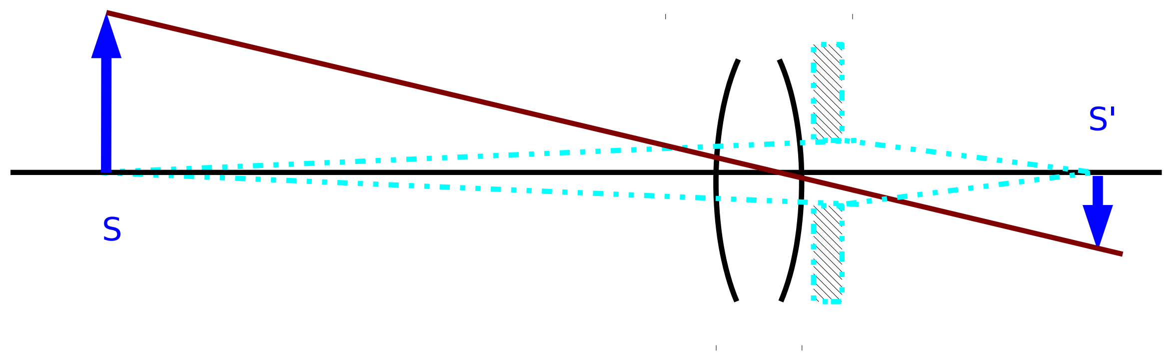
Intensity of Scattered Sunlight: 50W/sr

$$NA_{Object} \approx \frac{D}{2s} = \frac{f}{2Fs} = \frac{0.009\text{m}}{4 \times 1000\text{m}} = 2.25 \times 10^{-6}$$

$$\Omega \approx \pi NA^2 = 1.6 \times 10^{-11}\text{sr}$$

$$dP_{aperture} = dI\Omega = 50\text{W/sr} \times 1.6 \times 10^{-11}\text{sr} = 7.9 \times 10^{-10}\text{W}$$

Example: Camera (3)



$$dP_{aperture} = dI\Omega = 50\text{W/sr} \times 1.6 \times 10^{-11}\text{sr} = 7.9 \times 10^{-10}\text{W}$$

Photon Energy: $h\nu = hc/\lambda$

Photons (Lots of 'em!):

$$N = \frac{dP_{aperture}}{h\nu}\eta t$$

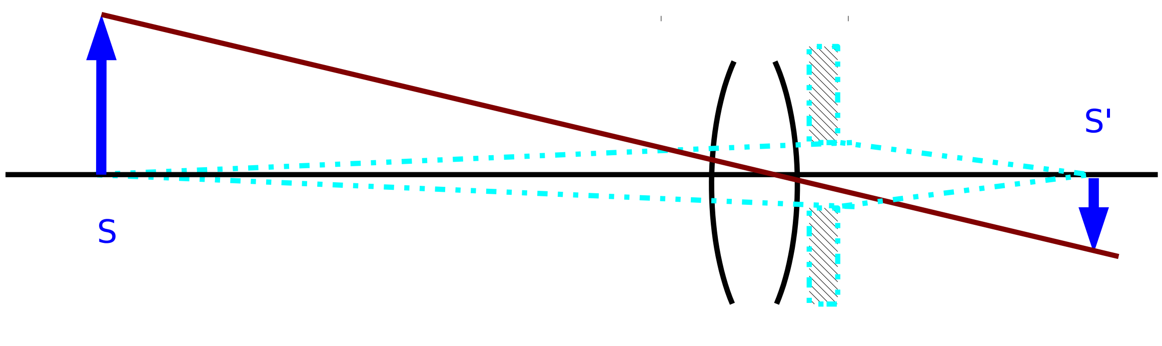
Wavelength (Green)	λ	500nm
Quantum Efficiency	η	0.4
Frame Time	t	1/30sec
Electrons	N	2.7×10^6

Camera Apertures

Abbe Invariant ($m_\alpha = 1/m$) Implies Constant Etendue (See Ch. 12)

$$A' = m^2 A \qquad \Omega' = \frac{1}{m^2}\Omega$$

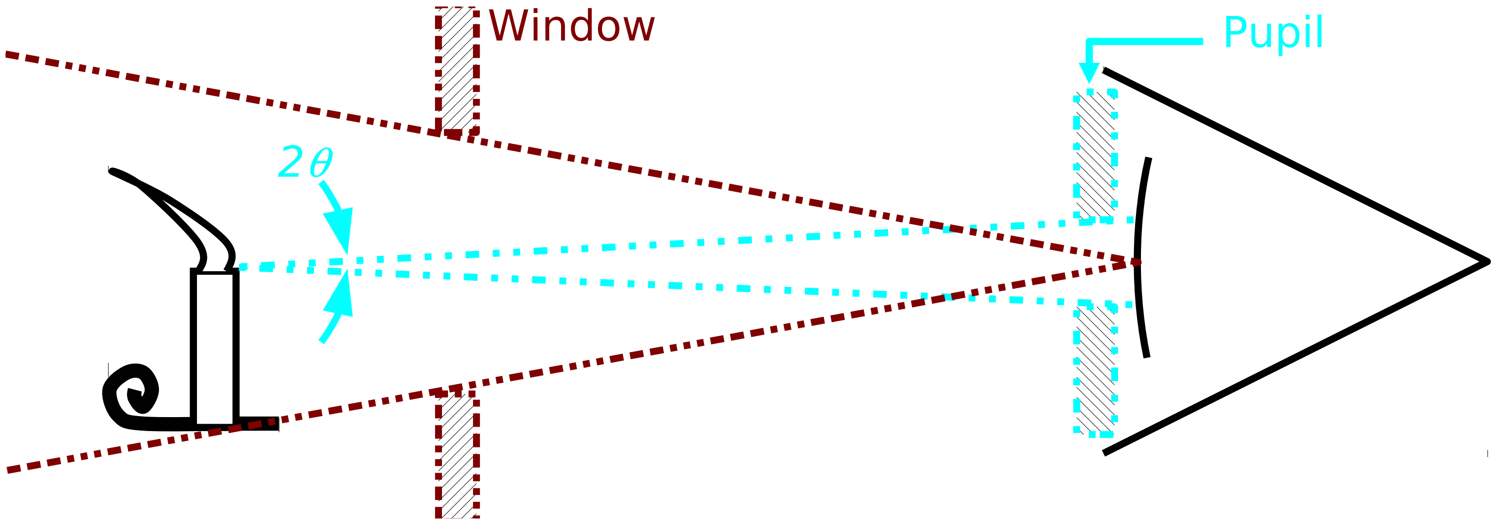
$$A\Omega = A'\Omega' \qquad LA\Omega = LA'\Omega'$$



Aperture Stops: Each Stop Is a Factor of 2

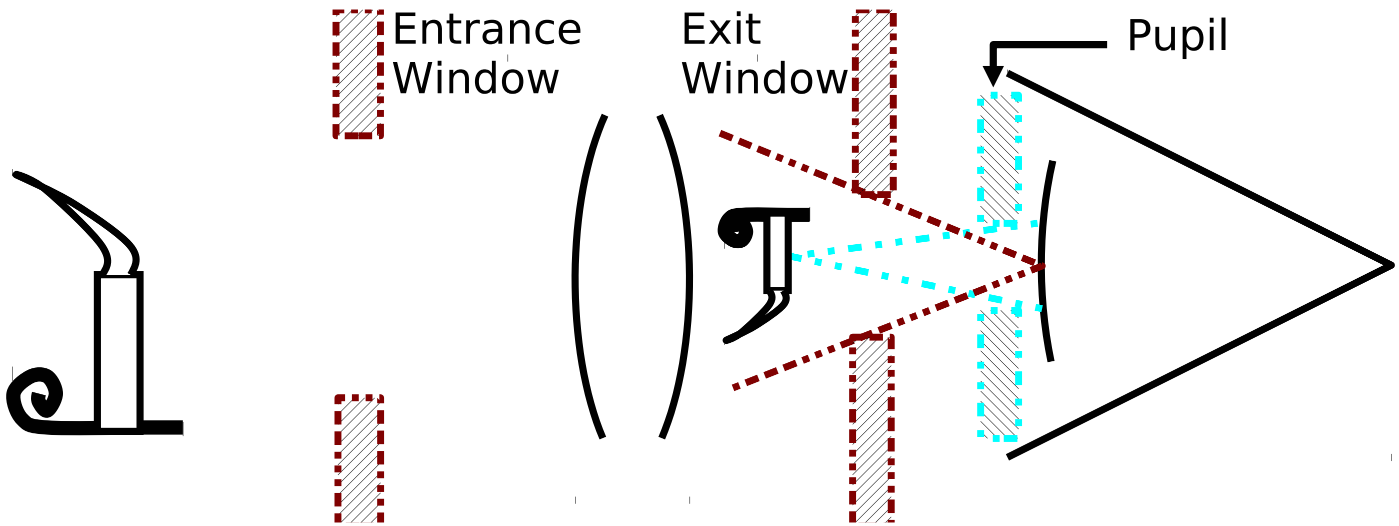
F, Indicated f–number	1.4	2	2.8	4	5.6	8	11
Actual f–number	$\sqrt{2}^1$	$\sqrt{2}^2$	$\sqrt{2}^3$	$\sqrt{2}^4$	$\sqrt{2}^5$	$\sqrt{2}^6$	$\sqrt{2}^7$
NA	0.3536	0.2500	0.1768	0.1250	0.0884	0.0625	0.0442
Ω, sr	0.3927	0.1963	0.0982	0.0491	0.0245	0.0123	0.0061

The Field Stop



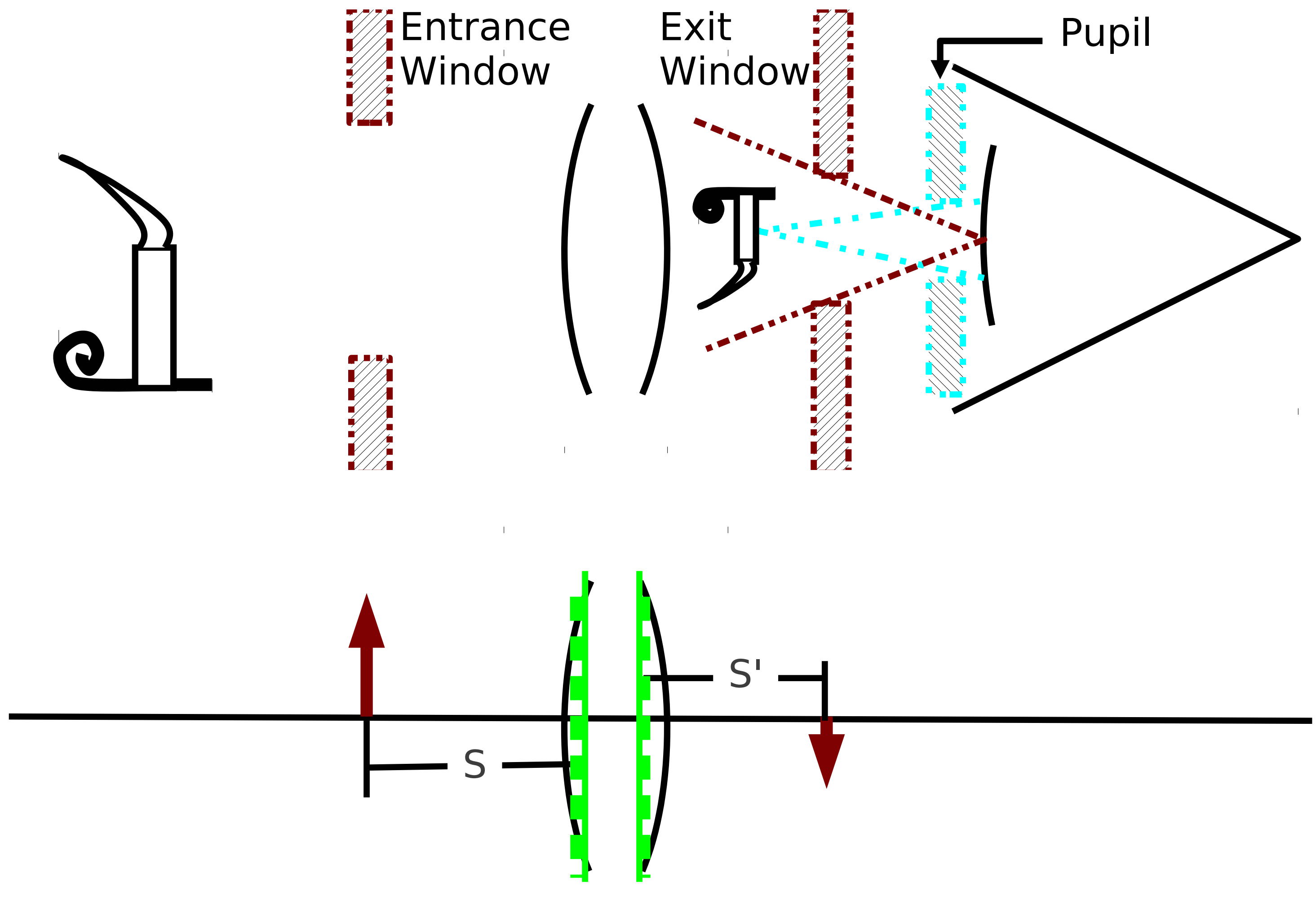
- Simple Example as Shown
  - Window = Field Stop
  - Called the Entrance Window
- Field of View
  - Limits Size of Object (Diameter or Angle)...
  - ...or Size of Image

Exit Window: Example



- Entrance Window Limits Size of Object
- Exit Window = Image of Field Stop
- Exit Window Limits Size of Image
- Location and Size from Imaging Equations

Finding the Exit Window



Use Imaging Equations: Location and Size

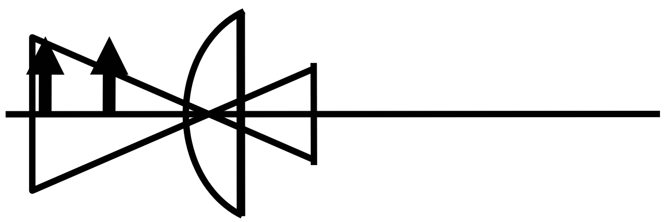
Camera FOV (1)

- Field of View Lmited in Image Rather than Object
  - Camera Chip is the Limit
  - 1/2.3in Compact Digital Camera
  - Diagonal Dimension = 11mm.
  - Image Field of View (Here Defined by Half Angle)

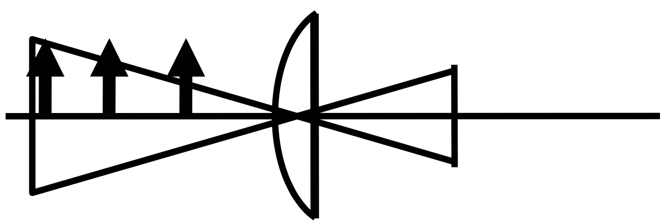
$$f = 10\text{cm} \qquad \text{(Normal Lens)} \qquad s \rightarrow \infty$$

$$FOV = 2\arctan\frac{11\text{mm}/2}{10\text{mm}} = 58^\circ$$

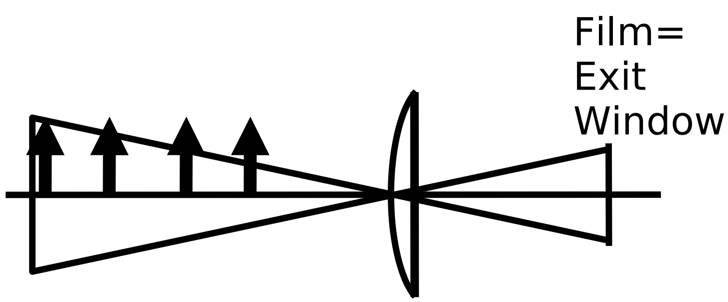
Camera FOV (2)



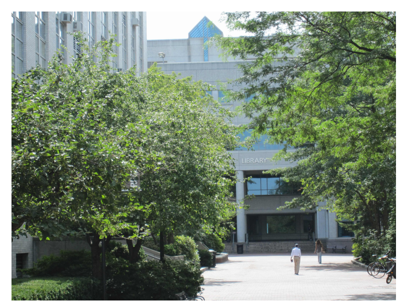
Wide–Angle Lens, $f = 5$mm



Normal Lens, $f = 10$mm



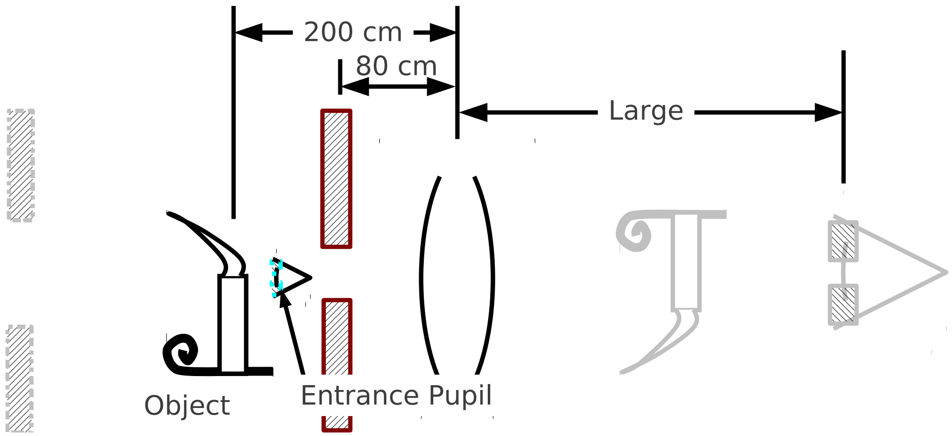
Telephoto Lens, $f = 20$mm



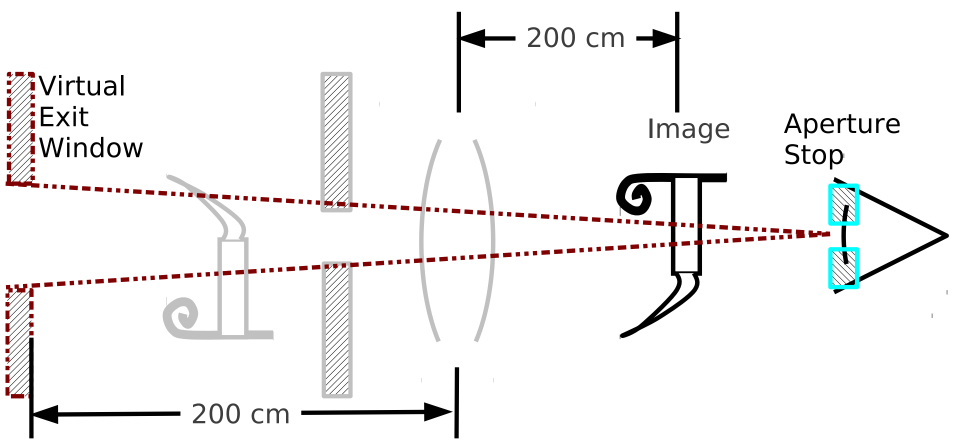
$$FOV = 2\arctan\frac{11\text{mm}/2}{f}$$

- Photographer Moved Away with Increasing f
- Same Linear FOV on the Building in Each Image
- Differences in Foreground Images

Another Example: Virtual Exit Window



$s = 200\text{cm},\ s_{fieldstop} = 80\text{cm}.$



$s' = 200\text{cm},\ s'_{fieldstop} = -400\text{cm}.$

$f = 100\text{cm}$

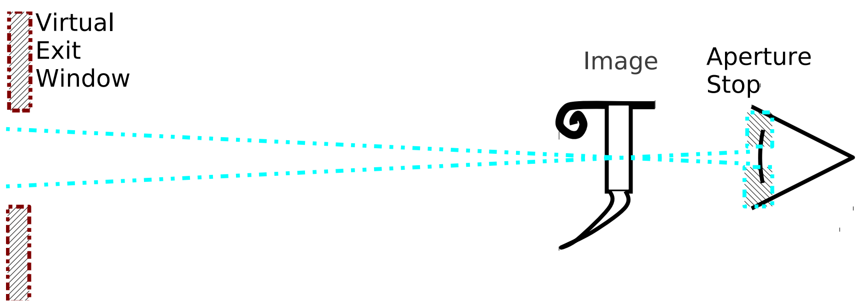
Field Stop Limits FOV (Top)

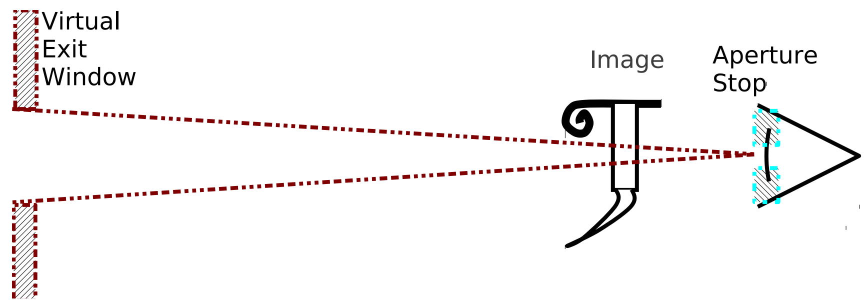
Virtual Exit Window Seen by Observer (Bottom)

Virtual Entrance Pupil in Object Space (Top)

Apertures Function Regardless of Apparent Order (Actual Sequence Matters).

Summary In Image Space

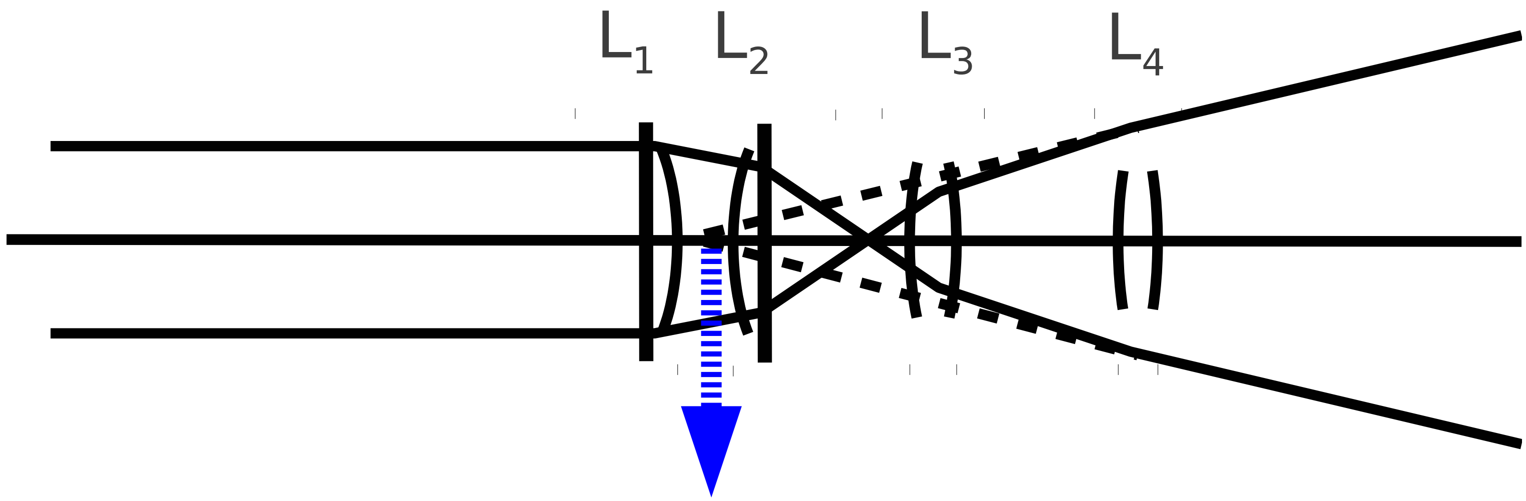




- Pupil Limits Light–Gathering Ability
- Cone of Rays From Image is Limited
- Solid Angle Determines Amount of Light Collected

- Window Limits Field of View
- Cone of Rays from Pupil is Limited
- FOV Defined by Angle or Linear Dimension

Where Are the Stops?



- Compound Lens
- Object to Left at Infinity
- Image as Shown
- Where Are the Stops Now?
  - Aperture Stop?
  - Field Stop?

Object Space, Image Space, and Stop Definitions

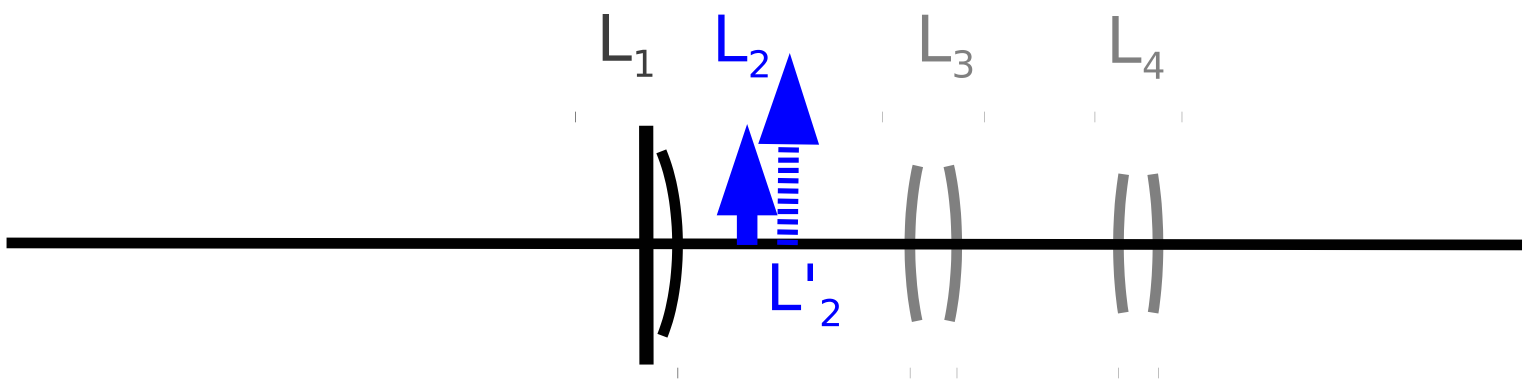
Mapping from Object Space to Image Space through the Compound Lens

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} \qquad x' = -\frac{s'}{s}x$$

Object Space	Physical Component	Image Space
Entrance Pupil: Image of Aperture Stop in Object Space. Limits Cone of Rays from Object	**Aperture Stop:** Limits Cone of Rays from Object which Can Pass Through the System.	**Exit Pupil:** Image of Aperture Stop in Image Space. Limits Cone of Rays from Image.
Entrance Window: Image of Field Stop in Object Space. Limits Cone of Rays From Entrance Pupil.	**Field Stop:** Limits Locations of Points in Object which Can Pass Through System	**Exit Window:** Image of Field Stop in Image Space. Limits Cone of Rays From Exit Pupil.

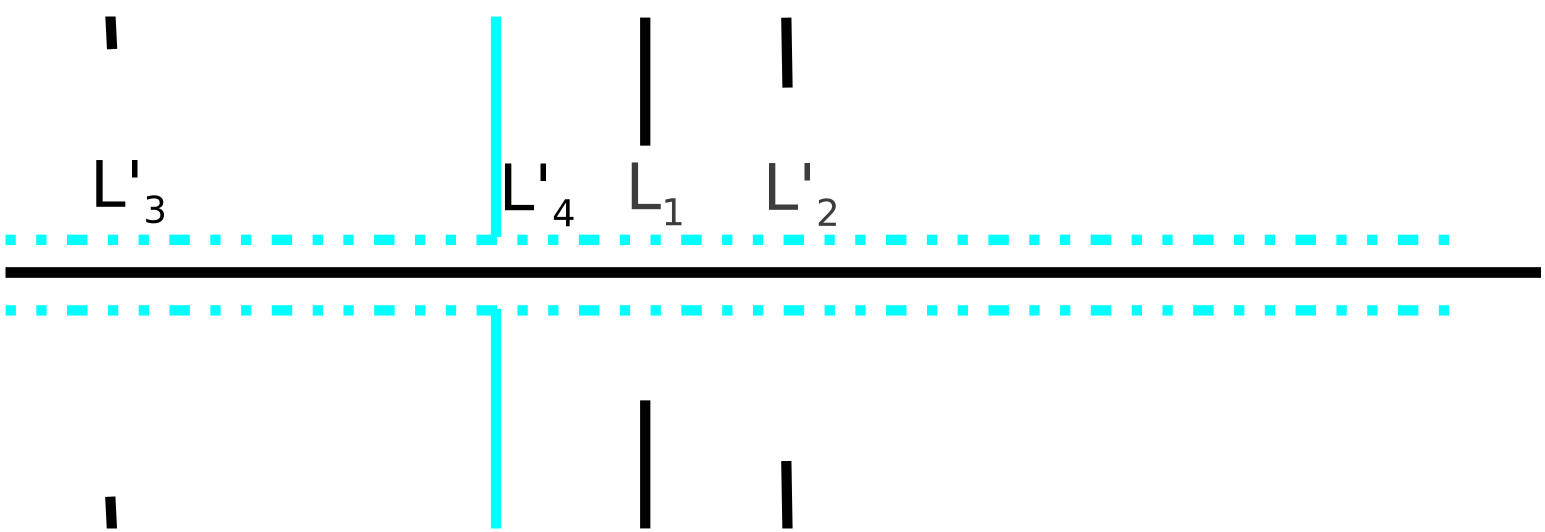
Finding the Stops in Object Space

- Find Each Lens as Seen in Object Space
  - Lens L_1
  - Lens L_2 as Seen Through L_1
  - Lens L_3 as Seen Through L_2 and L_1
  - Lens L_4 as Seen Through L_3, L_2 and L_1



Finding the Entrance Pupil

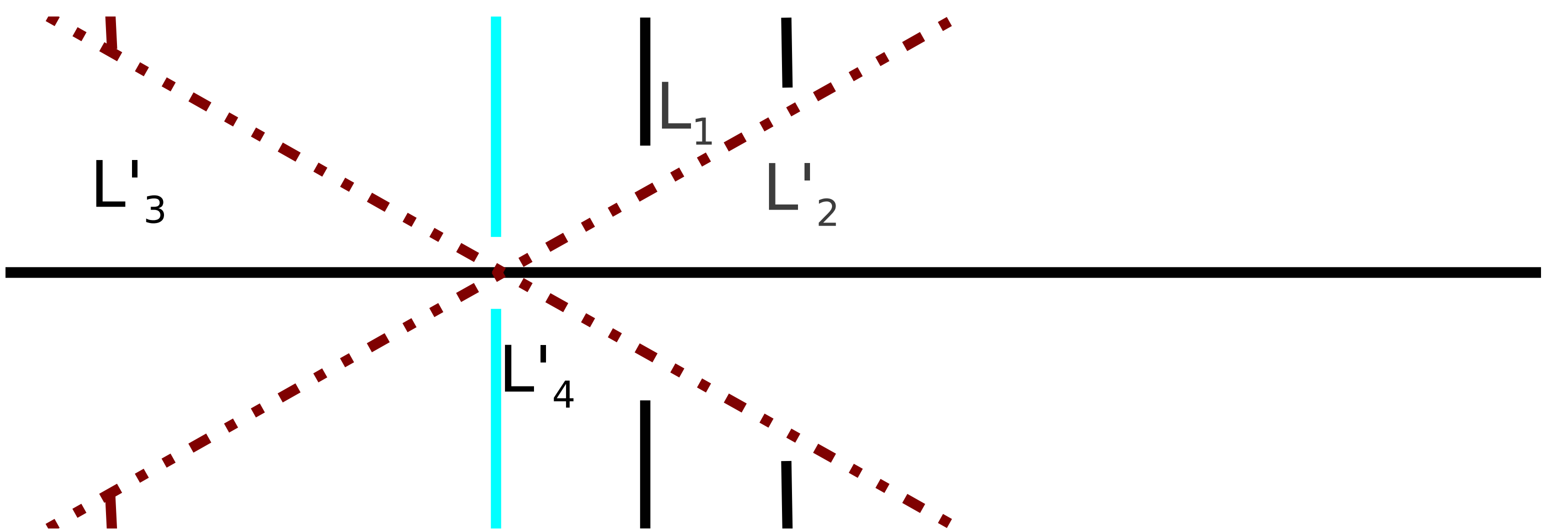
- Start at the Object (To the Left at Infinity Here)
- Find the Aperture that Limits Cone of Rays from Object
- Entrance Pupil is L'_4, Aperture Stop is L_4



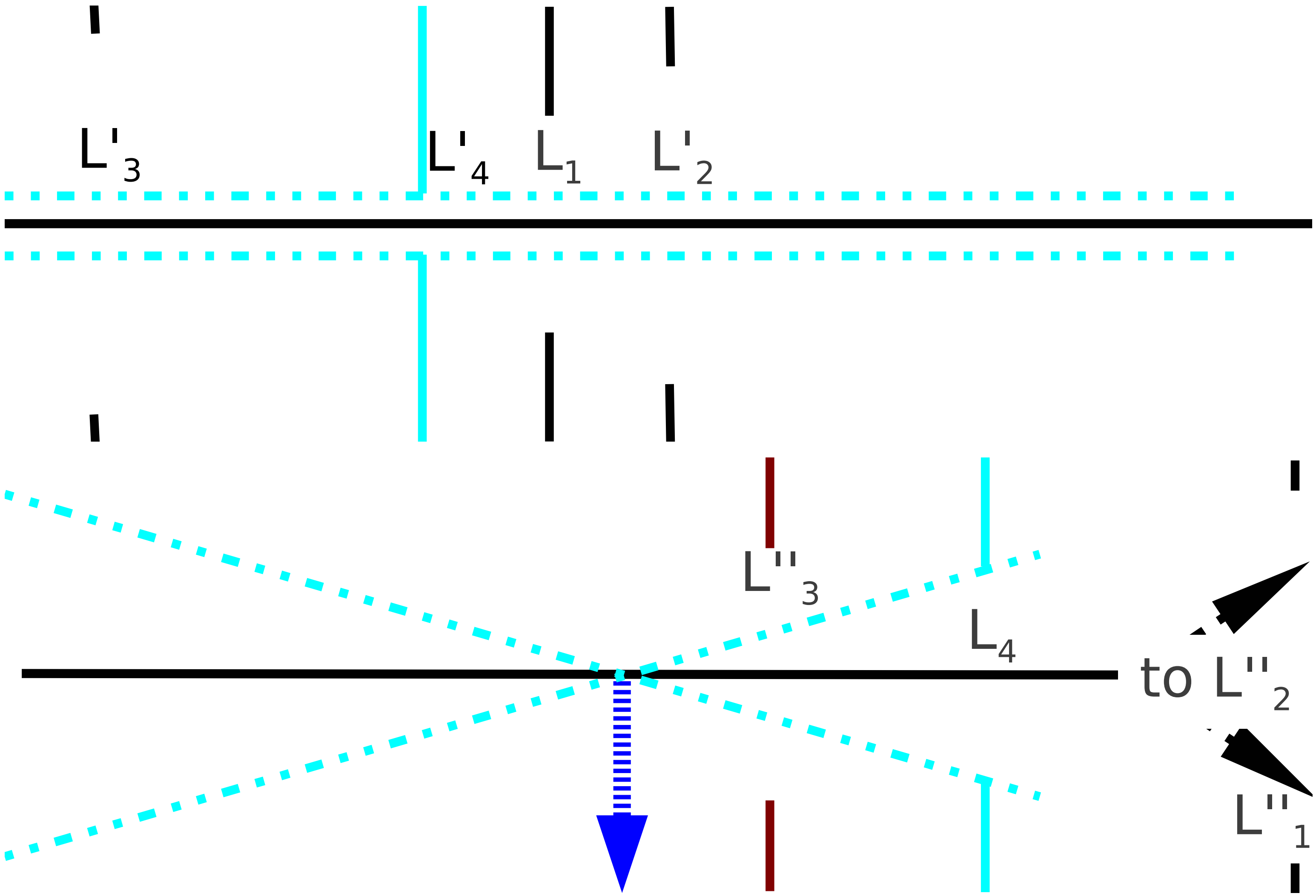
Finding the Entrance Window

- Start at the Entrance Pupil
- Find the Aperture that Limits Field of View from Pupil
- Entrance Window is L_3', Field Stop is L_3
- Remember Entrance Pupil is L_4', Aperture Stop is L_4
- The Remaining Apertures Don't Matter

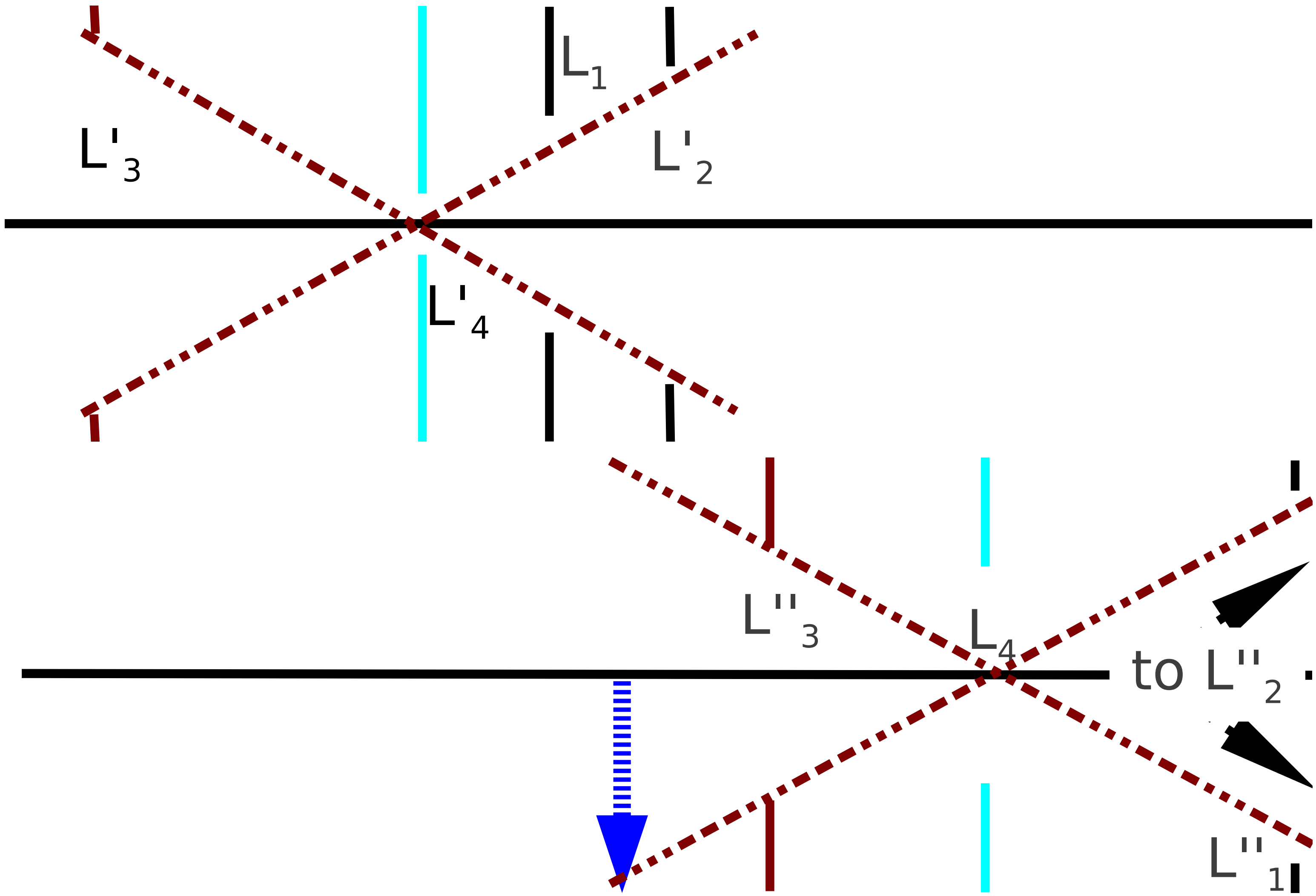
Fix figure in text (4-5-apexample.odg)



Object Space and Image Space: Entrance and Exit Pupils

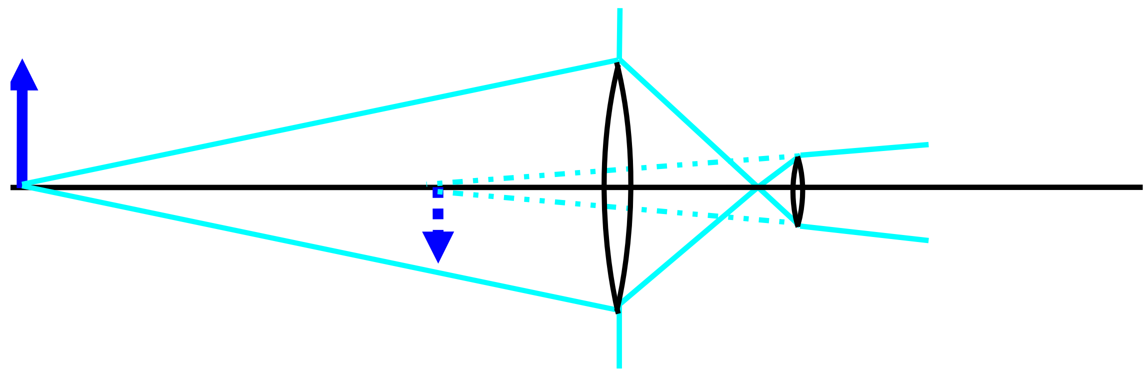


Object Space and Image Space: Entrance and Exit Windows

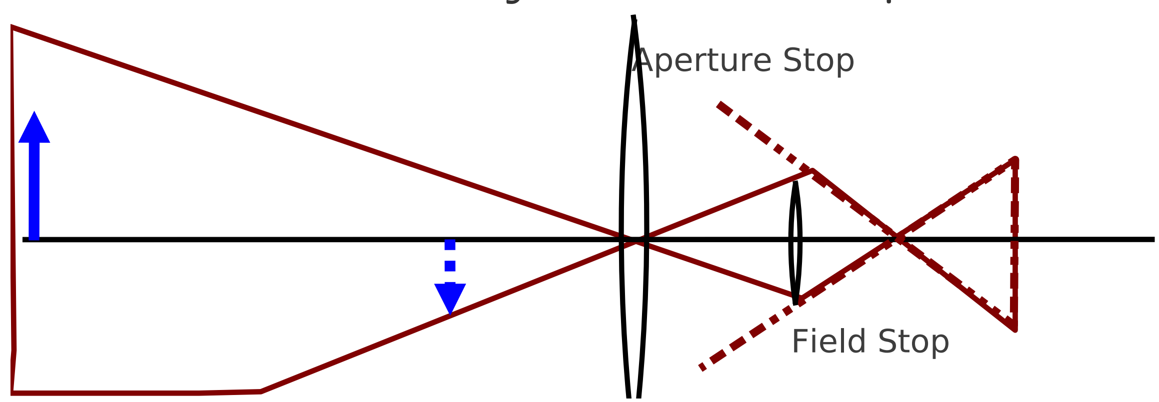


The Telescope

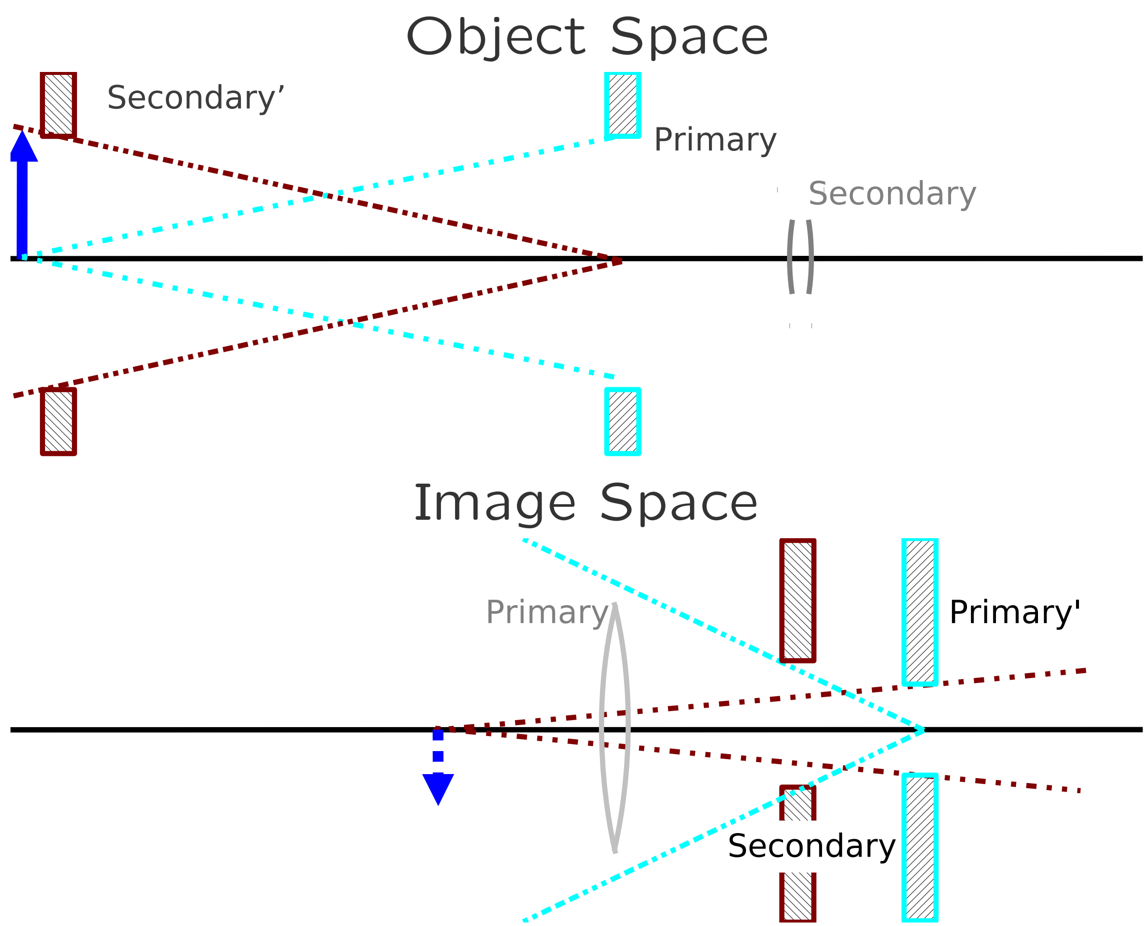
Primary as Aperture Stop



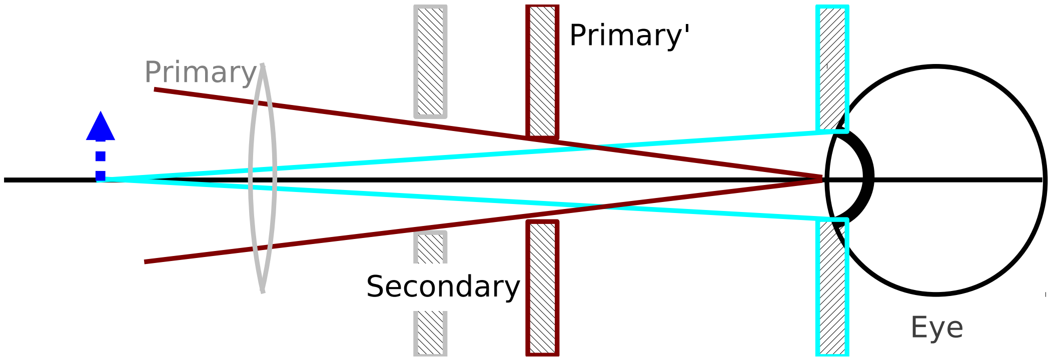
Secondary as Field Stop



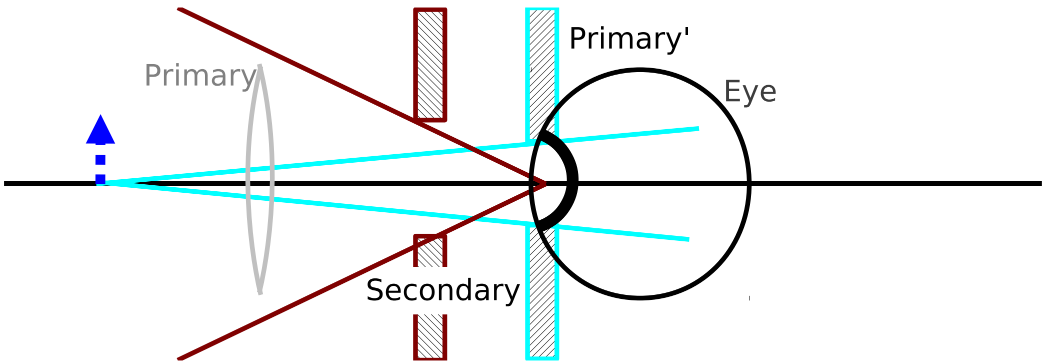
The Telescope: Object and Image Space



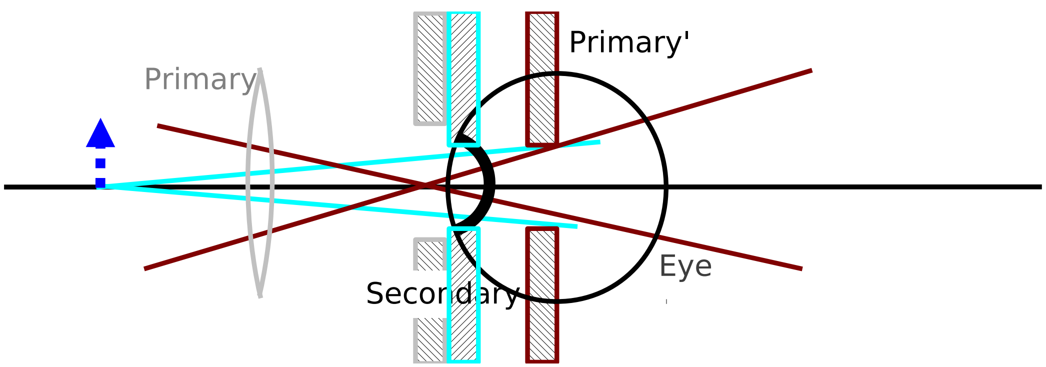
Eye Relief: Matching Pupils



Eye Too Far Away: Telescope Exit Pupil Becomes Field Stop and Is Visible.

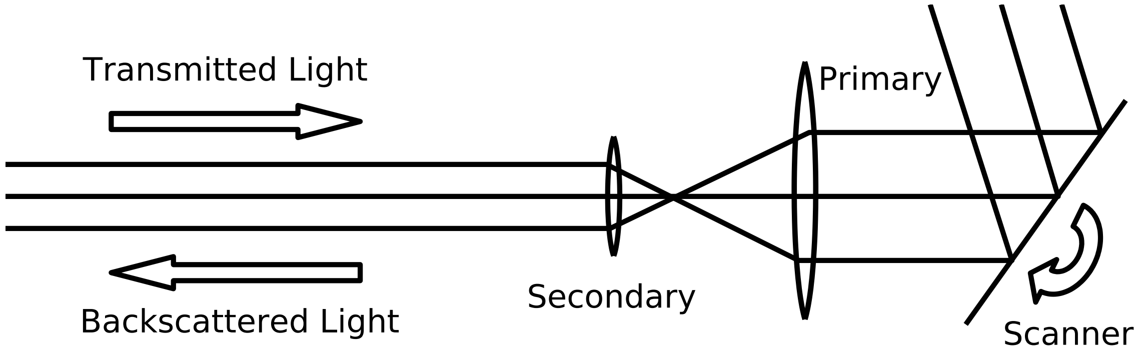


Pupils Matched: Actual Pupil is the Smaller of the Two.

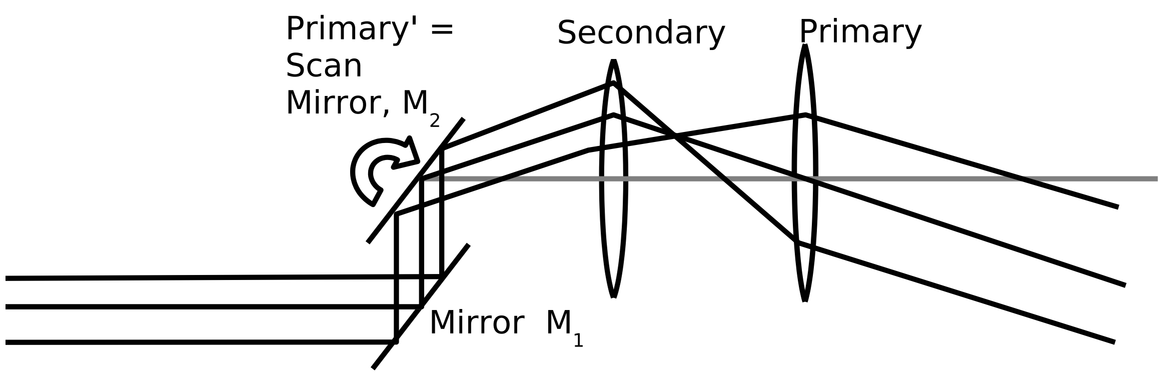


Eye Too Close. Telescope Exit Pupil Becomes Field Stop but is Blurred (Distance is Negative).

Scanning and Pupils



Post–Expander Scanning



Pre–Expander Scanning

Place the Scanning Mirrors in a Pupil Plane (or Close)
Small Mirrors Can Move Faster. . .
. . . but Remember the Angular Magnification

The Simple Magnifier

- The Simple Magnifier Has Practical Limitations

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s}$$

- Large s' for Large m; $s \approx f$

$$s' = -\frac{fs}{f-s} \approx -\frac{f^2}{f-s}.$$

- s Slightly Smaller than f for Positive m and negative s'

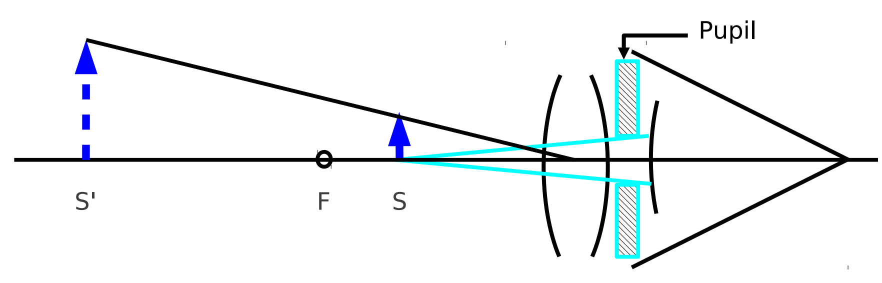
$$m = \frac{-s'}{s} > 0$$

- No Limit on m . . .
- But x'/s' is What Matters

- Define $M = m$ at $s' = 20\text{cm}$

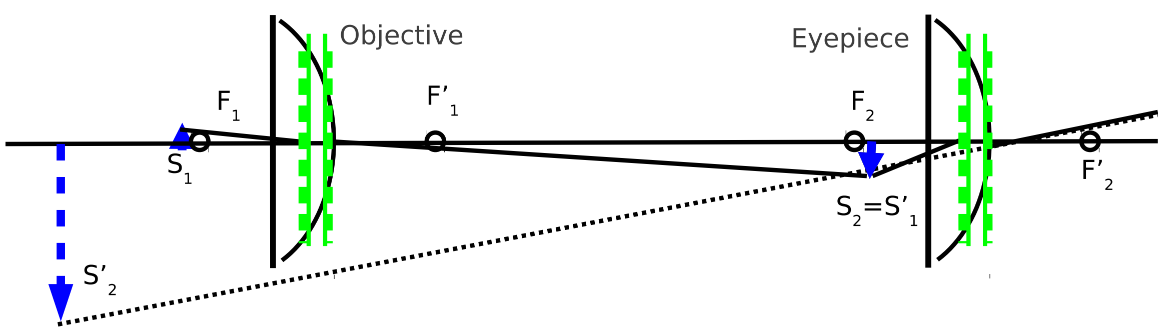
$$M = \frac{20\text{cm}}{f}$$

- Hard to Make f/d Small
- $d \ll d_{eye}$ Costs Light
- Hard to Make $f \ll 1\text{cm}$ (but Leeuwenhoek did it)
- Better Solution: Compound Microscope



Compound Microscope

- The Solution: Use Two (or More) Lenses
- Now We Really Need to Understand Pupils and Windows
- Two–Lens Microscope
  - Objective (First Lens) Provides High Magnification and Real Image
  - Eyepiece Acts as a Simple Magnifier
  - Both Are Usually Compound Lenses (See Ch. 5)

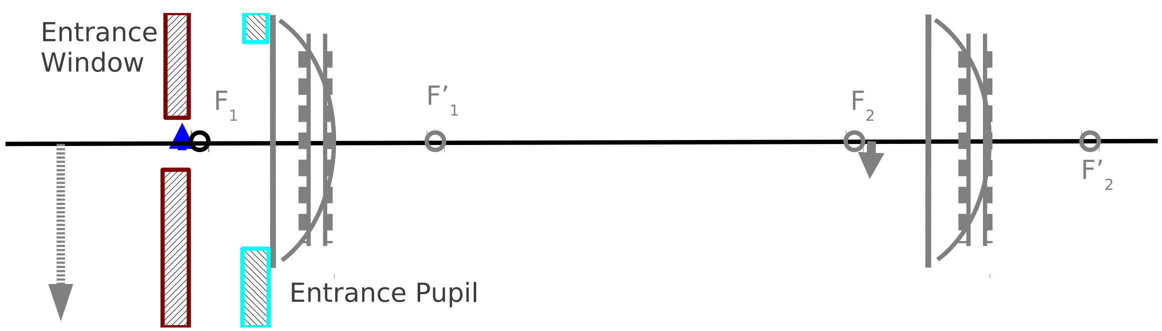


Compound Microscope: Object Space

- Objective Provides Magnification and Aperture Stop
  - Short Focal Length for High Magnification
  - High NA (Hopefully)
- Tube Length Provides Large Real Intermediate Image

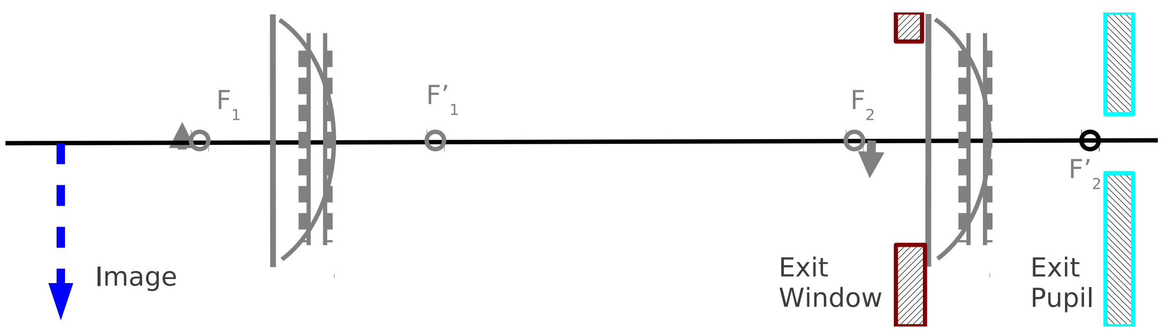
$$m_{objective} = -\frac{s'_{objective}}{f_{objective}} \approx -\frac{\ell_{tube}}{f_{objective}}$$

  - Standard Tube Length 160 mm
  - Many Variations
- Tube Length Makes Entrance Window Near Object



Compound Microscope: Image Space

- Eyepiece Acts as a Simple Magnifier and Field Stop
  - Moderate Focal Length for Moderate Magnification
  - Exact Magnification Not Critical (x'/s' Matters)
- Image Near Infinity (Virtual, Inverted)
- Tube Length Places Exit Pupil at Back Focus of Eyepiece
- Eye Relief for Pupil Matching

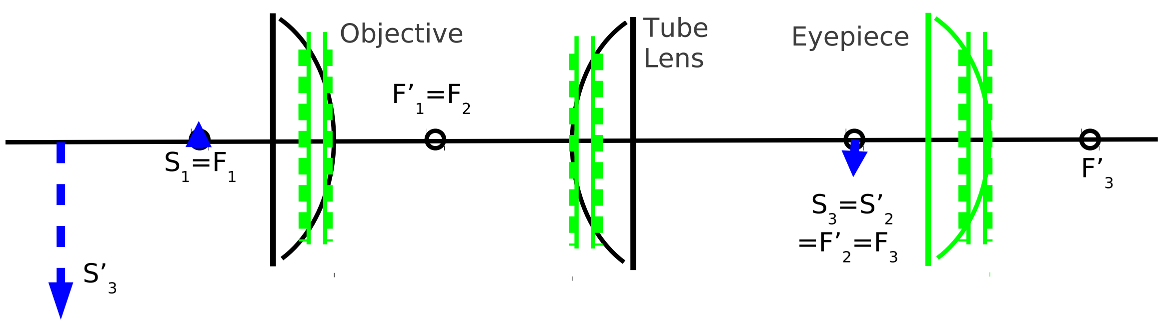


Infinity–Corrected Microscope

- Added Lens, Telecentric Configuration

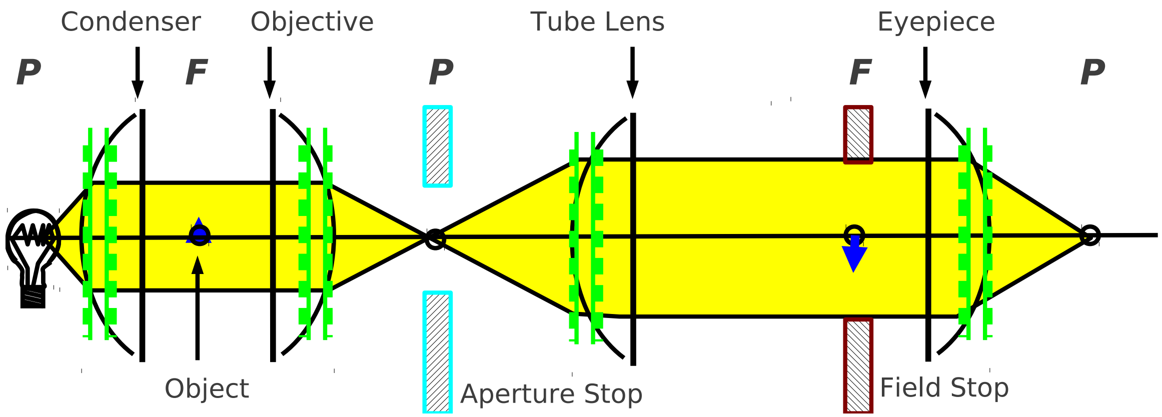
$$m_{objective} = -\frac{f_{tube}}{f_{objective}}$$

  - Improved Image Quality
- Infinity Space Between Objective and Tube Lens
  - Allows for Filters and Other Optics (Flat without Aberration: See Ch. 5)
  - Provides Real Pupil

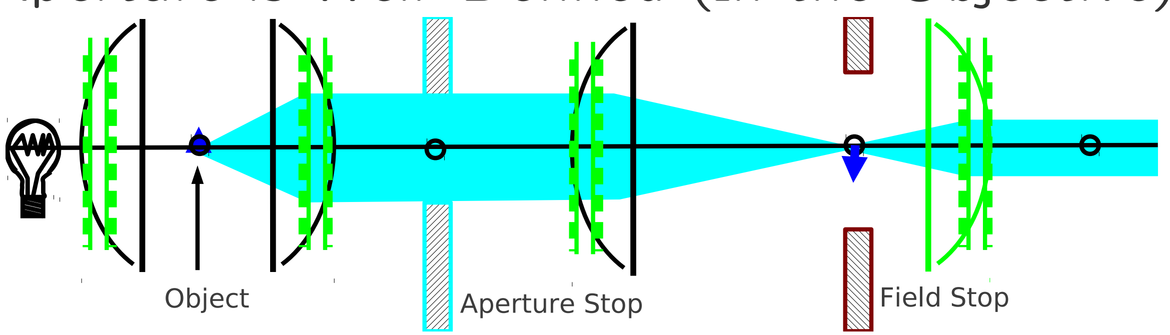


Infinity–Corrected Microscope

Illumination Should Match or Exceed FOV (& No Diffuser)

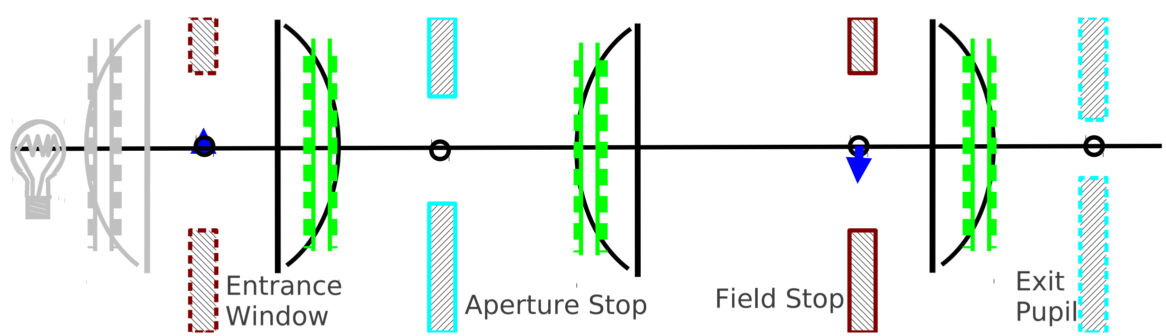


Aperture is Well–Defined (In the Objective)



Microscope Apertures

- Aperture Stop
  - In Back Focal Plane of Objective
  - Determines NA
  - Exit Pupil at Back Focal Plane of Eyepiece.
- Field Stop
  - At Intermediate Image
  - Often Used for Camera or Detector
  - Entrance Pupil at Object (Front Focal Plane of Objective
- All in Focal Planes



Köhler Illumination

- Light Source in a Pupil Plane
- Not Imaged (See Ch. 11 and Homework Problem There)
- Condenser Lens Determines FOV of Light Source