An Adaptive Extended Kalman Filter for State and Parameter Estimation in AUV Localization

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***Abstract*—Given their independence from operators and potentially unrestricted range of operations, Autonomous Underwater Vehicles (AUVs) are considered key enablers of a host of applications of the *Blue Economy*. A critical requirement for AUVs is that of being able to self-localize so that the data they collect are clearly marked with position information. Localization is challenging underwater, as GPS and other technologies that use radio frequencies do not work in water. This has brought to the development of solutions that often involve costly technology and operations that are impractical to use in many situations, such as when swift and affordable localization is required. In this paper, we present a method for localizing AUVs that lends itself to be used in such situations, while providing localization that is as accurate as that from more expensive methods. Our method is based on pre-deployed acoustic beacons (whose coordinates do not need to be known by the AUV) and on mainstream sensors usually available onboard most AUVs. It employs an adaptive Extended Kalman Filter (EKF) that exploits statistical techniques to overcome the inaccuracies of baseline EKF when the noise of the environment or of the instrumentation is time-varying or unknown. We demonstrate the effectiveness of our method for accurate AUV localization through simulations and experiments at sea with an AUV and commercial acoustic transducers. Our results show swift determination of the beacon positions and meter-level localization, suggesting that our method can be effectively used in most underwater applications.**

***Keywords*-Adaptive Kalman Filter, Autonomous Underwater Vehicles (AUVs), long baseline (LBL) localization.**

I. Introduction

Advances in technologies for real-time monitoring and control of underwater environments have provided unprecedented interest in deploying devices that enable the sustainable exploitation of oceans and waterways and meet the requirements of applications for the Blue Economy [1]. This is certainly the case of Autonomous Underwater Vehicles (AUVs) [2], which are increasingly used in critical missions in a wide variety of sectors. A fundamental requirement of each application is that the AUV should be aware of its position, so as to be precisely guided to loci of interests and to report the location of sensed events and data sources accurately. As radio frequency-based communication and navigation systems (e.g., GPS) cannot be used to communicate and guide vehicles in water, underwater devices have been provided with alternative technologies, e.g., acoustic transducers, which afford them robust communication and sensing with levels of accuracy previously unthinkable.

Over the years, AUV localization and navigation systems have been devised that can be divided into those based on dead-reckoning and Inertial Navigation Systems, and those based on external interaction with other devices. The first type of systems uses *proprioceptive detection* from sensors such as accelerometers and gyroscopes; the second type assists the vehicle by supplementing additional information, usually by means of acoustic communications. The latter include Long Baseline (LBL) acoustic positioning systems, which allow an AUV to self-localize by determining its position with respect to pre-deployed acoustic transceivers (beacons), whose coordinates are known or can be estimated by the AUV [3]. Although systems of this kind are usually more precise, a trade-off is to be made between accuracy and both cost (of devices and their deployment) and time (for deployment and calibration). Accurate LBL systems offer a clear example: setting the beacons up, and providing them with accurate ways to keep up-to-date coordinates can be costly, especially when their use is requested only for a limited period of time. This is why, when cost is an issue, and whenever a localization system is needed swiftly or temporarily, new methodologies are sought after that provide acceptable accuracy "on the fly," possibly using off-the-shelf components.

One of such methodologies uses optimized statistical processing of sensed data to cope with the nonlinear dynamics of the environment. Kalman filtering [4] and its variants play an essential role in these scenarios by aggregating the measures and the bias associated to the system components into a comprehensive estimate of the system state. However, the accuracy of the estimates obtained by using Kalman filters is highly dependent on the a priori knowledge of the noise statistics of the model. The use of incorrect statistics can lead to substantial estimation errors or even to filter divergence, especially if the noise is time-varying, as it is often the case underwater. Adaptive noise estimation techniques allow such issues to be obviated by estimating the error statistics of the vehicle state or of its sensors while moving. It follows that systems that are able to adapt to the time-varying noise conditions of the environment or of the instrumentation are less dependent on expensive sensor suites and costly equipment (that are more accurate and noise-resilient by design) if they favor, instead, active noise compensation strategies.

In this work, we aim at devising a technique of this kind to show that even for AUVs carrying common sensing equipment, remarkably accurate localization is possible. Specifically, we present an LBL system for localizing an AUV that only relies on an Inertial Measurement Unit (IMU) to determine its orientation, a pressure sensor to measure its depth, and an acoustic modem to communicate with the acoustic beacons deployed in the surveyed area. The estimation of the dynamic state of the system, comprehensive of the measurements from the beacons, relies on an adaptive EKF. Our proposed methodology enables the AUV to autonomously estimate the coordinates of the beacons and the statistical noise of the IMU data before starting the mission. As such, the beacon coordinates do not need to be known a priori. The AUV then self-localizes while moving, by constantly adapting its knowledge of the noise of the measures received from the beacons. We show that our methodology is more accurate than non-adaptive solutions [5]–[8] via simulation and tests at sea, where our solution can estimate the previously unknown coordinates of the beacons in about 3 minutes and can achieve meter-level mean localization accuracy, obtaining a 60% improvement over a similar non-adaptive version.

The rest of this paper is organized as follows. Work on adaptive filtering in AUV and beacon localization is described in Section II. In Section III we define our adaptive localization method. Its performance is evaluated and benchmarked in Section IV. Lastly, we draw our conclusions in V.

II. Previous Work

Several works exist in the literature on adaptive solutions to tackle the time-varying noise characteristics of both terrestrial [9]–[12] and underwater [13]–[17] dynamic systems. Among those concerning AUV localization in the underwater domain, the work by Wang et al. proposes an adaptive Sage-Husa-based Unscented KF (UKF) that allows estimating the process noise and measurement covariance matrices [13]. Instead of using an AUV, the method is validated by localizing a body that is towed by a vessel equipped with a costly suite of navigation sensors. While they report sub-meter Root Mean Square Error (RMSE) their setting is quite different from ours, which is concerned with actual low-cost AUVs and instruments. Xu et al. propose an Ultra-Short Baseline (USBL) positioning system based on adaptive Kalman Filtering to adjust the process noise and the measurement covariance matrices using the residuals of the latest measurement data [14]. An adaptive method is suggested by Shao et al. to adapt the process noise and measurement covariance matrices of an EKF and an AUV performing dead reckoning using measurements from a Doppler Velocity Log (DVL) and from an Attitude and Heading Reference Systems (AHRS) [15]. The comparison between the GPS ground truth and the filter estimates of these two works suggests a localization error higher than that from our approach. However, no localization error metric has been reported by the authors, which makes it difficult to compare their work to ours. In order to adapt the process noise covariance matrix to possibly varying process noise, Hajiyev et al. propose a UKF in which an adaptive process noise scale factor is put into the covariance matrix of the innovation sequence [16]. Their method is evaluated via simulations. As the authors do not apply adaptive strategies to the measurement covariance matrix and do not provide experiments at sea, a comparison with our method would be scarcely informative. Sun et al. propose a modified version of a Sage-Husa filter with an adaptive forgetting factor for an AUV using dead reckoning navigation [17]. They test their solution at sea, reporting a localization error up to 9 m, which we improve in this work.

Estimating unknown beacon coordinates is performed extensively in terrestrial applications and in Simultaneous Localization and Mapping (SLAM) [18]. It has also been attempted in the context of underwater localization. Vaganai et al., for instance, use the *Least Squares* method to home an AUV to a single beacon of unknown coordinates [19]. Instead of using GPS (as we do), the AUV uses dead reckoning while diving in a circular motion to disambiguate possibly collinear positions. Such approach is not directly comparable to ours, as no tests on localization accuracy are presented. Olson et al. use a voting scheme to determine the most likely beacon coordinates after filtering out outliers in range measurements [20]. They test their approach against a baseline EKF with knowledge of the beacon positions. The work concerns only beacon localization, and does not report on the accuracy of localizing AUVs. As such, a comparison with our work is not directly possible. Teixeira et al. propose an EKF-based estimation technique aimed at maximizing the determinant of the innovation covariance matrix to determine the most likely beacon coordinates. The authors do not provide direct localization accuracy metrics for comparison but they plot the mean euclidean error in time. It can be seen that, using an optimized AUV trajectory, they are able to converge to a localization error comparable to ours. However, their method is only tested by simulations and requires initial estimates of the beacon positions. Newman et al. propose a range-only based scheme to enable an AUV to determine the coordinates of the beacons while moving [21]. Their approach is tested against an EKF with knowledge of the true beacon positions. The authors provide error metrics concerning the lengths of the respective *inter-beacon* baselines (comprised between 3.7 m and 10 m) and a plot of the estimated vs. EKF "ground-truth" trajectories, which suggests that their localization accuracy is lower than ours. The latter two works consider the problem as a SLAM problem, i.e., they include the beacon coordinates in the state of the EKF.

III. Adaptive AUV Localization

Our localization method operates in marine scenarios where $N > 0$ LBL beacons have been statically deployed. The coordinates of the beacons do not need to be known. Each beacon has a unique ID. An AUV navigates in such scenario, with no knowledge about the beacon coordinates or of their IDs. We stipulate that the AUV uses data from its onboard IMU and pressure sensor, which provides depth information.

The aim of the AUV is to compute its approximate location through *inertial navigation*, using *yaw* measures computed through the IMU. In order to offset the inaccuracies due to accumulating integration errors (*drift*), which is common for inertial navigation, the AUV periodically calculates its distance from the beacons (*range*). The AUV relies on an EKF to fuse the approximate estimates of inertial navigation and the range measurements, in order to compute an estimate of its location. The EKF is assumed to have no knowledge of the true noise statistics related to the yaw and range measurements.

For the AUV to self-localize, we need to estimate the accurate coordinates of the beacons, the noise statistics of the IMU and those of the range measures. To this purpose, we divide our localization method operations into two phases.

• **GPS-aided Phase.** This phase occurs at the very beginning of the AUV mission, when the AUV has no knowledge of the beacon locations and no accurate belief on the accuracy of its IMU. The AUV (on the surface) will only rely on the accuracy of the GPS measurements to estimate the unknown coordinates of the beacons, derive the error on the yaw measures and embed it into the EKF formulation. These procedures are described in Sections III-B and III-C, respectively.

• **Inertial LBL Localization Phase.** After the location of at least $n \leq N$ beacons and the noise statistics of the IMU have been estimated, the AUV can start its actual underwater mission. From this point on, the location information from the GPS is no longer used, and the AUV only relies on the inertial navigation and range measures to estimate the evolution of its state in time in accordance with the model described in Section III-A. During this phase the AUV keeps estimating the noise statistics on the range measures (Section III-D).

A. Localization System Model

We define the state of the AUV at time k as $x_k = [p_k, v_k]$, where $p_k = (p_{x,k}, p_{y,k})$ is the position on the North and East relative coordinates of a North East Down (NED) reference frame,[1] and v_k is the speed of the AUV. We indicate the planar position of beacon i with $p^i = (p^i_x, p^i_y)$.

The AUV dynamics are modelled as a constant velocity model, formalized by the following recursive equation:

$$\begin{aligned} x_k &= f(x_{k-1}, \theta_k, q) = \\ &= \underbrace{\begin{pmatrix} 1 & 0 & \Delta t \cdot cos(\theta_k + q_\theta) \\ 0 & 1 & \Delta t \cdot sin(\theta_k + q_\theta) \\ 0 & 0 & 1 + q_v \end{pmatrix}}_{F} x_{k-1}, \end{aligned}$$

where $q = [q_\theta, q_v]$ is a zero mean Gaussian random variable with covariance matrix Q, and θ_k is the yaw value obtained by integrating the angular velocity reading from the IMU at time k.

At a frequency of $\frac{1}{\Delta t}\ Hz$ a new yaw reading is performed and the AUV computes a new estimate $x_{k|k-1}, P_{k|k-1}$ through the Predict Equations of the EKF:

$$\begin{aligned} x_{k|k-1} &= F \cdot x_{k-1|k-1} \\ P_{k|k-1} &= F \cdot P_{k-1|k-1} \cdot F^T + L_k \cdot Q \cdot L_k^T. \end{aligned}$$

[1] The high accuracy and low cost of pressure sensors, allow the navigation problem to be reduced to the 2D plane.

where $L_k = \frac{\partial f}{\partial q}\Big|_{\substack{x_k = x_{k|k-1}, \\ q=0}}$ is the Jacobian of f with respect to the error.

Asynchronously with respect to the time-evolution of the dynamics model, the AUV polls the beacons by broadcasting a range request packet (*rg_pkt*). Upon reception of a *rg_pkt*, the beacons reply with a packet containing their ID (*ID_pkt*). The AUV can calculate its distance (range) from beacon i using the two-way Time of Flight (ToF) as follows:

$$d^i_k = \frac{(t^{rg}_j - t^{ID^i}_{j'})c}{2}, \qquad j < j',$$

where t^{rg}_j and $t^{ID^i}_{j'}$ are the times at which the *rg_pkt* packet and the *ID_pkt* packet from beacon i are sent and received, respectively, and c is the speed of sound in water. The distance d^i_k from beacon i is related to the AUV state through the Measure Equation:

$$\begin{aligned} d^i_k &= h(p_k, p^i, r_b) + r_r = \\ &= \left|\left| \ [p_k - p^i + r_b, p_{z,k} - p^i_z] \ \right|\right|_2 + r_r \,, \end{aligned}$$

where r_b and r_r are Gaussian random variables representing the error of the beacon position and the ranging error, respectively. We indicate the overall error with $r = [r_b, r_r]$ and its covariance matrix with R. The depth measured by the AUV depth sensor and that of beacon i are indicated, respectively, as $p_{z,k}$ and p^i_z. Each time a range measure is computed, the AUV updates the state and covariance matrix estimates accordingly, through the Update Equations of the EKF:

$$\begin{aligned} S_k &= H_k \cdot P_{k|k-1} \cdot H_k^T + M_k \cdot R \cdot M_k^T \\ K_k &= P_{k|k-1} \cdot H_k^T \cdot S_k^{-1} \\ x_{k|k} &= x_{k|k-1} + K_k \cdot (d^i_k - h(x_{k|k-1}, p^i, 0)) \\ P_{k|k} &= (I - K_k H_k) \cdot P_{k|k-1}. \end{aligned} \tag{1}$$

Here $H_k = \frac{\partial h}{\partial x_k}\Big|_{\substack{x_k = x_{k|k-1} \\ r=0}}$ is the Jacobian of h with respect to the state, and $M_k = \frac{\partial h}{\partial r}\Big|_{\substack{x_k = x_{k|k-1} \\ r=0}}$ is the Jacobian of h with respect to the error.

B. GPS-aided Beacon Localization

During the GPS-aided phase, the planar coordinates and depth of the AUV are determined using GPS and the pressure sensor. Using the ranges computed during this phase, the estimation of the unknown coordinates of the beacons becomes a *Least Squares* optimization problem. Let p^*_i be the unknown true position of beacon i, $p^G_{0,k}$ and $r^i_{0,k}$ the collections of AUV positions measured using GPS and of range measurements, respectively, up until time k and relative to beacon i. Then:

$$p^*_i = ({x_i}^*, {y_i}^*) = \underset{\hat{p}_i}{\operatorname{argmin}} (r^i_{0,k} - ||\hat{p}_i - p^G_{0,k}||_2).$$

Since the number of steps needed for the above formulation to converge to an exact solution is inversely proportional to the accuracy of the GPS measures, $\hat{p}_i$ might not converge to p^*_i in a reasonable amount of steps. Because of this, we impose that $\Delta^i_p = ||p^*_i - \hat{p}_i||_2 \leq \tau$, where τ is a predefined threshold, that can be interpreted as an upper bound of the uncertainty on the position of the beacons. Thus, we use Δ_p^i as the initial value for the covariance term $\sigma_{b,i}^2$ on beacon i. Once $n = 3$ beacons have been localized, the AUV can start the actual mission. If subsequent ranges from previously unseen beacons are received, the procedure we just described is carried out using the position estimates of the EKF without GPS.

C. Initial Process Noise Covariance Estimation

The Process Noise Covariance Matrix Q is defined as:

$$Q = \begin{pmatrix} \sigma_\theta^2 & 0 \\ 0 & \sigma_v^2 \end{pmatrix},$$

where σ_θ^2 and σ_v^2 are the covariance on the yaw measures and on the speed estimates, respectively. We wish to leverage the precise location measurements during the GPS-aided phase, to calibrate the covariance term σ_θ^2 of the Q matrix of the EKF. Here, the aim is to find an initial fixed value for σ_θ^2 that is as close as possible to the real intrinsic time-invariant noise on the yaw readings from the IMU.

The following reasoning holds under a few assumptions: (I) the trajectory between pairwise subsequent GPS-acquired positions p_{k-1}^G and p_k^G, $k \geq 1$ is approximable as linear with sufficient confidence (i.e., the frequency of GPS measurements sufficiently high and is directly proportional to the speed of the AUV) and (II) noise factors (e.g., water currents) other than the time-invariant sensor noise are absent, known or estimated.

The yaw θ_k measured at time k, expresses the rotation of the AUV heading with respect to the z-axis of its own reference frame. Such rotation can also be derived using pairwise subsequent positions acquired by the GPS, as:

$$\hat{\theta}_k = atan2(p_{y,k}^G - p_{y,k-1}^G, p_{x,k}^G - p_{x,k-1}^G), \qquad k \geq 1$$

i.e., $\hat{\theta}_k$ is the angle in the Euclidean plane between the positive x-axis of the NED-referenced frame in position p_{k-1}^G and the ray from the origin of the same frame to the point p_k^G. Suppose the IMU was totally unbiased, then $\hat{\theta}_k$ and θ_k would be basically identical: the difference between them expresses how much θ_k deviates from the estimate of $\hat{\theta}_k$, which is supposed to be more accurate. Let $\Delta_\kappa^\theta = |\theta_i - \hat{\theta}_i|, 1 \leq i \leq \kappa$, be the collection of yaw residuals from the start to the end (at time κ) of the GPS-aided phase. The adjusted new value for the process noise σ_θ^2 relative to the yaw measures will be set as the covariance computed on Δ_κ^θ.

D. Adaptive Measurement Noise Covariance

The Process Noise Covariance Matrix $R \in \mathbb{R}^{(3N \times 3N)}$ is composed as such:

$$R = \begin{pmatrix} \sigma_{r,0}^2 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & \sigma_{b,0}^2 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \sigma_{b,0}^2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \sigma_{r,N}^2 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & \sigma_{b,N}^2 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & \sigma_{b,N}^2 \end{pmatrix}$$

where $\sigma_{r,i}^2$ and $\sigma_{b,i}^2$ are the variance on the range measurements and on the position of beacon i, respectively. In practice, since the k^{th} *Update* step of the filter is only computed with the subset of beacons whose measurements are received at time k, R is treated as N disjoint (3×3) matrices, each uniquely associated to a beacon. This grants the double advantage of both easing the computational burden of the algebraic calculations of the EKF, and being able to connotate the noise statistics of each beacon independently and with finer grain, since different noise levels could characterize them, depending on the environmental conditions at their location.

At time k, let the *innovation* $\rho_k = d_k^i - h(x_{k|k-1}, p^i, 0)$ (in Equation 1), be the difference between the received range d_k^i and the distance $h(\cdot)$ only computed using the *a priori* belief of the filter about the position of the AUV and of beacon i. Let $\rho^{(k-m,k)}, k \geq m$ the collection of the last m innovations, and σ_k^ρ the covariance between its elements. Such quantity captures the variations of the behavior of the innovation within a sliding window of size m across the entire mission time. To avoid the overestimation of error contributions relative to measurements that are less trusted by the filter (i.e., that cause an increase of the Innovation Covariance S), we allow the measurement noise to be considered only as much as the filter values the measurements themselves. Thus, we weigh σ_k^ρ by the Kalman Gain K, i.e., the confidence that the EKF puts in the incoming range measures. The adapted measurement covariance on the range measures at time k, will then be:

$$\sigma_{r,i}^2 = \sigma_k^\rho \ K.$$

IV. Performance Evaluation

To evaluate our method, we performed two different sets of experiments using synthetically simulated (Section IV-A) and real data from field tests at sea (Section IV-B).

The aim of the simulations is that of individually validating the effectiveness of each of the optimizations proposed in Sections III-B, III-C and III-D. A separate simulation is carried out for each optimization, in which no bias on the measurements has been introduced, except on the measurements concerning the evaluated optimization. We then evaluate the accuracy of the overall method by using it on data collected in actual measurement campaigns at sea [8]. To do so, we compute the MSE between the estimated location and a GPS ground truth (localization error).

Both simulation and field experiments model a similar scenario, which is described in the following. After the GPS-aided phase, the AUV computes the *Predict* step of the EKF each time a yaw measure is derived from the IMU, at a frequency of 10 Hz. Once every 2 s, the AUV polls the beacons broadcasting a *rg_pkt* and executes the *Update* step of the EKF every time that an *ID_pkt* is received from the beacons, to compute range.

Table I shows the values of the threshold on beacon position accuracy τ, the size m of the sliding window for range noise estimation, and the initial values of the process and measurement noise covariance matrices, for both simulations and tests at sea.

TABLE I
SIMULATION AND FIELD TESTS PARAMETERS.

	Simulations	Field Tests
τ	0.2 m	0.2 m
m	20	20
σ_v^2	0.1 m^2/s^2	0.1 m^2/s^2
initial σ_θ^2	0.01 rad^2	0.1 rad^2
initial σ_r^2	0.5 m^2	1.5 m^2
initial σ_b^2	1 m^2	2 m^2

A. Simulation Experiments

The simulated scenario consists of three beacons and an AUV moving on the surface according to a lawnmower path at a median speed of about 0.2 m/s, in an area of about 30 $m \times 60$ m. We evaluate each of the proposed optimization techniques in such a scenario.

a) ***Unknown Beacon Coordinates Estimation:*** We artificially shift each beacon location by about 5 m North West. For the first t seconds, the AUV gets its position using GPS measurements arriving at 10 Hz. Its state at time k, $k < t$ will then be $x_k = [p^G_{x,k-1}, p^G_{y,k-1}, v_k]$. The procedure can be seen as a *multilateration* in which each computed range is the radius of the circumference with the estimated AUV GPS position $(p^G_{(x,k-1)}, p^G_{(y,k-1)})$ as the center. The intersection of such circumferences (with a tolerance of τ m) is the estimated position of the beacon. In the considered scenario, this method can successfully localize all the beacons after about $t = 200s$.

Fig. 1 shows the estimated trajectory of the AUV after the completion of the beacon localization phase, annotated with the estimated beacon locations (in green), versus the trajectory (light blue) estimated by the same filter with biased beacon location beliefs. As can be seen from the trajectory plot, the

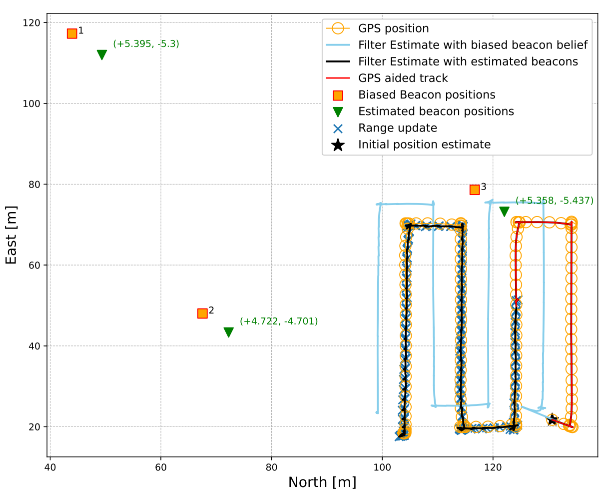

Fig. 1. Estimated trajectory after beacon localization (black solid) vs. estimated trajectory with biased beliefs about the beacon locations (light blue).

trajectory estimate that relies on biased beacon coordinates is significantly shifted with respect to the ground truth, while a proper beacon location belief ensures a correct trajectory estimation (black line).

b) ***Adaptive Process Noise Covariance:*** In this experiment, the simulated yaw measures used during the predict step of the EKF have been perturbed with random normal noise $\sim \mathcal{N}(0, 0.2)$. At the end of the GPS-aided phase, the AUV computes the new estimate for σ_θ^2 using the collected residuals, as seen in Section III-C. Fig. 2 (main plot) shows the comparison between the artificially induced Gaussian noise on the yaw measures and the noise estimated by our method. The estimated additive noise on yaw measures throughout the GPS-aided phase (yellow) closely follows the distribution of the artificial noise induced in simulation (green). In fact, the

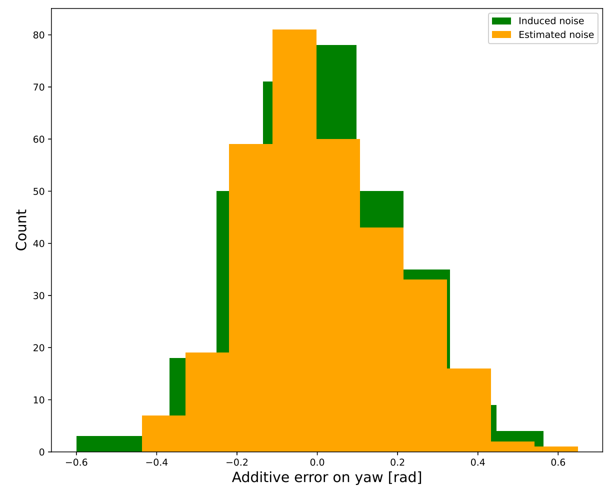

Fig. 2. Comparison between the artificially induced Gaussian noise perturbing the yaw measures (green) and the noise estimated by our method (orange).

estimated value of σ_θ (0.18 rad) is very close to the standard deviation of the artificially induced noise on the yaw measures (0.2 rad). Consequently, we can conclude that σ_θ^2 closely describes the noise that the *yaw* measures underwent during the experiment.[2]

c) ***Adaptive Measurement Noise Covariance:*** In this experiment, each ranging value received by the AUV from the beacons has been artificially perturbed with additive noise following a Half-Normal distribution. This kind of induced noise has been selected to reflect the fact that the only range-related noise that could affect the localization of the AUV, once the beacons have been localized, is that affecting the range measures themselves, i.e., r_r. Specifically, assuming the beacons do not move around more than τ m,[3] a range measure can never be lower than the direct path we are trying to estimate. In fact, while multipath and reflections can cause ranges to overestimate the actual distance, the ToF used to compute them can never be lower than the speed of sound. Thus, in cases (like ours) where the noise on range measures is

[2] In case of non-negligible water dynamics effects (e.g., currents), this method would incorporate them in σ_θ^2. In that case, estimating water dynamics would help in ruling out noises that are independent of the instrument.

[3] If they do, σ_r^2 would also be affected by the uncertainty that should be attributed to σ_b^2. However, the noise estimations would still ensure proper adaptation.

zero-mean Gaussian, the Half-Normal distribution well models the possible errors.

Fig. 3 shows the comparison between the artificially induced half-normal noise and the noise estimated by our method, relative to each of the involved beacons. The estimated noise on range measures throughout the mission (yellow), closely follows the distribution of the artificial noise that has been induced in simulation (green) for all the beacons, confirming the effectiveness of the proposed solution.

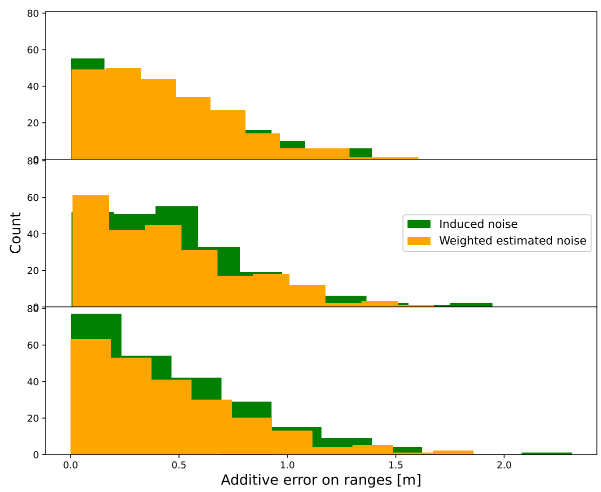

Fig. 3. Comparison between the artificially induced half-normal noise perturbing the range measures (green) and the noise estimated by our method (orange) on each beacon (top to bottom).

B. Experiments with Data Collected at Sea

We have evaluated the accuracy of our localization method using data collected in July 2021 in the shallow waters of the Mediterranean Sea [8]. We conducted a campaign where the Zeno AUV [22], [23] was operated from a boat, moving randomly in an area of $80\ m \times 100\ m$. The total time of the AUV mission, including the GPS-aided phase (which lasted about 3 minutes) was about 40 minutes. The system consisted of four WSense's LBL beacons whose coordinates have been measured with a low precision handheld GPS ($\pm$ 4 m) during deployment (however, the AUV does not use this information). Each beacon is attached to a buoy by a rope long 1.5 m (i.e., the approximate depth of the beacons is known), and is anchored to the seabed.

All system elements were programmed with the described system logic before being deployed. The proposed localization approach was compared against a baseline, namely a non-adaptive EKF with the same initial model of the noise statistics and parameters (Table I). Fig. 4 shows the trajectory estimate and the localization error of the baseline EKF.

In the baseline setting, the AUV only knows the biased beacon locations measured with an imprecise GPS and adopts a fixed initial belief of the noise statistics of range and yaw measures. These assumptions cause the whole trajectory estimate to be subject to some shifting and inaccuracies,

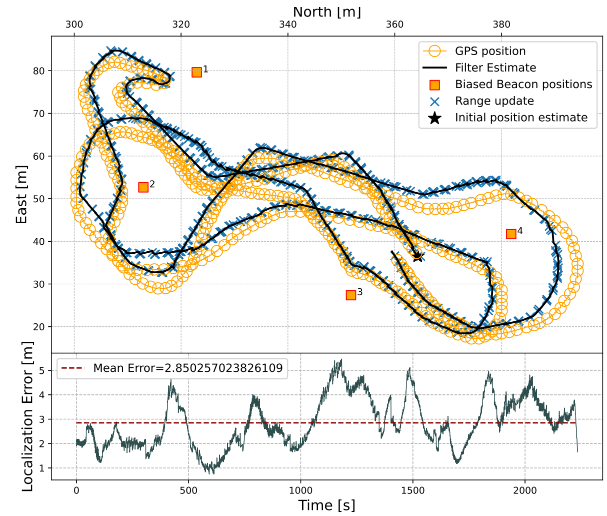

Fig. 4. Trajectory estimate (top) and mean localization error (bottom) of the baseline method.

similar to what we have observed in Section IV-A. The overall mean localization error with the baseline approach is 2.85 m.

Fig. 5 shows the estimated trajectory and the localization error obtained instead by the solution proposed in this paper.

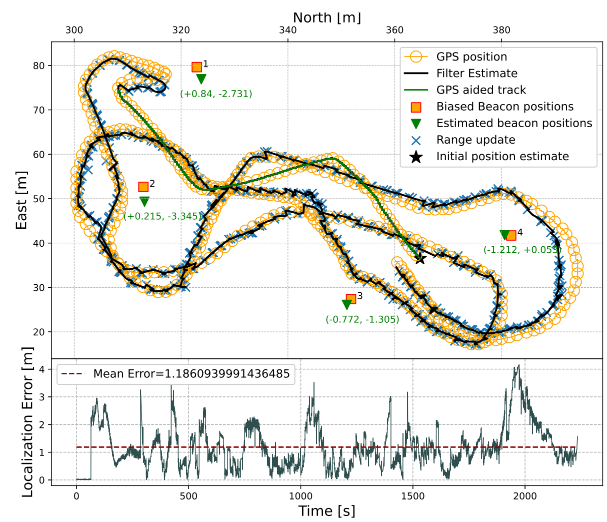

Fig. 5. Trajectory estimate (top) and mean localization error (bottom) of our method.

Some irregularities in the overall trajectory estimates by the filter can be noticed compared to the baseline. We attribute this to the fact that the noise statistics of the measurement noise covariance matrix are constantly updated in real time as the AUV moves. Our method allows the AUV to self-localize with an error of 1.18 m, a 60% improvement over the baseline EKF. As our method includes the initial GPS-aided phase, the computation of the localization error does not include the transient state, i.e., the data collected during the first 3 minutes (time needed to localize the first 3 beacons), for fair comparison against the baseline, whose first 3 minutes of trajectory were therefore not considered.

V. Conclusions

We design and evaluate the performance of a set of adaptive techniques to overcome the limitations of the classical EKF in LBL localization scenarios, allowing an AUV to independently estimate the unknown location of beacons and the noise statistics related to the yaw and range measurements. We show that the proposed method allows accurate localization of an AUV that only relies on a minimal set of low-cost sensors, namely on an IMU for orientation information, a depth sensor, and acoustic ranging information, without the need for costly or time-consuming calibrations. We evaluate the performance of our approach both by means of simulations and field tests at sea, evaluating the effectiveness of the noise and beacon location estimation techniques, and comparing the accuracy of the AUV location estimates against a GPS ground truth. We show that our method allows the AUV to localize the LBL beacons in around 3 minutes and to self-localize with a mean localization error of $1.18\ m$, a 60% improvement over a non-adaptive method in a similar configuration.

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