Cramer's Rule

ECE U400 – Linear Circuits
Supplemental Handout
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Given a set of linear equations

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

consider the determinant

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

If $\mathbf{d} = \mathbf{0}$, then the system has nondegenerate solutions (i.e., solutions other than (0, 0, 0)) only if D = 0 (in which case there is a family of solutions). If $\mathbf{d} \neq \mathbf{0}$ and D = 0, the system has no unique solution.

If instead $\mathbf{d} \neq \mathbf{0}$ and $D \neq 0$ then solutions are given by[1]:

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{D},$$

$$y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{D},$$

$$z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{D}.$$

This procedure can be generalized to a set of n equations. So, given a system of n linear equations in n unknowns

[1] This will be the case for nearly 100% of the problems you will have in this class! If this is NOT the case, you have probably set up one or more of your equations incorrectly.

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n &= d_1 \\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n &= d_2 \\
&\vdots \\
a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \cdots + a_{nn}x_n &= d_n
\end{aligned}
$$

let

$$
D = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}.
$$

Again, if $\mathbf{d} = \mathbf{0}$, then nondegenerate solutions exist only if D = 0. If $\mathbf{d} \neq \mathbf{0}$ and D = 0, the system has no unique solution.

If instead $\mathbf{d} \neq 0$ and $D \neq 0$, compute[2]:

$$
D_k = \begin{vmatrix} a_{11} & \cdots & a_{1(k-1)} & d_1 & a_{1(k+1)} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2(k-1)} & d_2 & a_{2(k+1)} & \cdots & a_{2n} \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{n(k-1)} & d_n & a_{n(k+1)} & \cdots & a_{nn} \end{vmatrix}
$$

Then,

$$
x_k = \frac{D_k}{D} \text{ for } 1 \le k \le n.
$$

Note that for this class, your unknown variables x_k could consist of say unknown voltages, currents, resistance, capacitance, and/or inductance values.

[2] Again, this will be the case for nearly 100% of the problems you will have in this class! If this is NOT the case, you have probably set up one or more of your equations incorrectly.