Formula Sheet

$$x(t) = x_f + \left[x(t_o) - x_f\right]e^{-(t-t_o)/\tau}$$, general response for 1st-order circuit, $\tau = \frac{L}{R}$ or $\tau = RC$

$v_L = L\frac{di}{dt}$	$i_L = \frac{1}{L}\int_{t_o}^{t} v(x)dx + i(t_o)$	$p_L = Li\frac{di}{dt}$	$\omega_L = \frac{1}{2}Li^2$
$v_C = \frac{1}{C}\int_{t_o}^{t} i(x)dx + v(t_o)$	$i_C = C\frac{dv}{dt}$	$p_C = Cv\frac{dv}{dt}$	$\omega_C = \frac{1}{2}Cv^2$

$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$ $\quad \omega_o^2 = \frac{1}{LC}$ $\quad \alpha = \frac{1}{2RC}$ (parallel) $\quad \alpha = \frac{R}{2L}$ (series)

- The response of a second-order circuit is overdamped, underdamped, or critically damped as follows:

THE CIRCUIT IS	WHEN	QUALITATIVE NATURE OF THE RESPONSE
Overdamped	$\alpha^2 > \omega_o^2$	The voltage or current approaches its final value without oscillation
Underdamped	$\alpha^2 < \omega_o^2$	The voltage or current oscillates about its final value
Critically damped	$\alpha^2 = \omega_o^2$	The voltage or current is on the verge of oscillating about its final value

- In determining the **natural response** of a second-order circuit, we first determine whether it is over-, under-, or critically damped, and then we solve the appropriate equations as follows:

DAMPING	NATURAL-RESPONSE EQUATIONS	COEFFICIENT EQUATIONS ~~OVERDAMPED~~
Overdamped	$x(t) = A_1e^{s_1t} + A_2e^{s_2t}$	$x(0) = A_1 + A_2$; $dx/dt(0) = A_1s_1 + A_2s_2$
Underdamped	$x(t) = (B_1\cos\omega_d t + B_2\sin\omega_d t)e^{-\alpha t}$	$x(0) = B_1$; $dx/dt(0) = -\alpha B_1 + \omega_d B_2$, where $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$
Critically damped	$x(t) = (D_1t + D_2)e^{-\alpha t}$	$x(0) = D_2$, $dx/dt(0) = D_1 - \alpha D_2$

- In determining the **step response** of a second-order circuit, we apply the appropriate equations depending on the damping, as follows:

DAMPING	STEP ~~NATURAL~~-RESPONSE EQUATIONS[1]	COEFFICIENT EQUATIONS ~~OVERDAMPED~~
Overdamped	$x(t) = X_f + A_1'e^{s_1t} + A_2'e^{s_2t}$	$x(0) = X_f + A_1' + A_2'$; $dx/dt(0) = A_1's_1 + A_2's_2$
Underdamped	$x(t) = X_f + (B_1'\cos\omega_d t + B_2'\sin\omega_d t)e^{-\alpha t}$	$x(0) = X_f + B_1'$; $dx/dt(0) = -\alpha B_1' + \omega_d B_2'$
Critically damped	$x(t) = X_f + D_1'te^{-\alpha t} + D_2'e^{-\alpha t}$	$x(0) = X_f + D_2'$; $dx/dt(0) = D_1' - \alpha D_2'$

[1] where X_f is the final value of $x(t)$.