

Discrete S, equally likely events $\Rightarrow P(event) = \frac{\# \text{ outcomes in event}}{\text{total \# outcomes in S}}$

Some useful counting methods via combinatorics are shown below.

4 ways to choose a sample of k elements from a set of n distinct objects

Sample type	Order matters?	Repetition allowed?	Number of ways to select sample
Permutation with replacement	Yes	Yes	n^k
Permutation	Yes	No	$\frac{n!}{(n-k)!}$
Combination	No	No	$\binom{n}{k} = \frac{n!}{k!(n-k)!}$
Combination with replacement	No	Yes	$\binom{n+k-1}{k}$

Permutations: order matters (i.e., abc, cba, bca are all considered “different”)

Combinations: order is NOT taken into account (i.e., $ab = ba$)

“n factorial” = $n! = n(n-1)(n-2)\cdots(2)(1)$. Note $0! = 1$.

“n choose k” = $\binom{n}{k} = \frac{n!}{k!(n-k)!} = \text{binomial coefficient}$

Stirling’s formula: For large n , $n! \approx \sqrt{2\pi n} n^{n+\frac{1}{2}} e^{-n}$

Multinomial coefficient: If you have n objects, where n_1 are the same, n_2 are the same, ..., n_j are the same such that $n_1 + n_2 + \cdots + n_j = n$,

then the number of distinguishable permutations of these n objects = $\binom{n}{n_1 n_2 \dots n_j} = \frac{n!}{n_1! n_2! \cdots n_j!}$. Equivalent statement: