ECE U468 Noise & Stochastic Processes—Lecture 9
Discrete Probability Distributions or Probability Mass Functions (pmf)
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Uniform Distribution

Experiment obeys: all outcomes equally probable
Random variable: X = outcome
Probability distribution: if k is the number of possible outcomes,

$$p_X(x) = \begin{cases} \frac{1}{k} & \text{if } x \text{ is a possible outcome} \\ 0 & \text{otherwise} \end{cases}$$

Example: tossing a fair die ($k = 6$).

Bernoulli Distribution

Experiment obeys:
- a single trial with two possible outcomes (success and failure)
- P[{trial is successful}] = p

Random variable: X = number of successful trials (zero or one)
Probability distribution: $p_X(x) = p^x(1-p)^{1-x}$
Mean and variance: $\mu = p$, $\sigma^2 = p(1-p)$
Example: tossing a fair coin once

Binomial Distribution

Experiment obeys:
- n repeated independent Bernoulli trials, i.e.:
- each trial has two possible outcomes (success and failure)
- P[{i^{th} trial is successful}] = p for all i
- trials are independent

Random variable: X = number of successful trials. $X \sim \mathcal{B}(n, p)$
Probability distribution: $p_X(x) = \binom{n}{x} p^x (1-p)^{n-x}$
Mean and variance: $\mu = np$, $\sigma^2 = np(1-p)$
Example: tossing a fair coin n times
Approximations: Poisson approximation: $\mathcal{B}(n, p) \approx \mathcal{P}(\lambda = np)$ if $p << 1$, $x << n$
Normal approximation: $\mathcal{B}(n, p) \approx \mathcal{N}(\mu = np, \sigma^2 = npq)$ if $np >> 1$, $nq >> 1$

Geometric Distribution

Experiment obeys:
- indeterminate number of independent repeated Bernoulli trials, i.e.:
- each trial has two possible outcomes (success and failure)
- P[{i^{th} trial is successful}] = p for all i
- trials are independent
- keep going until 1st success

Random variable: X = trial number of 1st successful trial.
Probability distribution: $p_X(x) = p(1-p)^{x-1}$, $x = 1, 2, 3, \ldots$
Mean and variance: $\mu = \frac{1}{p}$, $\sigma^2 = \frac{1-p}{p^2}$
Example: repeated attempts to start an engine, playing lottery until you win

Variation on Geometric Distribution

Experiment obeys:
- indeterminate number of independent repeated Bernoulli trials, i.e.:
- each trial has two possible outcomes (success and failure)
- P[{i^{th} trial is successful}] = p for all i
- trials are independent
- keep going until 1^{st} success

Random variable: X = number of failures until 1^{st} success

Probability distribution: $p_X(x) = p(1-p)^x$, $x = 0, 1, 2, \ldots$

Mean and variance: $\mu = \frac{1-p}{p}$, $\sigma^2 = \frac{1-p}{p^2}$

Example: queueing models

Negative Binomial Distribution

Experiment obeys:
- indeterminate number of independent repeated Bernoulli trials, i.e.:
- each trial has two possible outcomes (success and failure)
- P[{i^{th} trial is successful}] = p for all i
- trials are independent
- keep going until r^{th} success

Random variable: X = trial number on which r^{th} success occurs

Probability distribution: $p_X(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$

Mean and variance: $\mu = \frac{r}{p}$, $\sigma^2 = \frac{r(1-p)}{p^2}$

Example: fabricating r nondefective computer chips

Poisson Distribution

Experiment obeys:
- events occur "completely at random"
- events occur at a point in time or space
- # of events in 1 region is independent of the # occurring in any disjoint region
- probability of more than 1 event occurring at the same point is negligible
- probability of n events in region 1 = probability of n events in region 2, when regions have the same size

Random variable: X = number of events occurring in a given time interval or region of space
$X \sim \mathcal{P}(\lambda)$

Probability distribution: $p_X(x) = \frac{\lambda^x}{x!} e^{-\lambda}$, where λ is the average number of events in the given region

Mean and variance: $\mu = \lambda$, $\sigma^2 = \lambda$

Example: telephone calls arriving at a switchboard in a specified one-hour period