The property of stationary increments means that:

$$(\blacktriangle) \quad P[N(t-d)=j] = P[N(t)-N(d)=j]. \qquad \text{(TRUE!)}$$

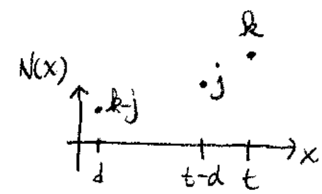
This does <u>NOT</u>, however, hold true within a conditional or joint probabality. Let's see why...

$$\text{Is } P[N(t-d)=j,\ N(t)=k] \overset{?}{=} P[N(t)-N(d)=j,\ N(t)=k] \quad (*)$$

$(*)$ can only be true iff:

$$P[N(t)=k \mid N(t-d)=j] \overset{?}{=} P[N(t)=k \mid N(t)-N(d)=j] \quad (**)$$

$$\left(\text{since } P[A \cap B] = \frac{P[B|A]}{P[A]}\right)$$



But $(**)$ says that

$$P[N(t)=k \mid \underbrace{N(t-d)=j}_{\text{here, we're Given Something close to } N(t)}] \overset{?}{=} P[N(t)=k \mid \underbrace{N(d)=k-j}_{\text{whereas here we're conditioning over Something far from } N(t)}] \quad (**)$$

suppose d=small...

These two probabilities are NOT equal.
Hence $(*)$ and $(**)$ are NOT TRUE statements.

(ie, you can only use the property $(\blacktriangle)$ exactly as written... when the single probability value is isolated by itself... as done in HW Problem 5.6)