A Probabilistic Approach of Designing Driving Circuits for Strings of High-Brightness Light Emitting Diodes

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***Abstract*-Often, High Brightness LEDs (HB-LED) are connected in series to create strings. According to their data sheets, the HB-LEDs have a variation in their forward voltage drop. This forward voltage variation may create a non-uniform illumination if the strings are connected in parallel. This paper proposes a probabilistic approach for modeling the forward voltage drop across the HB-LEDs and determining the value of the resistance needed in each string to control the current. The results of this paper show that when the probabilistic models are used, the value of the added balancing resistance is reduced compared to when using standard worst case models.**

I. INTRODUCTION

Within the last few years, new LED materials and improved production processes have resulted in bright LEDs in colors throughout the visible spectrum with efficacies greater than incandescent lamps. These High-Brightness LEDs (HB-LED) are brighter, more efficacious, colorful and are already being utilized for a wide variety of colored applications such as traffic signals, signage, external automotive lighting and backlights for mobile phones. White lighting applications are also increasing rapidly [1]. Several advantages of HB-LEDs are : 1. Incredibly long life, lasting between 50,000 to 100,000 hours, 2. Save energy, 3. Work on dimmable switches, 4. Quick turn on and turn off, 5. Environmentally friendly, 6. Good color saturation, 7. Able to withstand shock, vibration and environmental extremes [2], 8. HB-LED efficiency is already 100 Lumens/Watt [3].

LEDs are low voltage light sources that operate with DC current. Depending on wavelength, HB-LEDs have forward voltage 2-4V, and forward current is 350mA or more. For many applications like automotive backlighting, traffic lights, home lighting etc., single HB-LEDs are normally connected in series to form HB-LED strings. The brightness of the HB-LEDs in each string is proportional to the DC current flowing through them and is controlled by regulating that DC current. When LED strings are connected in parallel, as shown in Fig. 1, the difference in forward voltages will cause different currents to flow through the branches. Those in the branch with lower forward voltage drop will glow brighter while the strings of LEDs with higher forward voltage will be dim. Unfortunately, the forward voltage drops of typical commercial HB-LEDs have wide variation, even up to 33%, for LEDs with the same wavelength (color) specification. If not properly compensated for, these voltage variations lead to unacceptable, nonuniform illumination noticeable to the human eye.

The purpose of this paper is to provide a new modeling viewpoint for strings of HB-LEDs. By modeling forward voltage drops probabilistically, it is possible to see benefits of stringing LEDs that could not be seen before when design was based on worst case scenarios (with maximum or minimum forward voltage drops). As a result of these benefits, a designer can justify the use of a smaller value balancing resistor, thus improve power efficiency.

Specifically, the research contributions of this paper include:

- A stochastic modeling approach is proposed for the forward voltage drop of LEDs. The benefits of this (simple) modeling approach is discussed and compared to the standard worst case analysis approach.
- Design algorithms based on the probabilistic models are presented in order to calculate the balancing resistor value for each LED string. The approach can be applied to parallel LED strings driven by either a voltage source or could be extended to a current source (although we only present voltage source in this paper). In both cases, the balancing resistor value added to each string as well as the requirement of the source voltage becomes reduced compared to when worst case models are used.
- Experimental results are presented that justify the probabilistic models and verify their design benefits.

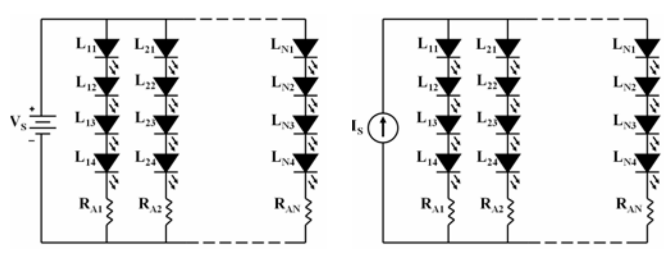

Fig. 1. *Strings of LEDs in Parallel, Either Driven by a Voltage Source (left) or a Current Source (right).The source could be thought of as a single DC-DC converter, a battery, etc.*

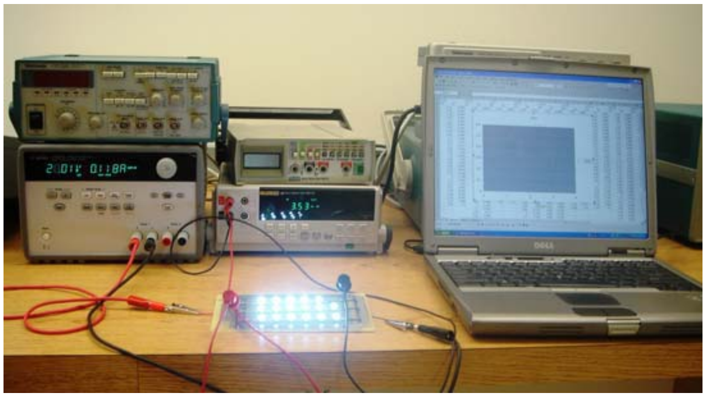

Fig. 2. *Strings of HB-LEDs in Parallel are being tested at laboratory.*

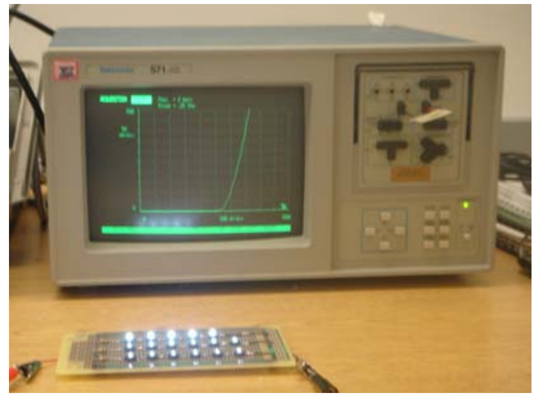

Fig.3. *A curve tracer is used to measure the I-V characteristics curve of HB-LEDs*

We remark that to cope with the wide variation in each forward HB-LED voltage, several strategies are commonly utilized:

1. Provide active control to regulate the (averaged) current through each string of LEDs [7]. Essentially, this is equivalent to having an individually regulated current source, perhaps a DC-DC converter, across each string and to not connect any HB-LED strings in parallel. This approach is the costliest of all approaches because of the number of power converters and control ICs required (one for each string), but it is effective in minimizing the effect of LED voltage variation.
2. Manufacturers or users individually test the forward voltage of each LED and then bin each LED into groups with smaller voltage drops. For example, in [4] white HB-LEDs have a specified voltage drop (V_D) at 350mA in the range of 2.79V<V_D<3.99V. However, for a premium cost, it is possible to purchase the LEDs from voltage "bins" that have guaranteed voltage ranges less than 0.24V, for example, two separate bins specified are 2.79V< V_D< 3.03V (Bin G) and 3.75V< V_{LED}<3.99V (Bin L)[5]. Of course, it is significantly more costly to purchase voltage binned LEDs, since each LED's voltage must be tested individually.
3. Adding a resistor in each string in order to balance the current flow in each branch [6]. The wider the voltage range of the LED, the larger the balancing resistor necessary to keep the currents to a specified range. (In fact, a balancing resistor is also often necessary with voltage binned LEDs, as above, but can be of lower value). Obviously, adding a large resistor to LED string increases power loss and often negates the benefits of their high luminous efficacy. However, this is the lowest cost solution and is widely utilized in applications. This is the approach primarily discussed in this paper.

II. Conventional Linear Modeling and Conventional Worst Case Design with HB-LEDs [5]

The linear model of the LED approximates the I-V characteristic by a straight line that is tangent to the actual curve at the DC bias point. Fig. 4 shows the curve with the tangent line at the point (V_D, I_D). The curve intersects the horizontal axis at the voltage V_D and the vertical axis at the point I_D and this leads to a voltage plus resistance model (V_D=V_{FD}+I_DR_D). Fig. 4(b) presents the simplified model of the HB-LED: $V_D = I_DR_D + V_{FD}$ where V_D is the voltage across the LED, R_D is the Dynamic Resistance (the inverse slope of the curve) of LED and V_{FD} is the Forward voltage drop taken at the knee of the curve.

Fig. 5 uses a curve tracer to plot a random sample of thirty V_D vs I_D curves for the typical white 350mA HB-LEDs which are not binned by the manufacturer. These curves verify: 1) HB-LEDs follow the typical diode equation curve. 2) For HB-LEDs there is wide variation in forward voltage drop. 3) A linear model or a constant voltage drop (V_{FD}) plus dynamic resistance (R_D) model is a reasonable approximation of the HB-LEDs too at sufficiently high currents.

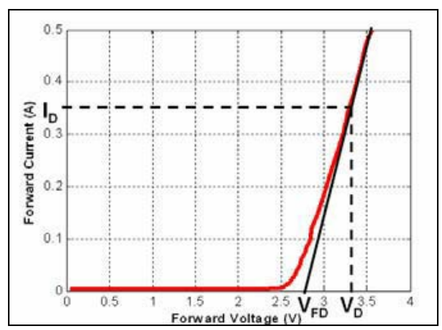

Fig.4 (a). I-V characteristic curve of an LED

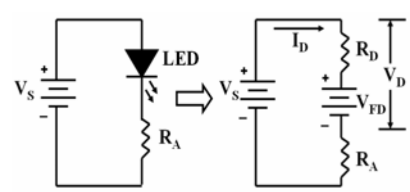

Fig. 4(b). Simplified model of the LED

Notice in Fig. 5, there is a large variation in LED forward voltage V_D for HB-LEDs which are not binned. For maximum current of 500mA, the V_D varies from 3.52V<V_D<4.24V depending on the LED. Fig. 6 plots a random sample of twenty V_D vs I_D curves for white 350mA binned HB-LEDs.

Even for voltage binned LEDs in Fig. 6, the voltage varies from 3.75V<V_D<4V.

TABLE I [4]
TEST CURRENT 350MA

RADIATION PATTERN		LAMBERTIAN
COLOR		WHITE
LED FORWARD VOLTAGE (V_D)	MIN	3.75V
	MAX	3.99V
DYNAMIC RESISTANCE		1Ω

Table 1 presents typical characteristics found in data sheets for voltage binned HB-LEDs [4]. The white LEDs were purchased at a premium cost so that they could be voltage binned (Bin L) between 3.75V< V_{LED}<3.99V for a 350mA test current.

As has been reported in Table 1 and can be seen in Fig. 5 and Fig. 6, notice that the dynamic resistance, R_D does not vary much from one HB-LED to the other and can be assumed approximately constant. Thus, the variation in forward voltage drop of the HB-LEDs can primarily be assumed to be due to variation in V_{FD}. Therefore, $V_{FDMAX} = V_{DMAX} - I_D R_D$ and $V_{FDMIN} = V_{DMIN} - I_D R_D$ where V_{DMAX} and V_{DMIN} are specified in the datasheet of the LED. Fig.5 show that the dynamic resistance of the unbinned LED is $R_D \approx 0.56\Omega$ with V_{FDMAX} = 3.444 and V_{FDMIN} = 3.024 and for the binned LEDs R_D =1, V_{FDMAX} = 3.64V and V_{FDMIN} = 3.4V.

III. DESIGN APPROACH FOR BALANCING RESISTOR USING CONVENTIONAL WORST CASE MODELING

To design a HB-LED string, a designer must first specify a range of acceptable current variation within each paralleled string in Fig. 1, i.e, $I_{DMIN} < I_D < I_{DMAX}$. The goal of the designer is to add sufficient balancing resistor in each branch to guarantee that this condition will always hold true (assuming that each string does not have a separate current controller). We can define a current regulation constant, $\gamma = \frac{I_{DMIN}}{I_{DMAX}}$. Here 0<γ<1, and a typical value of γ may be around 0.9 (90%), which is about the maximum variation in current allowable before the human eye may be bothered by differences in brightness of the paralleled LED strings.

Consider a string of M numbers of HB-LEDs in series. Assume that each string may be in parallel with other similar strings and is being driven by a voltage source, V_S, as in Fig. 1. With given γ , I_{DMIN} , I_{DMAX} and V_{DMAX}, V_{DMIN} and R_D from the data sheet, the designer now must determine the value of the balancing resistor, R_A to keep the currents within $I_{DMIN} < I_D < I_{DMAX}$. The advantage of driving the strings with a voltage source is that each string can be designed independently, and so, in Fig1(a) $R_{A1} = R_{A2} = ... = R_{AN} \equiv R_A$. Specifically, for each LED string, if standard worst case models are assumed for the system in Fig 1(a). $I_{DMAX} = \frac{V_S - M \cdot V_{FDMIN}}{R_T}$ and $I_{DMIN} = \frac{V_S - M \cdot V_{FDMAX}}{R_T}$ where $R_T = R_A + M{*}R_D$ is the total equivalent series string resistance.

Using the fact that the designer specifies $I_{DMIN} = \gamma I_{DMAX}$, where 0<γ<1, this leads to the specification of the required voltage source and balancing resistor to be:

$$V_S = \frac{M(V_{FDMAX} - \gamma V_{FDMIN})}{1-\gamma} \quad (1) \qquad R_T = \frac{M(V_{FDMAX} - V_{FDMIN})}{(1-\gamma)I_{DMAX}} \quad (2)$$

For example, suppose four (M=4) HB-LEDs from Table 1 are stringed in a branch with maximum 10% current difference in each branch, i.e, γ = 0.9. Using I_{DMAX} = 350mA, V_{FDMAX} = 3.64V, V_{FDMIN} = 3.4V and equation (1) leads to a required source voltage V_S = 23.2V and total resistance R_T = 27.43Ω. Therefore, the balancing resistor, $R_A = R_T - R_D$ = 23.43 Ω need to be added to each string to keep the current between 315mA and 350mA. From this worst case analysis (even though the forward voltage is tightly binned) the required balancing resistor leads to a substantial and unwanted power loss $I_D^2 R_A$ = 2.87W or an additional 23% power loss.

Similarly, if four (M=4) HB-LEDs which are not binned are connected in series when γ = 0.9, I_{DMAX} = 350mA, V_{FDMAX} = 3.444V, V_{FDMIN} = 3.024V, then the required source voltage, V_S = 28.90V and the total resistance is 48 Ω . So the balancing resistor needs to be added is 44Ω leading to an additional 53% power loss.

IV. NEW APPROACH – MODELING V_D STOCHASTICALLY

Instead of designing solely on worst case voltage drops, we propose a different philosophy that is based on probability theory. If the forward voltage drop of the HB-LED is assumed a random variable with a value somewhere in the range of the bin, then it has a mean and a standard deviation associated with it. Stringing LEDs in series would, therefore, be equivalent to summing random variables, and this has the effect of lowering the normalized standard deviation of the voltage across the entire string. Thus, from a probabilistic point of view, the chances of having the LED string voltage to be at its minimum or maximum worst case rated value can become very low. The designer can therefore decide to reduce the balancing resistor with only mild risk. As the number of LEDs in each string increases, the size of the balancing resistor becomes smaller and smaller.

Suppose there is a voltage source input as in Fig.1(a). We propose the following new design procedure to select R_A, the balancing resistor for a string of M number of HB-LEDs:

Step 1: As before, the designer first specifies γ, I_{DMIN}, I_{DMAX} for the LED strings. Calculate V_{FDMIN} and V_{FDMAX} using data sheet values of V_{DMAX}, V_{DMIN}, R_D and the given test current I_D i.e. $V_{FDMIN} = V_{DMIN} - I_D R_D$, etc. Next, the designer must specify an acceptable probabilistic certainty, ***P***, that the current in each string is between $I_{DMIN} < I_D < I_{DMAX}$, e.g. 99% (***P*** = 0.99) etc.

Step 2: Assume that each HB-LED has its forward voltage V_{FD} as a random variable with mean $\mu_{VFD} = (V_{FDMIN} + V_{FDMAX})/2$. Further the designer must assume a probability

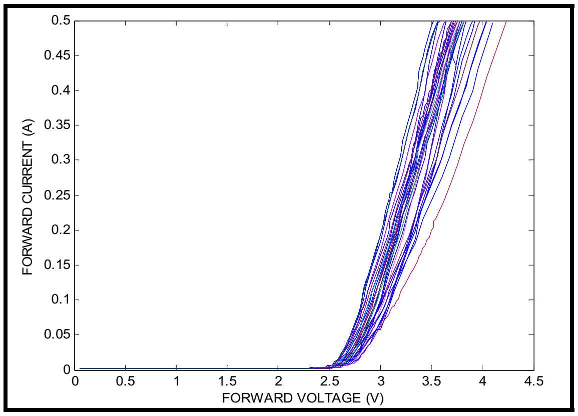

Fig. 5. Measured *Current-Voltage Characteristics of 30 unbinned HB-LEDs*

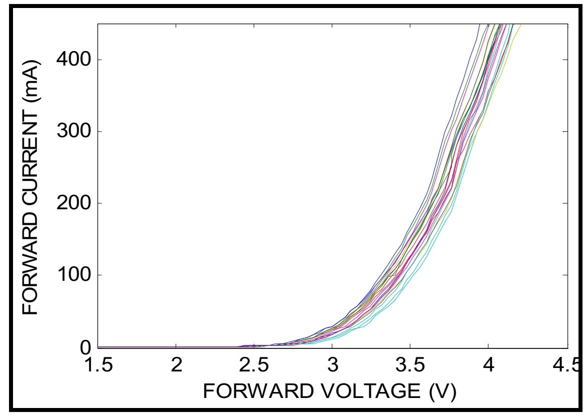

Fig. 6. Measured *Current-Voltage Characteristics of 20 binned HB-LEDs*

distribution function associated with V_{FD} . For example, Fig. 7 shows a histogram of the forward voltage drops from the curve tracer. From this histogram, a simple approximation may be to assume a Gaussian distribution [9]. Alternatively, it is also reasonable to assume a uniform distribution of V_{FD} over [V_{FDMIN} , V_{FDMAX}]. From this distribution, derive a standard deviation value. For example, if a Gaussian distribution is assumed, then we might assume $\sigma_{VFD} = (V_{FDMAX} - V_{FDMIN})/6$, which sets the minimum and maximum values to be at the $\pm 3\sigma_{VFD}$ points. In a uniform distribution, the standard deviation would be $\sigma_{VFD} = (V_{FDMAX} - V_{FDMIN})/\sqrt{(12)}$. Alternatively, the standard deviation could be calculated directly from a sample of HB- LEDs to be used.

Step 3: Compute the mean and standard deviation for the series sum of M number of HB-LEDs by defining $V_{FDSTRING}$ to be the random variable $V_{FDSTRING} = M^* V_{FD}$. For a Gaussian distribution, this leads to just another Gaussian distribution with new mean $\mu_{VFDSTRING}$ = M μ_{VFD} and a new standard deviation of $\sigma_{VFDSTRING} = \sqrt{(M)}*\sigma_{VFD}$. If a uniform distribution was assumed, then the new random variable is obtained by convolving V_{FD} with itself M-times. (MATLAB easily performs this convolution.) In fact, as M becomes larger the convolution is often approximated by a Gaussian distribution which is justified by the Central Limit Theorem. Essentially, we are creating the new probability distribution function (PDF) for $V_{FDSTRING} = M^*V_{FD.}$

Step 4: Given ***P*** in Step 1 and the PDF of the random variable $V_{FDSTRING}$ in Step 3, solve for V_{MIN} and V_{MAX} such that there is a probability ***P*** that $V_{FDSTRING}$ satisfies $V_{MIN} < V_{FDSTRING} < V_{MAX}$, selecting V_{MIN} and V_{MAX} symmetrically around the mean, i.e. keeping $\mu_{VFDSTRING} = (V_{MIN} + V_{MAX})/2$.

Step 5: Select the balancing resistor and source voltage according to the formula:

$$R_A = \frac{M(V_{MAX} - V_{MIN})}{(1-\gamma)I_{DMAX}} - MR_D \; (3), \quad V_S = \frac{M(V_{MAX} - \gamma V_{MIN})}{1-\gamma} \; (4)$$

Example 1: M=4 LEDs from binned LEDs inTable 1

Returning to the previous design example of binned LEDs, we will assume that $\gamma = 0.9$, I_{MAX} = 350mA, and V_S = 23.84V. Let ***P*** = 0.9998 in Step 1, i.e. a 99.98% certainty that the current is within the desired bounds. This example presents an Approach for LED Arrangement for Driving Design In Normal pdfs (ALADDIN). We will assume a Gaussian distribution of the forward voltage drop of each HB-LED in Table 1, with $\mu_{VD} = (V_{FDMIN} + V_{FDMAX})/2 = 3.52$, and standard deviation, $\sigma_{VFD} = (V_{FDMAX} - V_{FDMIN})/6 = 0.04$. From Step 3, this leads to the normal random variable $V_{FDSTRING}$ with mean $\mu_{VFDSTRING}$ =14.08 and standard deviation $\sigma_{VFDSTRING} = 0.08$.

A sketch of the probability distribution function of $V_{FDSTRING}$ is shown in Fig. 8. Using basic probability theory for this curve [8], we derive that V_{MAX} = 14.38V and V_{MIN} = 13.79V. Thus there is a 99.98% probability that $V_{FDSTRING}$ lies between these two bounds. Using equation (3) and (4), this leads to the necessary balancing resistor of only $R_A = 12.77\Omega$ and a source voltage of V_S = 19.66V. Notice that this balancing resistor is approximately half the value necessary when the conventional worst case design in used. This corresponds to almost half of the $I_D^2 R_A$ power loss when worst case design is used, that is we reduced the balancing resistor power loss from 2.87W to 1.52W compared to worst case design. Simple protection circuitry could still be added to assume that I_{DMAX} is not exceeded. Alternatively, the probability of failure is so low, it becomes more economical to test the entire lighting system (all strings together) than each individual LED (as is done now).

Example 2: M=4 LEDs (not binned)

Let us design a similar string of 4 unbinned LEDs, whose characteristics are shown in Fig.5 and Fig.7, for $\gamma = 0.9$, I_{MAX} = 350mA. As we are using LEDs those are not tightly binned, we considered 97% probability would be a fair choice. So let ***P*** = 0.97 in Step 1, i.e. a 97% chance that the current is within our desired requirements. As before, assuming Gaussian distribution, we calculate mean as $\mu_{VD} = (V_{FDMIN} + V_{FDMAX})/2 = 3.234$ and standard deviation, σ_{VFD} =(V_{FDMAX} -

$V_{FDMIN})/6$ = 0.07. From Step 3, this leads to $\mu_{VFDSTRING}$ =12.936 and $\sigma_{VFDSTRING}$ = 0.14.

In this case when 4 unbinned LEDs are connected in a string the probability distribution function in the bottom of Fig. 8 is created. This leads to V_{MAX} = 13.2398V and V_{MIN} = 12.6322V. Using equation (3) the necessary balancing resistor is only R_A = 15.12Ω whereas in the worstcase calculation, the added resistance value is 45.76Ω. When the reduced balancing resistor value is used the power loss in the case of worst case calculation is 5.61W where with 97% probability the power loss is 1.85W. Similarly, using (4), the required source voltage becomes V_S = 18.71V, compared to V_S = 28.90V in the worst case design approach.

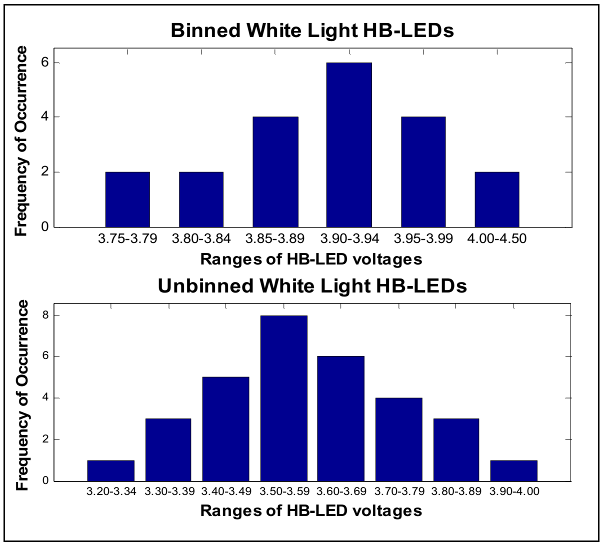

Fig. 7. *Histogram of the frequency of V_{FD}*

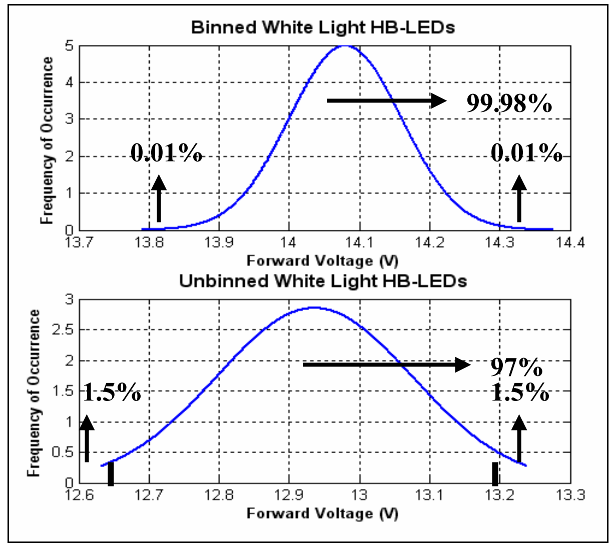

Fig. 8 *Normal PDF of forward voltage of four LEDs in series*

Table II Worstcase Calculations for the voltage binned HB-LEDs

Number of LEDs	V_{FDMAX} (V)	V_{FDMIN} (V)	V_S (V)	R_A (Ω)	R_D (Ω)	Power loss for R_A (W)	Power loss for R_D (W)	Power loss total (W)	System Efficiency
5	18.2	17	29	29.2857	5	10.25	1.75	12.00	58.62%
10	36.4	34	58	58.5714	10	20.50	3.5	24.00	58.62%
15	**54.6**	**51**	**87**	**87.8571**	**15**	**30.75**	**5.25**	**36.00**	**58.62%**
20	72.8	68	116	117.1429	20	41.00	7.00	48.00	58.62%
25	91.0	85	145	146.4286	25	51.25	8.75	60.00	58.62%
30	109.2	102	174	175.7143	30	61.50	10.50	72.00	58.62%
35	127.4	119	203	205.0000	35	71.75	12.25	84.00	58.62%
40	145.6	136	232	234.2857	40	82.00	14.00	96.00	58.62%
45	163.8	153	261	263.5714	45	92.25	15.75	108.00	58.62%
50	182.0	170	290	292.8571	50	102.50	17.50	120.00	58.62%
55	200.2	187	319	322.1429	55	112.75	19.25	132.00	58.62%
60	218.4	204	348	351.4286	60	123.00	21.00	144.00	58.62%
65	236.6	221	377	380.7143	65	133.25	22.75	156.00	58.62%
70	254.8	238	406	410.0000	70	143.50	24.50	168.00	58.62%

Table III Stochastical Calculations for the voltage binned HB-LEDs

Number of LEDs	V_{FDMAX} (V)	V_{FDMIN} (V)	V_S (V)	R_A (Ω)	R_D (Ω)	Power loss for R_A (W)	Power loss for R_D (W)	Total power loss	System Efficiency
5	17.9326	17.2762	23.8404	13.7549	5	4.8142	1.75	6.5642	72.47%
10	35.6704	34.7421	44.0253	16.5235	10	5.7832	3.5	9.2832	78.91%
15	**53.3761**	**52.2392**	**63.6088**	**17.4845**	**15**	**6.1196**	**5.25**	**11.3696**	**82.13%**
20	71.0653	69.7524	82.8809	17.5098	20	6.1284	7.00	13.1284	84.16%
25	88.7438	87.2760	101.9540	16.9373	25	5.9280	8.75	14.6780	85.60%
30	106.4148	104.8069	120.8859	15.9400	30	5.5790	10.50	16.0790	86.70%
35	124.0801	122.3434	139.7106	14.6208	35	5.1173	12.25	17.3673	87.57%
40	141.7408	139.8842	158.4506	13.0469	40	4.5664	14.00	18.5664	88.28%
45	159.3979	157.4287	177.1213	11.2647	45	3.9427	15.75	19.6927	88.88%
50	177.0519	174.9761	195.7340	9.3082	50	3.2579	17.50	20.7579	89.39%
55	194.7032	192.5261	214.2972	7.2030	55	2.5211	19.25	21.7711	89.84%
60	212.3523	210.0784	232.8175	4.9689	60	1.7391	21.00	22.7391	90.23%
65	229.9993	227.6326	251.3002	2.6218	65	0.9176	22.75	23.6676	90.58%
70	247.6446	245.1885	269.7496	0.1745	70	0.0611	24.50	24.5611	90.89%

V. Further Discussion and Results

For the conventional worst case design, Table II shows the calculated values of required source voltage, added resistance, power loss and the efficiency of the circuit when binned LEDs are used. It is visible from Table II that the added resistance value increases linearly with the increment of HB-LED numbers in series. In Table III, we have shown the calculated values of the same parameters as Table II using the stochastical model of strings of binned LEDs for 99.98% probability. It can be seen from Table III that the value of the added current balancing resistance increases until around 20 HB-LEDs are connected in series. After that the value of the added resistance starts to decrease. Notice that even for less than 20 HB-LEDs in series, the value of the added resistance is much less than the resistance added at the time of worst case analysis. It is noticeable in Table III that when around 70 LEDs are connected in series, the value of the added resistance is negligible. This means that the dynamic resistances of the HB-LEDs are compensating for the added resistance required.

Let us further explore the benefits of the stochastic modeling approach with an example of strings of 15 HB-LEDs. We can see from Table II that the power loss for the added resistance to the string of 15 HB-LED is 30.75W. Similarly, if we look into Table II, we can see that for 15 LEDs in a string the power loss for the added resistance is 6.1196W, which is almost one fifth of the loss when worst case analysis is used. In fact, the system efficiency is 82.15% when the probabilistic design approach is used.

Suppose, for comparison purposes, that each string utilizes a DC-DC converter for current control as in Fig.9. Now, even if 90% efficient DC-DC converters are used, then, also, the circuit efficiency becomes 82.11%. This result shows us that with a probability of 99.98% with a battery operated source, we can achieve circuit efficiency comparable, and sometimes even higher than a system operated with active current control method, yet at substantially reduced cost. Thus stringing large number of HB-LEDs together may be a practical, cost efficient and power efficient design approach when precise current regulation in each string is not required and a proper DC bus voltage or a battery is available, e.g., street lamps. We remark that this is true even if only a single converter is used to power all the paralleled strings, as in [10].

Fig. 10 plots the data of Tables I and II to show the added resistance requirement of the HB-LED strings. It is clear from the figure that the requirement of the added resistance gradually decreases to zero when the stochastical method is applied. Fig. 11 shows the significant difference in total power loss of the circuit with HB-LED strings at worst case calculation and also at stochastic calculation. Fig. 12 and Fig. 13 show circuit efficiency curves. Fig. 12 shows that the circuit efficiency of the stochastical model is much better than the worstcase model. Fig. 13 shows the important scenario that with the increase of the number of LEDs in string, the circuit efficiency becomes better when the balancing resistor designed with the probabilistic model compared to DC/DC converters approach as in Fig.9.

We now revisit Example 2 (ALADDIN) with 4 LEDs in a string with R_A=15.12Ω and V_S=18.71V. The 4 LEDs were randomly selected from a box of 30 LEDs and then connected in series. The current in the string was then measured. The process was repeated 50 times to create an experimental histogram as shown in Fig.14. As explained in section IV, a 97% probability was considered at the time of experiment where I_{DMAX} and I_{DMIN} were taken as 350mA and 315mA respectively. More importantly, the value of the balancing resistor used at the experiment was 15.12Ω NOT 44Ω as in the worst case model. 50 different combinations of unbinned HB-LEDs (4 randomly selected LEDs in a string) were tested.

The histogram current for the designed string of the unbinned HB-LEDs in 48 out of 50 trials met the designer specification, exceeding 350mA only twice. Thus, 96% of the time the current is between 315mA and 350mA. Our model predicted that this would occur 97% of the time. If the number of LED samples to choose from increased and the number of trials increased, the accuracy of the predictions would improve even more.

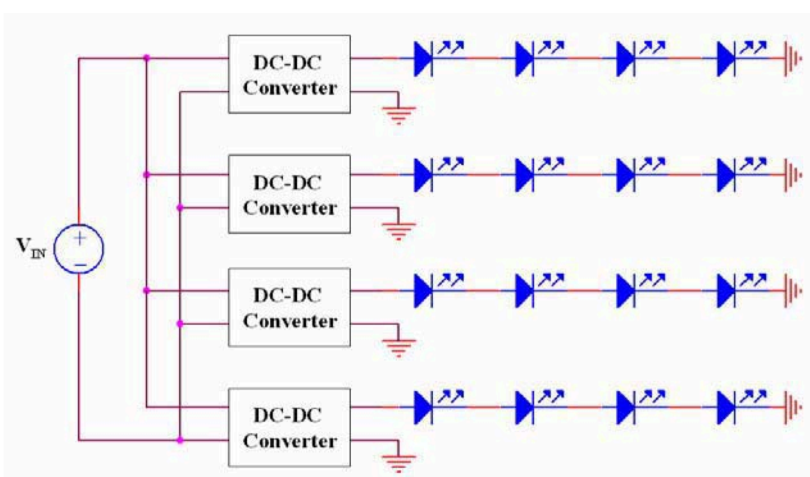

Fig.9.*Strings of LEDs connected with DC/DC converters*

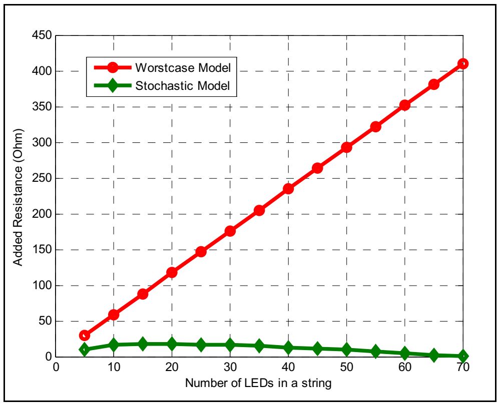

Fig. 10 . *Number of LEDs Vs Added resistance curve*

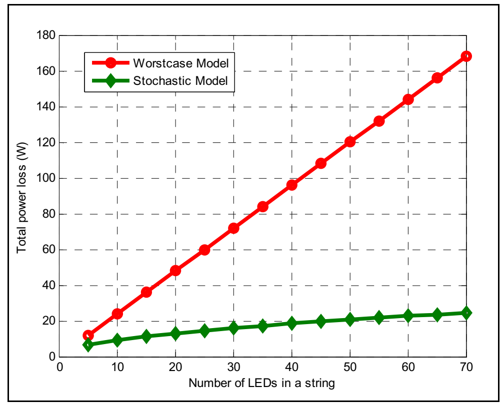

Fig. 11. *Number of LEDs Vs total power loss curve*

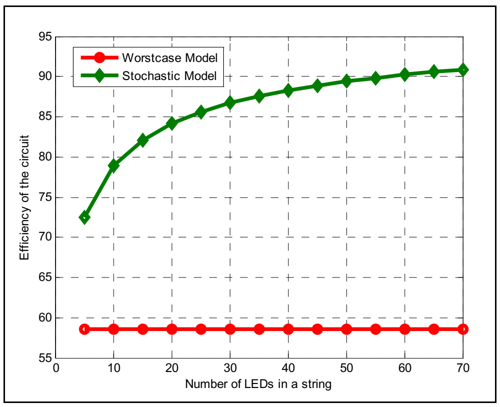

Fig. 12 . *Difference of efficiency for worstcase stochastical model*

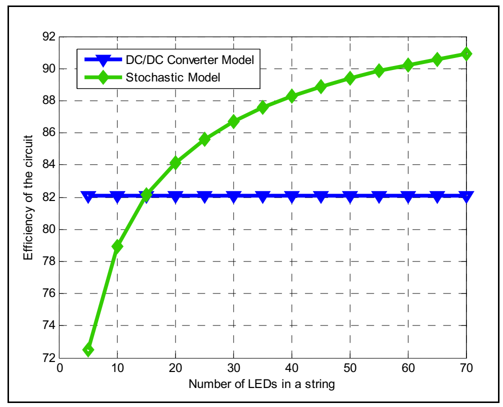

Fig. 13. *Difference of efficiency for DC/DC converter model and and stochastical model*

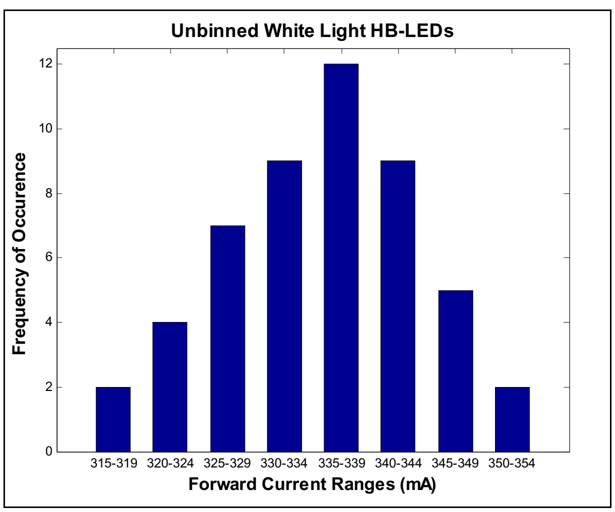

Fig. 14. *Experimental result of the forward current of 30 unibinned HB-LEDs*

VI. Conclusion

A design procedure to calculate the balancing resistor and source voltage value for LED strings based on the probabilistic models is presented. The approach yields lower balancing resistor value compared to conventional worst case design and thus improve power system efficiency. The methods are also compared with the approach of individually controlling each HB-LED string with an individual DC-DC converter. Experimental results are shown that justify the probabilistic model and prove the benefits of the design.

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