Realizing Data driven and Hampel preprocessor based Adaptive filtering on a Software Defined Radio testbed: A USRP case Study

Anu Jagannath and Ashwin Amanna
ANDRO Advanced Applied Technology, ANDRO Computational Solutions, LLC, Rome NY
{ajagannath, aamanna}@androcs.com

Abstract—Adaptive filtering is a critical component of modern Direct Sequence Code Division Multiple Access (DS-CDMA) systems that operate in a highly dynamic environment. Adaptive filtering algorithms such as Sample Matrix Inversion (SMI) have been widely studied for Multiple Access Interference (MAI) suppression but is infamous for its strenuous matrix operations. The limited processing time and resources call for Auxiliary Vector (AV) filtering, a computationally efficient alternative that has been studied well in literature. Due to the extreme challenges associated with realization of these techniques on hardware, most of the work has been limited to simulation based analysis. Corroborating the simulation results through hardware implementation is necessary before these filtering techniques can be adopted by different tactical and commercial radios. Therefore, the objective of this work is to surpass the hurdles of implementation and demonstrate the significance of these adaptive receivers on an actual radio framework. Accordingly, we analyze the computationally exhaustive SMI against the mathematically efficient AV to examine the tradeoffs. In our analysis, the supervised and blind J-divergence rules showed remarkable performance at good signal strength scenario while cross-validated minimum output variance rule outperformed at low signal strength cases. Blind J-divergence rule proved to be a better choice for DS-CDMA systems with limited data record and binary phase shift keying modulation (or its variants). Additionally, we also examine the improvement achieved by introducing the Hampel preprocessor for both AV and SMI receivers.

I. INTRODUCTION AND BACKGROUND

Spectral efficiency is an inevitable factor in designing future wireless communication systems [1]. Supporting more users (\(U\)) in the system for a given spreading code length (\(\nu\)) is key to increase the system spectral efficiency. The ability of a Direct Sequence Code Division Multiple Access (DS-CDMA) system to support more users than the spreading code length (\(U > \nu\)) to operate simultaneously at acceptable Bit-Error-Rate (BER) levels is referred to as system overload. Tactical communication systems adopt DS-CDMA technology for its narrow band jam resistance capability and low probability of intercept/low probability of detection support. As the number of active users in the system increases, the Multiple Access Interference (MAI) experienced by each user will increase significantly. This motivates the need for a receiver with good MAI suppression capability. In these scenarios, static filter design that does not consider the varying MAI statistics causes higher BER. Thus, a plethora of adaptive filtering and adaptive spreading code assignment algorithms has been proposed and studied in the past several years [2]–[8].

In this paper, we focus on adaptive filtering algorithms and perform extensive feasibility study using Software Defined Radios (SDRs). To ensure feasibility, an adaptive receiver must possess the following attributes: (1) superior MAI suppression capability, (2) low computational complexity, and (3) the ability to adapt under short/limited data records (sample size).

One of the commonly used adaptive filtering approach is the Sample Matrix Inversion (SMI) method [2] where the MAI is estimated by sample averaging. The sample averaged autocorrelation estimate of the MAI disturbance matrix is inverted in this approach to obtain the filter coefficients. This can be computationally demanding especially as the size of the matrix grows. Meanwhile, the recursive Auxiliary Vector (AV) filtering [3] approach was introduced as a computationally efficient and robust solution which provides superior small sample support to the system. The data record based AV filtering criterion [4] serves as a tool to choose the number of AVs required to estimate the filter coefficients.

The heart of an SMI and AV adaptive filter lies in the disturbance autocorrelation matrix which constitutes the MAI as well as the noise distortion to the signal samples. In simulations [9], where the simulated characteristics are known, this disturbance autocorrelation matrix can be perfectly computed and the receiver is assumed to have perfect knowledge of the disturbance matrix. Simulation cuts down the development time and is an efficient method to validate proposed solution albeit under several assumptions. Therefore, all previous work have been limited to simulations and overlook the implications of realizing the proposed filtering on hardware testbed. In a realistic scenario, where channel characteristics are unknown the disturbance autocorrelation matrix has to be estimated from the received signal samples as in section (II-A). The challenges involved in the realization of any algorithm on an actual radio framework would entail bridging the gap between the assumptions considered during design or simulations and unknown artifacts (caused by lack of synchronization, hardware limitation, channel conditions among others) that affects the actual over the air (OTA) signal. Therefore, the major contributions of this paper can be summarized as follows,

- We implement both SMI and data-record based AV filtering to study the robustness of these algorithms in a MAI environment. To the best of our knowledge, this is the first work that implements and validates the SMI and AV
filtering methods on an actual SDR framework.
- Next, to establish the significance of employing a Hampel preprocessor in an impulsive noise environment, we realize the AV and SMI based Hampel adaptive receive chains [8] on the SDR testbed.
- All these techniques are rigorously evaluated OTA using SDRs to compare the performance of SMI, AV with and without Hampel preprocessing. This study using actual hardware aims to validate the performance in realistic scenarios which will enable rapid transitioning of these techniques to commercial hardware.

The rest of the paper is organized as follows. In II, we will elaborate on the Minimum-Variance-Distortionless-Response (MVDR) class of filters. The experimental framework, setup and results will be explained in III. Finally, we conclude this paper by summarizing the findings in IV.

II. MVDR FILTERS

Consider a DS-CDMA system containing $U$ users with spreading gain or spreading signatures of length, $G$. The received signal vector $y \in \mathbb{C}^G$ is given by

$$y = \sum_{u=1}^{U} \sqrt{\sigma_u} s_u + n$$

(1)

where $s_u$, $u = 1, 2, ..., U$, is the $G$-dimensional user signal, $\sigma_u$ is the received signal energy, $i_u \in \{-1, +1\}$ is the information bit of the $u$-th user and $n$ is the additive white Gaussian noise. If user-1 is the UOI, the signals from the remaining users act as MAI.

MVDR filter computes a linear filter that minimizes the noise variance at its output by maintaining a unity response, i.e., distortionless in the target or look direction of interest. The work in [10] demonstrated the extra interference (caused by synchronization errors) suppression property of MVDR filtering. If the complex input vector to the filter is $y$ and $F \in \mathbb{C}^{G \times G}$ refers to the $G$-tap filter coefficients, then the filter output variance is $F^HDF$, where $D = E\{yy^H\} \in \mathbb{C}^{G \times G}$, is the input autocorrelation matrix ( $E[.]$ denotes statistical expectation operation and $y^H$ is the conjugate transpose, Hermitian transposition, of $y$ ). The constraint vector (look direction of interest), $v$, can be obtained by performing the statistical cross-correlation between the desired output (pilot/training sequence), $r$, and received input vector $r$, that is given by, $v = E\{yr^T\}$. For a given constraint vector $v$ and perfectly known input autocorrelation matrix $D$, the ideal MVDR filter solution is given by $F_{MVDR} = (D^{-1}v)/(v^HD^{-1}v)$ where $D^{-1}$ denotes the inverse of $D$.

A. SMI MVDR Filters

In a realistic radio environment, $D$ is unknown and is therefore estimated from snapshots of received input vector $y$. The snapshots are collectively termed as data record and so a data record of $P$ points can be represented by $[y_1, y_2, ..., y_P]$. From a data record of $P$ points, $D$ is sample average estimated as $\hat{D}(P) = (1/P) \sum_{p=1}^{P} y_p y_p^H$. The filter obtained by using the sample-averaged estimate $\hat{D}(P)$ is known as the SMI MVDR filter and is given by $F_{SMI} = (\hat{D}(P)^{-1}v)/(v^H\hat{D}(P)^{-1}v)$ where, $F_{SMI} \in \mathbb{C}^G$ is the SMI filter coefficients. In actual implementation, the $G$-tap coefficients returned by $F_{SMI}$ form the taps of the finite impulse response (FIR) filter. The bit detection can be represented by the mathematical expression $\hat{\ell}(i) = \text{sign}(\Re\{F_{SMI(\text{i})}y\})$ where $\hat{\ell}(i)$ denotes the $l$-th bit of $u$-th user and sign($\cdot$) is the sign mathematical operation which will return a $\pm 1$ for a scalar value greater than or less than 0 respectively. $\Re(x)$ returns the real part of the complex number $x$. The sample average estimate $\hat{D}(P)$ is a complex matrix of size $G \times G$ which implies as $G$ increases, the matrix size will grow with which comes in the disadvantage tailored to matrix inversion operation. The matrix inversion has a complexity of $O(n^3)$, i.e., the run time may grow cubic, making them less favorable for practical communication systems.

The authors of [3] showed that a sample covariance matrix is positive definite and hence invertible with probability 1 only if data record size from which it is estimated is larger than the dimensionality of the matrix, i.e., $P \geq G$. $P$ has to be adequately large (multiple times $G$) for SMI to perform reasonably well. Thus, in a swiftly varying radio environment, SMI filter would render impractical leaving way for a computationally efficient filter estimation which will exhibit substantial performance improvement under short data record. This served as the underlying motivation behind the formulation of AV filters which was introduced and extensively studied in [4]. In our study, we consider a data record size $P = G$ as a short data record.

B. AV Filters

AV filter estimation algorithm is a recursive procedure that generates a sequence of vectors (linear AV filters) which converges to the MVDR solution. The algorithmic formulation and convergence was studied in [3] where it was shown that the sequence of AVs $\{g_d\}, d = 1, 2, …$, converges to the 0 vector and hence $F_d \rightarrow F_{MVDR}$. The AV filter sequence is generated as shown in Algorithm 1. This iterative procedure

**Algorithm 1 Recursive AV filter algorithm**

1. **$F_0 := \frac{v}{|v|^2}$**
2. **for** $d = 1, 2, ..., do$ **do**
3. **$g_d := (I - \frac{v v^H}{|v|^2}) \hat{D}(P) F_{d-1}$**
4. **if** $g_d = 0$ **then** **EXIT**
5. **end if**
6. **$\mu_d := \frac{g_d^H \hat{D}(P) F_{d-1}}{g_d^H \hat{D}(P) g_d}$**
7. **$F_d := F_{d-1} - \mu_d g_d$**
8. **end for**

is a greedy and mathematically simpler technique which involves no explicit matrix inversion, diagonalization and Eigen decomposition. The algorithm intakes sample averaged auto-correlation matrix $\hat{D}(P)$ and $v$ to recursively generate filter sequences. The recursive procedure aims at minimizing the variance/energy $E\{F^Hv|v|^2\}$ at the output of the filter while retaining the UOI by imposing the constraint $F^Hv = 1$. Minimization of the corresponding filter output variance attenuates
the interference and noise thus resulting in a maximum SINR solution [7]. Each AV $g_d$ accounts for the interference and noise in the system. The formal derivation of the scalar $\mu_d$ that minimizes the filtered output energy was presented in [3]. This would curb all signal components that are not in the direction of interest. The authors of [4] proposed three data driven search criterion to select the most successful AV filter from the sequence for a finite $P$, namely; (i) Cross-validated minimum output variance (CV-MOV) rule, (ii) Supervised Output J-divergence rule, and (iii) Blind Output J-divergence rule. In the following subsections, we will have a closer look at each of these rules.

1) CV-MOV Filters: The CV-MOV rule minimizes the cross-validated sample average variance at the AV filter output and applies to general filter estimation problems. Cross-validation is a widely used statistical validation technique [11], of which the particular case that is used in [4] is the “Leave one out” method. The CV-MOV rule can be summarized by, $\hat{d}_c = \arg\min_a \{ \sum_{n=1}^{N} \| p_{k} \mathbf{y}_{n} - \mathbf{F}_{d}(p_{k}) \| \}$ where, $\hat{d}_c$ is the number of AVs that minimizes the cross-validated output variance, subplot $(\hat{P})$, denotes the AV filter estimator that is evaluated from the $P$-point data record after removing the $k$-th snapshot. Let us shed more light into estimating $\mathbf{F}_{d}(p_{k})$. Removing the $k$-th sample from the $P$-point data record would change the input autocorrelation matrix and constraint vector to

$$\hat{\mathbf{D}}(\hat{P}) = (1/P) \sum_{p=1}^{P} \mathbf{y}_{p} \mathbf{y}_{p}^H$$

and

$$\hat{\mathbf{v}}(\hat{P}) = (1/P) \sum_{p=1}^{P} \mathbf{y}_{p} t_{p}^H$$

where $\hat{\mathbf{D}}(\hat{P})$ and $\hat{\mathbf{v}}(\hat{P})$ are the leave-$k$ out input auto- correlation matrix and constraint vector respectively which when applied to the recursive AV algorithm will give the corresponding $\mathbf{F}_{d}(p_{k})$. Thus, for a given finite $P$-point data record, CV-MOV filter computes the filter estimator $\mathbf{F}_{d}\hat{y}$. 

2) Supervised Output J-divergence Filters: The supervised and blind output J-divergence rules are tailored specifically for binary hypothesis testing (binary phase shift keying (BPSK) type detection) problems. J-divergence a.k.a symmetrized Kullback-Leibler divergence is the measure of dissimilarity between two probability distributions. BPSK detection can be viewed as a binary hypothesis testing problem where the detector has to decide between $\pm 1$. For any binary hypothesis testing problem, the probability of error of the optimum Bayesian detector is lower bounded by a monotonically decreasing function of the J-divergence between the two conditional distributions ($p_0$ and $p_1$ under hypothesis $H_0$ and $H_1$) [4]. Thus, minimizing the error probability problem translates into an equivalent maximizing J-divergence ($J(d)$) problem. Supervised Output J-divergence assumes the availability of a training sequence to compute dissimilarity between the AV filter output conditional distributions which is measured as $J_{\text{train}}(d) = (\hat{\gamma}(d) - \hat{\delta}^2(d))$ where, $\hat{\gamma}(d)$ and $\hat{\delta}^2(d)$ are the minimum variance unbiased estimator of the conditional mean and variance under either hypothesis and are estimated as follows:

$$\hat{\gamma}(d) = (1/P) \sum_{p=1}^{P} (i(p) \mathbf{F}_{d}^H \mathbf{y}_{p})$$

and

$$\hat{\delta}^2(d) = (1/P) \sum_{p=1}^{P} i(p) \mathbf{F}_{d}^H \mathbf{y}_{p} - \hat{\gamma}(d)^2$$

where, $i(p)$ is the training sequence. Now the supervised output J-divergence rule of maximizing J-divergence or minimizing error probability of optimum Bayesian detector follows in a straightforward manner such that $d_{\text{train}} = \arg\max_d J_{\text{train}}(d)$ where, $d_{\text{train}}$ is the number of AVs required to generate filter estimator $\mathbf{F}_{d}\hat{y}$.

3) Blind Output J-divergence Filters: The blind or unsupervised output J-divergence rule does not require a training sequence in the computation of J-divergence. Having looked at II-B2, the supervised implementation can be easily modified to derive its blind counterpart by substituting the training sequence $\{i(p)\}_{p=1}^{P}$ with detected bits to arrive at the effective divergence measure,

$$J_{\text{blind}}(d) = \frac{4 \sum_{p=1}^{P} |\mathbf{R}(\mathbf{F}_{d}^H \mathbf{y}_{p})|^2}{\frac{1}{P} \sum_{p=1}^{P} |\mathbf{R}(\mathbf{F}_{d}^H \mathbf{y}_{p})|^2 - \frac{1}{P} \sum_{p=1}^{P} |\mathbf{R}(\mathbf{F}_{d}^H \mathbf{y}_{p})|^2}$$

[4, Proposition 2] states when the filter output SINR is substantially higher than 0 dB, the blind output J-divergence $J_{\text{blind}}(d)$ nearly equals the J-divergence between the AV filter output conditional distributions ($p_0$ and $p_1$). As studied in [12], the other widely used divergence measures in the design of experiments are, Bhattacharya distance $B(p_0,p_1)$ and Kullback-Leibler distance (KLD) $\mathcal{K}(p_0,p_1)$. Similar to J-divergence expression in [4, Equation. 10], the corresponding Bhattacharya and KLD distance measures can be written as $B(d) = 2 \rho(\mathcal{d})$ and $\mathcal{K}(d) = 2 \rho(\mathcal{d})$. Therefore, another set of equivalent divergence rules hence follows which aims to maximize the Bhattacharya distance and KLD respectively. Since all the divergence measures in its entirety differs only by a scaling factor, the final returned filter parameter of interest $\hat{d}$ will be the same: $\hat{d} = \arg\max_d J_{\text{train}}(d) = \arg\max_d B(d) = \arg\max_d \mathcal{K}(d)$.

C. Hampel Preprocessor

![Fig. 1: Hampel-AV adaptive receiver.](image)

The authors of [8] proposed utilizing a Hampel preprocessor prior to AV filtering of the received inphase-quadrature (IQ) samples for scenarios involving impulsive noise in addition to the MAI. The proposed receiver structure (Fig. 1) with Hampel preprocessor prior to AV filter will jointly suppress the effect of noise and MAI from the received samples. Each element of the received vector is mapped such as

$$H(y,v_1,v_2,v_3) = H(y,v_1,v_2,v_3) \cdots H(y,v_1,v_2,v_3)$$

where the $H(.;v_1,v_2,v_3)$ transformation is given by

$$H(y,v_1,v_2,v_3) = \begin{cases} y & \text{if } |y| < v_1, 0 < v_1 \\
\rho_1 & \text{if } v_1 \leq |y| \leq v_2, 0 < v_1 \leq v_2 \\
\rho_2 & \text{if } v_2 < |y| < v_3, 0 < v_1 \leq v_2 \leq v_3 \\
0 & \text{otherwise} \end{cases}$$

(4)
The Hampel processed IQ samples are fed into the AV filter which will perform the AV filtering algorithm as discussed in Algorithm 1. Replacing the AV filter with SMI filter would yield the Hampel-SMI receiver which will be used in our experimental comparisons.

III. EXPERIMENTAL EVALUATION

Fig. 2: Indoor SDR Testbed.

The experimental testbed comprised of two USRP-N210s (SBX daughterboard, USRP Hardware Driver (UHD) version 3.8.2); a $U$-user transmitter (TX) and a UOI receiver (RX) separated by $\approx 13$ ft. as depicted in Fig. 2 in an indoor laboratory environment. The TX and RX are controlled by a Linux PC. The system is primarily implemented using C++ and Python utilizing the GNU Radio development toolbox (version 3.7.5). The system operates at 925 MHz and uses VERT900 antennas. The frame structure of our system consists of a preamble followed by payload. The preamble is a 127-bit Gold sequence and is used to achieve frame/packet synchronization. In our implementation, the preamble serves a second purpose by acting as the training sequence, $\{i(p)\}_{p=1}^{127}$. This training sequence is used to estimate $v$ and $J_{\text{train}}(d)$. Thus, the maximum value of $P$ is 127. The payload size is set to 220 bytes and sending 30 packets per transmission. Each data point on the BER curve is an average over 30 independent repetitions (each repetition involves a transmit-receive session).

Transmitter Setup: The $U$-user DS-CDMA transmitter is emulated such that a single USRP-N210 will transmit the multiplexed signal from all $U$ users. The data bits of each user is differential BPSK modulated and spread with the desired spreading signature ($s_u$). The spreading is achieved using an interpolating FIR filter whose taps form the spreading signature bits. The signal stream from each user branch is added to obtain the multiplexed $U$-user stream. The multiplexed signal is root raised cosine (RRC) filtered to obtain the pulse shaping and is finally transmitted OTA.

Parallel Adaptive Receiver Setup: Our receiver has two parallel filtering branches which can process the synchronized IQ samples with two filters of choice. This is done to serve as a direct comparison between two types of filtering techniques. We encourage the readers to refer [13] to learn about our transmitter design and synchronization technique. The filtered and detected bits are maintained in two isolated queues, the bit errors corresponding to each queue will be used to assess a filter with respect to the other.

Discussion of results: Figure 3 evaluates performance of UOI as the system is loaded by increasing the total number of users $U$ from 10 to 16 and $G = 31$. This is denoted by the user loading factor $U/G$ along the x-axis. The USRP TX and RX gains are set at 0 dB and 24 dB respectively. The supervised and blind J-divergence filter rules demonstrate similar performance and outperforms CV-MOV, fixed-AV (AV filter that uses a fixed number of AVs) and SMI filters for increasing $U/G$. CV-MOV rule outperforms both fixed-AV and SMI. Figure 4 plots the BER performance of a UOI for varying USRP TX gain. The system uses a Gold spreading sequence matrix of $G = 31$ and 9 users in the system. At lower TX gains, CV-MOV outperforms both supervised and blind filter rules, and vice-versa at higher TX gains. This corroborates the Proposition 2 in [4] that for low filter output SINR values, the J-divergence approximation is less accurate. All three filter rules are superior to both fixed-AV and SMI filters validating the benefit of choosing the appropriate number of AVs. Next, we evaluate the BER performance of a UOI versus data record size $P$ which is varied from lowest setting ($P = G = 31$) to 124 in steps of 31. The fixed-AV in our testing uses 20 AVs to estimate filter coefficients. In Fig. 5, for a short data record ($P = 31$), supervised and blind filter rules are superior to CV-MOV, fixed-AV and SMI but as $P$ increases CV-MOV perform slightly better than supervised filter rule. Blind filter rule outperforms all the filters for varying $P$. The CV-MOV based AV filter outperforms the fixed-AV and SMI filters.

To evaluate the robustness of Hampel based receiver against impulsive noise, we inject impulsive noise following Gaussian noise mixture model as in [8] to the received IQ samples. In [8], authors do not compare the Hampel-AV’s performance against plain-AV (AV receiver without the Hampel preprocessor). Identifying and validating the performance improvement if any in an impulsive noise scenario, is paramount to motivate the need for a Hampel preprocessor in the system. Thus, we implement and compare the performances of plain-SMI, Hampel-SMI, plain-AV and Hampel-AV receivers in the presence of impulsive noise. In Figs. 6, 7 and 8, we denote AV receiver utilizing 2 AVs as plain-2 AV and Hampel based AV receiver which requires 2 AVs as Hampel-2 AV. Here again, the DS-CDMA system is constituted by Gold spreading sequence matrix of $G = 31$ and 9 users in the system where 1 user is considered the UOI. The USRP RX gain was fixed at 8 dB and the TX gain was varied from 0 dB to 20 dB in steps of 4 dB for the Fig. 6 and Fig. 7. Figure 6 evaluates the performance under short data record ($P = G = 31$). The small $P$ setting caused severe data starvation in plain-SMI and Hampel-SMI although the BER curve of Hampel-SMI is slightly better than plain-SMI in the low TX gain region. Evidently, Hampel-2 AV is performing better than plain-2 AV implying the impulsive noise suppression capability of the Hampel preprocessor. Now, we repeat the experiments for a larger $P$ and plot the performance in Fig.7. Notably, the larger $P$ have mitigated the data starvation issue of plain-SMI and Hampel SMI. The higher $P$ has also improved the performance of plain-2 AV filter in contrast to its performance under small
In all the three cases, Hampel-AV exhibited superior BER performance (as is the case with Hampel-SMI and plain-SMI). In the varying data record case with blind J-divergence filter being the best of all. Thus, blind J-divergence filter can provide exceptional BER performance for DS-CDMA systems (formulated as a binary hypothesis based detection) at a good signal-to-noise ratio (i.e., high TX gain) and limited data record (i.e., $31 < P < 124$) scenario. The non-requirement of a training sequence for blind J-divergence rule makes it an even better choice. The evaluation of Hampel based AV receiver exhibited better BER in impulsive noise scenario as compared to the one without Hampel preprocessor. Additionally, we demonstrated the data starvation caused by short data record in SMI and Hampel-SMI receivers and how it was corrected by large data record.

### References


