Defeating jamming with the power of silence: a game-theoretic analysis

Salvatore D'Oro, *Student Member, IEEE,* Laura Galluccio, *Member, IEEE,* Giacomo Morabito, Sergio Palazzo, *Senior Member, IEEE,* Lin Chen, *Senior Member, IEEE,* and Fabio Martignon, *Senior Member, IEEE,*

***Abstract*—The timing channel is a logical communication channel in which information is encoded in the timing between events. Recently, the use of the timing channel has been proposed as a countermeasure to reactive jamming attacks performed by an energy-constrained malicious node. In fact, whilst a jammer is able to disrupt the information contained in the attacked packets, timing information cannot be jammed and, therefore, timing channels can be exploited to deliver information to the receiver even on a jammed channel.**

Since the nodes under attack and the jammer have conflicting interests, their interactions can be modeled by means of game theory. Accordingly, in this paper a game-theoretic model of the interactions between nodes exploiting the timing channel to achieve resilience to jamming attacks and a jammer is derived and analyzed. More specifically, the Nash equilibrium is studied in the terms of existence, uniqueness, and convergence under best response dynamics. Furthermore, the case in which the communication nodes set their strategy and the jammer reacts accordingly is modeled and analyzed as a Stackelberg game, by considering both perfect and imperfect knowledge of the jammer's utility function. Extensive numerical results are presented, showing the impact of network parameters on the system performance.

***Index Terms*—Anti-jamming, Timing Channel, Game-Theoretic Models, Nash Equilibrium.**

I. INTRODUCTION

A *timing channel* is a communication channel which exploits silence intervals between consecutive transmissions to encode information [1]. Recently, use of timing channels has been proposed in the wireless domain to support low rate, energy efficient communications [2, 3] as well as covert and resilient communications [4, 5].

In this paper we focus on the resilience of timing channels to jamming attacks [6, 7]. In general, these attacks can completely disrupt communications when the jammer continuously emits a high power disturbing signal, i.e., when *continuous jamming* is performed. However, continuous jamming is very costly in terms of energy consumption for the jammer [8–10]. This is the reason why in most scenarios characterized by energy constraints for the jammer, e.g., when the jammer is battery powered, non continuous jamming such as *reactive jamming* is considered. In this case the jammer continuously listens over the wireless channel and begins the transmission of a high power disturbing signal as soon as it detects an ongoing transmission activity. Effectiveness of reactive jamming has been demonstrated and its energy cost analyzed in [6, 10–12].

Timing channels are more - although not totally [4] - immune from reactive jamming attacks. In fact, the interfering signal begins its disturbing action against the communication only after identifying an ongoing transmission, and thus after the timing information has been decoded by the receiver. In [4], for example, a timing channel-based communication scheme has been proposed to counteract jamming by establishing a low-rate physical layer on top of the traditional physical/link layers using detection and timing of failed packet receptions at the receiver. In [5], instead, the energy cost of jamming the timing channel and the resulting trade-offs have been analyzed.

In this paper we analyze the interactions between the jammer and the node whose transmissions are under attack, which we call *target node*. Specifically, we assume that the target node wants to maximize the amount of information that can be transmitted per unit of time by means of the timing channel[1], whereas, the jammer wants to minimize such amount of information while reducing the energy expenditure[2]. As the target node and the jammer have conflicting interests, we develop a game theoretical framework that models their interactions. We investigate both the case in which these two adversaries play their strategies simultaneously, and the situation when the target node (the leader) anticipates the actions of the jammer (the follower). To this purpose, we study both the Nash Equilibria (NEs) and Stackelberg Equilibria (SEs) of our proposed games.

The main contributions of this paper can be therefore summarized as follows: 1) we model the interactions between a jammer and a target node as a jamming game; 2) we prove the existence, uniqueness and convergence to the Nash equilibrium (NE) under best response dynamics; 3) we prove the existence and uniqueness of the equilibrium of the

S. D'Oro, L. Galluccio, G. Morabito and S. Palazzo are with the CNIT Research Unit at University of Catania, Italy (e-mail: {name.surname}@dieei.unict.it). L. Chen is with the Laboratoire de Recherche en Informatique (LRI), Université Paris-Sud, Paris, France. F. Martignon is with the Laboratoire de Recherche en Informatique (LRI), Université Paris-Sud, Paris, France, and is also with the Institut Universitaire de France (IUF).

This work has been supported by the European Commission in the framework of the FP7 Network of Excellence in Wireless COMmunications NEWCOM# (Grant agreement no. 318306) and partially supported by the French ANR in the framework of the Green-Dyspan project.

[1]Note that in this context energy is not a concern for the target node, since by exploiting the timing channel, a significant reduction in the energy consumption can be obtained as demonstrated in [2].

[2]Up to now, despite the wide literature in this context, a universal model describing how jammers and target nodes behave in real adversarial scenarios is missing. Therefore, in our study we tried to propose a high-level model that describes rational and realistic behavior of each player, by considering several elements that are related to hardware parameters and the energy/power concerns.

Stackelberg game where the target node plays as a leader and the jammer reacts consequently; 4) we investigate in this latter Stackelberg scenario the impact on the achievable performance of *imperfect knowledge* of the jammer's utility function; 5) we conduct an extensive numerical analysis which shows that our proposed models well capture the main factors behind the utilisation of timing channels, thus representing a promising framework for the design and understanding of such systems.

Accordingly, the rest of this paper is organized as follows. Related work is presented in Section II. In Section III the proposed jamming game model is presented. A theoretical study of the existence and uniqueness of the NE as well as of the convergence of the game to that equilibrium under best response dynamics is derived in Section IV. Existence and uniqueness of the SE are discussed in Section V, together with some considerations relevant to imperfect knowledge scenarios. Then, numerical results are illustrated in Section VI. Finally, in Section VII conclusions are drawn.

II. Related Work

Wireless networks are especially prone to several attacks due to the shared and broadcast nature of the wireless medium. One of the most critical attacks is *jamming* [6, 7]. Jamming attacks can partially or totally disrupt ongoing communications, and proper solutions have been proposed in various application scenarios [6, 9, 10]. *Continuous* jamming attacks can be really expensive for the jammer in terms of energy consumption as the transmission of jamming signals needs a significant, and constant, amount of power. To reduce energy consumption while achieving a high jamming effectiveness, *reactive* jamming is frequently used [5, 11–13]. In [12] and [13] the feasibility and detectability of jamming attacks in wireless networks are analyzed. In these papers above, methodologies to detect jamming attacks are illustrated; it is also shown that it is possible to identify which kind of jamming attack is ongoing by looking at the signal strength and other relevant network parameters, such as bit and packet errors. In [11] Wilhelm et al. investigate the feasibility of reactive jamming attacks by providing a real implementation of a reactive jammer in a software-defined radio environment where a reactive jammer prototype is implemented on a USRP2 platform and network users are implemented on MICAz motes. Authors show that reactive jamming attacks are feasible and efficient, and that low reaction times can be achieved; then, they highlight the need to investigate proper countermeasures against reactive jamming attacks.

Several solutions against reactive jamming have been proposed that exploit different techniques, such as frequency hopping [14, 15], power control [16] and unjammed bits [17] (see [6, 7] for surveys). However, such solutions usually rely on users' cooperation and coordination, which might not be guaranteed in a jammed environment. In fact, the reactive jammer can totally disrupt each transmitted packet and, consequently, no information can be decoded and then used to this purpose.

Timing channels have been frequently exploited to support covert low rate [1], energy efficient [2, 3] and undetectable communications [18]. Also, they have been proposed as anti-jamming solutions [4, 5]. More specifically, in [4] Xu et al. propose an anti-jamming timing channel that exploits inter-arrival times between jammed packets to encode information to be transmitted, showing how timing channels are suitable to guarantee low rate communications even though a reactive jammer is disrupting transmitted packets. Actually, in [4] two constraining assumptions are made, that is, i) to perform an attack, the jammer first has to recognize the preamble of a packet, and ii) the jamming signal is transmitted as long as the jammer senses activity on the channel.

In [5] an analysis of energy consumption and effectiveness of a reactive jammer attack against timing channels is presented. Moreover, it is shown how a trade-off between energy consumption and jamming effectiveness can be sought. It is also demonstrated that continuous jamming can be very costly in terms of energy consumption.

Since the jammer and the target node(s) have opposite interests and the actions of the ones depend on those of the others, game theory is a valid tool to study such scenarios [15, 16, 19, 20]. An anti-jamming stochastic game in cognitive radio networks is proposed in [15], where authors provide learning mechanisms for users to counteract jamming attacks; also, it is shown that users can exploit frequency hopping to avoid jamming attacks by taking hopping decisions depending on the channel state. Often the jammer has to adapt its attack depending on network operations; hence, in literature it is frequently assumed that the jammer plays as a *follower* after the *leader*, i.e. the target node, has manifested its strategy. Such a scenario can be modeled as a Stackelberg game. For example, in [16] a Stackelberg game is proposed to model the interactions between target nodes and a smart jammer that is able to vary its transmission power to maximize its own utility function. In [19] Altman et al. analyze a game where both the target node and the jammer have energy constraints. Finally, as specifically relevant to our work, we mention the study carried out by Sengupta et al. [20] on a power control game modeling a network of nodes exploiting the timing channel, which maximize SINR and throughput by properly setting the transmission power level and the silence duration. In [20] however, although game theory is applied to timing channel networks, jamming issues are not considered.

As compared to the solutions proposed so far in the literature, our paper is the first together with [21] by Anand et al. to develop a game-theoretical model of the interactions between the jammer and a target node exploiting the timing channel. The main differences between our work and [21] can be summarized as follows:

- in [21] the target node focuses on deploying camouflaging resources (e.g., the number of auxiliary communications assisting the covert communication) to hide the underlying timing channel. In our work, instead, the target node establishes a timing channel that exploits the silence period between the end of an attack and the beginning of a subsequent packet transmission to counteract an ongoing jamming attack;
- in [21], only the Nash Equilibrium (NE) is studied, whereas in our work we study both the NE and SE

(Stackelberg Equilibrum). Furthermore, we compare the achievable performance of each player, and find that the SE dominates the NE (i.e., both players improve their achieved utilities), thus allowing each player to improve its own utility;

- in our work, the target node is able to transmit covert information even if the jammer has successfully disrupted all the bits contained in a packet. On the other hand, the authors in [21] assume that the jamming attack is successful if the Signal-to-Interference ratio (SIR) of the attacked node measured at the receiver side is higher than the one of the target node. In our approach, instead, we do not make any assumption on the SIR as, by exploiting our proposed timing channel implementation, it is possible to transmit some information even when the jammer has successfully corrupted each packet.

In addition, we only assume that the jammer is aware of timing channel communications ongoing between the target node and the perspective receiver, whereas we relax the two assumptions in [4]. Specifically, we assume that i) to start an attack the jammer has only to detect a possible ongoing transmission activity (e.g., the power on the monitored channel exceeds a given threshold), and ii) the transmission of the jamming signal does not necessarily stops when the packet transmission by the target node ends, that is, the jammer is able to introduce some transmission delay in timing channel communications by extending its jamming signal duration.

III. Game model

Let us consider the scenario where two wireless nodes, a transmitter and a receiver, want to communicate, while a malicious node aims at disrupting their communication. To this purpose, we assume that the malicious node executes a reactive jamming attack on the wireless channel. In the following we refer to the malicious node as the *jammer*, J, and the transmitting node under attack as the *target node*, T.

The jammer senses the wireless channel continuously. Upon detecting a possible transmission activity performed by T, J starts emitting a jamming signal. As shown in Fig. 1, we denote as T_{AJ} the duration of the time interval between the beginning of the packet transmission and the beginning of the jamming signal emission. The duration of the interference signal emission that jams the transmission of the j-th packet can be modeled as a continuous random variable, which we call Y_j. To maximize the uncertainty on the value of Y_j, we assume that it is exponentially distributed with mean value y.

We assume that when no attack is performed the target node communicates with the receiver by applying traditional transmissions schemes; on the other hand, when it realizes to be under attack, it exploits the timing channel to transmit part of (or all) the information[3]. The latter is encoded in the duration of the interval between the instant when the jammer J terminates the emission of the jamming signal and the

[3]Attack detection can be achieved by the target node either by means of explicit notification messages sent back to T by the receiver or by inference after missing reception of ACK messages. Details on attack detection operations are however out of the scope of this paper.

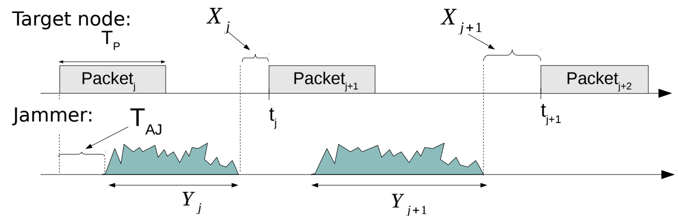


Fig. 1: Interactions between the jammer and the target node.

beginning of the transmission of the next packet. Hence, it is possible to consider a discrete time axis and refer to each timing channel utilization by means of an integer index j. The silence period duration scheduled after the transmission of the j-th packet and the corresponding jamming signal can be modeled as a continuous random variable, X_j, uniformly distributed[4] in the range $[0, x]$. The amount of information transmitted per each use of the timing channel depends on the value of x and the precision Δ of the clocks of the communicating nodes as shown in [2]. In our model we assume that the parameters Δ and T_{AJ} which are hardware dependent are known a-priori to both the target node and the jammer, whereas the strategies x and y are estimated by means of a training phase. This is consistent with the complete information assumption which is common in game theoretic frameworks.

To model the interactions between the target node and the jammer we propose a jamming game framework, defined by a 3-tuple $\mathcal{G} = (\mathcal{N}, \mathcal{S}, \mathcal{U})$, where $\mathcal{N}$ is the set of players, $\mathcal{S}$ is the strategy set, and $\mathcal{U}$ is the utility set. The set $\mathcal{N}$ is composed by the target node T and the jammer J, while the strategy set is $\mathcal{S} = \mathcal{S}_{\mathcal{T}} \times \mathcal{S}_{\mathcal{J}}$, where $\mathcal{S}_{\mathcal{T}}$ and $\mathcal{S}_{\mathcal{J}}$ are the set of strategies of the target node and the jammer, respectively.

In our model we assume that the jammer is energy-constrained, e.g., it is battery-powered; hence, its choice of y (i.e., the average duration of the signal emission that jams the packet transmission) stems from a trade-off between two requirements, i.e., i) reduce the amount of information that the target node T can transmit to the perspective receiver, and ii) keep the energy consumption as low as possible. Observe that requirement i) would result in the selection of a high value for y, whereas requirement ii) would result in a low value for y. On the other hand, the target node has to properly choose the value of x (i.e., the maximum silence period duration scheduled following the transmission of the $j-th$ packet and the subsequent jamming signal) in order to maximize the achievable capacity $\mathcal{C}(x, y)$, i.e., the amount of information that can be sent by means of the timing channel, while minimizing its energy consumption. Therefore, it is reasonable to consider that the values of x and y represent the *strategies* for the target node T and the jammer J, respectively. Accordingly, the set of strategies for both players, $\mathcal{S}_{\mathcal{T}}$ and $\mathcal{S}_{\mathcal{J}}$, can be defined as the set of all the feasible strategies x and y, respectively.

[4]The uniform distribution assumption is due to the fact that, as well known, this distribution maximizes the entropy, given the range in which the random variable is defined.

The utility set of the game is defined as $\mathcal{U} = (\mathcal{U}_{\mathcal{T}}, \mathcal{U}_{\mathcal{J}})$, where $\mathcal{U}_{\mathcal{T}}$ and $\mathcal{U}_{\mathcal{J}}$ are the utility functions of the target node and the jammer, respectively. As already said, the target node aims at maximizing its own achievable capacity, $\mathcal{C}(x, y)$ while also minimizing its energy consumption. The jammer, on its side, aims at reducing the capacity achieved by the target node by generating interference signals, whose duration is y (in average), while keeping its own energy consumption low. Accordingly, the utility functions $\mathcal{U}_T(x, y)$ and $\mathcal{U}_J(x, y)$ to be maximized are defined as follows:

$$\begin{cases} \mathcal{U}_T(x, y) &= +C(x, y) - c_{T^*} \cdot T_P \cdot P_T \\ \mathcal{U}_J(x, y) &= -C(x, y) - c_T \cdot y \cdot P \end{cases} \quad (1)$$

where P_T and P_J are the transmission power of the target node and the jammer, respectively, T_P is the duration of a transmitted packet in seconds, c_{T^*} and c_T are positive transmission costs expressed in [bit/(s · J)] which weight the two contributions in the utility functions and therefore, in the following will be referred to as *weight parameters*. Note that while the energy consumption of the jammer varies as a function of the strategy y of the jammer itself, on the contrary the energy consumption of the target node during a cycle only depends on the duration T_P of the packet and not on the strategy. Furthermore, a low value of c_T means that the jammer considers its jamming effectiveness more important than its energy consumption, while a high c_T value indicates that the jammer is energy-constrained and, as a consequence, it prefers to save energy rather than reducing the capacity of the target node. We observe that $c_T = 0$ models the case of continuous jamming without any energy constraint, which is of limited interested and out of the scope of this paper, since we focus on studying the trade-off between the achievable capacity and the consumed energy.

Let us now calculate the capacity $C(x, y)$ which appears in the utility function (1). To this purpose, we denote the interval between two consecutive transmissions executed by T as a *cycle*. The expected duration of a cycle is

$$t_{\text{Cycle}} = T_{AJ} + y + x/2 \quad (2)$$

The capacity $C(x, y)$ can be derived as the expected value of the information transferred during a cycle, $c_{\text{Cycle}}(x, y)$, divided by the expected duration of a cycle, t_{Cycle}. It is easy to show that $c_{\text{Cycle}}(x, y)$ is approximately

$$c_{\text{Cycle}} = \log_2 (x/\Delta) \quad (3)$$

Note that at each timing channel utilization the target node T is expected to transmit at least one bit; then, from eq. (3) it follows that $x \geq 2\Delta$.

Eqs. (2) and (3) can be exploited to calculate the capacity $C(x, y)$, i.e.,

$$C(x, y) = \frac{\log_2 (x/\Delta)}{T_{AJ} + y + x/2} \quad (4)$$

Hereafter we illustrate a simple numerical example that refers to the same realistic scenarios addressed in [11]. The considered parameter settings are reported in Table I. It is also assumed that both the target node and the jammer transmit their respective signals by using the maximum allowed transmitting power, i.e., $P_T = P_J = P$.

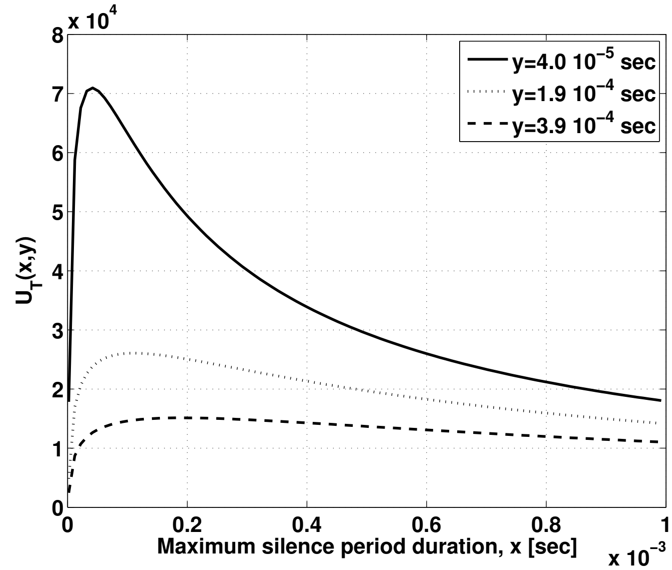


Fig. 2: Utility function of the target node ($\mathcal{U}_T(x, y)$) as a function of x for different values of the average jamming signal duration y ($c_{T^*} \cdot P = 2 \cdot 10^6$).

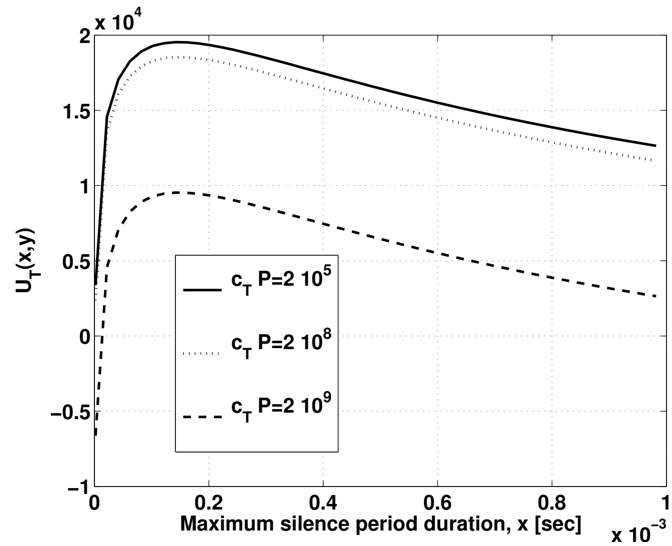


Fig. 3: Utility function of the target node ($\mathcal{U}_T(x, y)$) as a function of x for different values of the product $c_{T^*} \cdot P$ ($y = 2.8 \cdot 10^{-4} s$).

Fig. 2 shows the utility function of the target node T as a function of x, for different values of y.

We note that $\mathcal{U}_T(x, y)$ increases when x increases until it reaches a threshold after which the utility function starts decreasing. This is due to the fact that, when x is higher than such a threshold, the silence duration is large enough to cause an increase in the transmission delay and, consequently, a decrease in the transmission capacity. This is a well known result in timing channel communications [22]. In Fig. 2 we also note that the achievable performance noticeably depends on the jamming signal duration y. In fact, when y increases, the capacity of the target node decreases as the jamming attack forces the transmitter in delaying its timing channel communications by increasing x. Figure 3 shows the impact

Name	Value	Unit
T_{AJ}	15	μs
Δ	1	μs
P	2	W
T_P	50	μs

TABLE I: Parameter settings used in our simulations.

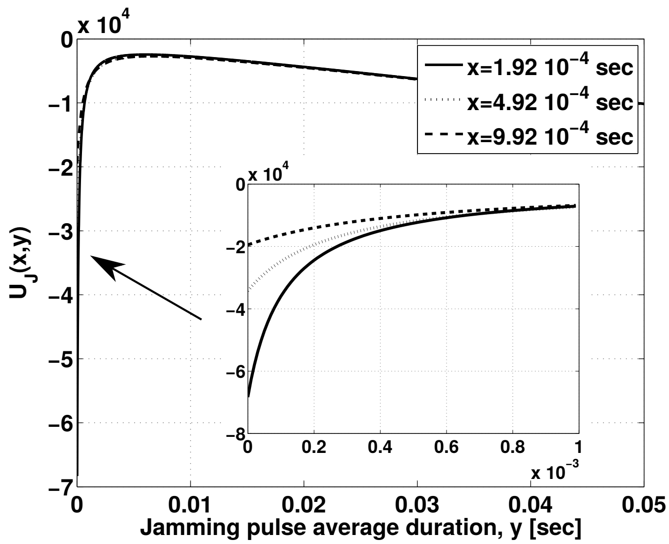


Fig. 4: Utility function of the jammer ($\mathcal{U}_J(x, y)$) as a function of y for different values of the maximum silence period duration x ($c_T \cdot P = 2 \cdot 10^6$).

of the energy consumption on the utility achieved by the target node. As expected, the higher the product $c_T \cdot P$ is, the lower the achieved utility is. Note that, as the energy consumption in any cycle is constant and does not depend on either x or y, the energy cost of the target node $\mathcal{U}_T(x, y)$ would only result in a slight shift in the utility function of the target node.

Fig. 4 shows instead the utility function of the jammer $\mathcal{U}_J(x, y)$ vs. y for different values of x.

Note that for high values of y the utility function $\mathcal{U}_J(x, y)$ does not practically depend on x. This is because high y values imply $C(x, y) \approx 0$ regardless of the specific value of x. Such a behavior is evident in Fig. 4. We observe that for high values of y the capacity achieved by the target node $C(x, y)$ is negligible and, thus, the utility function of the jammer can be approximated as $\mathcal{U}_J(x, y) \simeq -c_T \cdot y \cdot P$. In other words, the utility of the jammer decreases linearly with y. To this purpose, in Fig. 5 we show the utility of the jammer $\mathcal{U}_J(x, y)$ for different values of the product $c_T \cdot P$. It is evident that, as expected, when the cost of transmitting the interference signal at the jammer is high (i.e., $c_T \cdot P$ is high) the utility function $\mathcal{U}_J(x, y)$ decreases rapidly and linearly.

IV. Nash Equilibrium analysis

In this Section we solve the game described in Section III, and we find the *Nash Equilibrium points* (NEs), in which both players achieve their highest utility given the strategy profile of the opponent. In the following we also provide

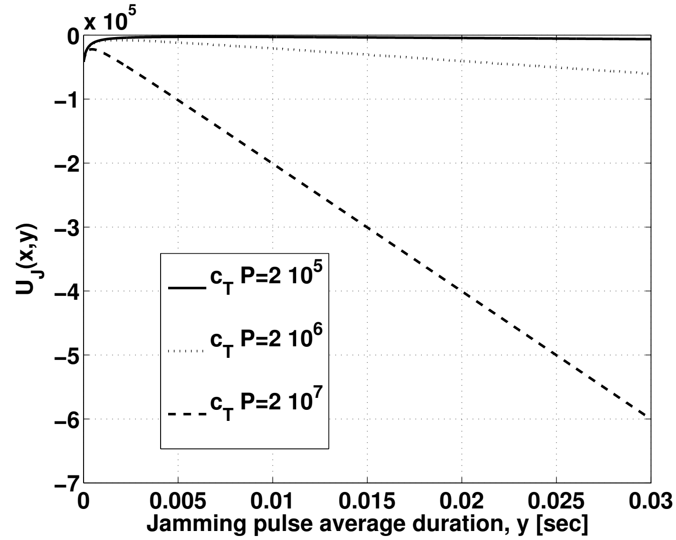


Fig. 5: Utility function of the jammer ($\mathcal{U}_J(x, y)$) as a function of y for different values of the product $c_T \cdot P$ ($x = 5 \cdot 10^{-4} s$).

proofs of the *existence, uniqueness* and *convergence* to the Nash Equilibrium under best response dynamics.

Let us recall the definition of Nash equilibrium:

Definition 1. *A strategy profile* $(x^*, y^*) \in \mathcal{S}$ *is a Nash Equilibrium (NE) if* $\forall (x', y') \in \mathcal{S}$

$$\mathcal{U}_T(x^*, y^*) \geqslant \mathcal{U}_T(x', y^*)$$
$$\mathcal{U}_J(x^*, y^*) \geqslant \mathcal{U}_J(x^*, y')$$

that is, (x^*, y^*) *is a strategy profile where no player has incentive to deviate unilaterally.*

One possible way to study the NE and its properties is to look at the *best response functions* (BRs). A best response function is a function that maximizes the utility function of a player, given the opponents' strategy profile. Let $b_T(y)$ be the BR of the target node and $b_J(x)$ the BR of the jammer. These functions can be characterized as follows:

$$b_T(y) = \arg\max_{x \in \mathcal{S}_\mathcal{T}} \mathcal{U}_\mathcal{T}(x, y)$$

$$b_J(x) = \arg\max_{y \in \mathcal{S}_\mathcal{J}} \mathcal{U}_\mathcal{J}(x, y)$$

In our model it is possible to analytically derive the closed form of the above BRs by analyzing the first derivatives of $\mathcal{U}_\mathcal{T}(x, y)$ and $\mathcal{U}_\mathcal{J}(x, y)$, and imposing that $\frac{\partial}{\partial x}\mathcal{U}_\mathcal{T}(x, y) = 0$ and $\frac{\partial}{\partial y}\mathcal{U}_\mathcal{J}(x, y) = 0$.

It is easy to see that $\frac{\partial}{\partial x}\mathcal{U}_\mathcal{T}(x, y) = 0$ leads to

$$\frac{1}{x} - \frac{1}{2}\log\left(\frac{x}{\Delta}\right)\frac{1}{T_{AJ} + y + \frac{x}{2}} = 0 \tag{5}$$

Eq. (5) can be rewritten as follows:

$$\frac{2(T_{AJ} + y)}{e\Delta} = \frac{x}{e\Delta} \cdot \log\frac{x}{e\Delta} \tag{6}$$

Note that eq. (6) is in the form $\beta = \alpha \log \alpha$, and, by exploiting the definition of *Lambert W-function*, say $W(z)$, which, for any complex z, satisfies $z = W(z)e^{W(z)}$, it has solution $\alpha =$

$$(x_{\mathrm{NE}}, y_{\mathrm{NE}}) = \begin{cases} \left(\Delta e^{\frac{1}{2}W(\frac{8}{\eta\Delta^2})}, \frac{\Delta}{2}[\frac{1}{2}W(\frac{8}{\eta\Delta^2}) - 1]e^{\frac{1}{2}W(\frac{8}{\eta\Delta^2})} - T_{AJ}\right) & \text{if } c_T < \tilde{c_T} \\ \left(\Delta e^{W(\frac{2T}{e\Delta})+1}, 0\right) & \text{otherwise} \end{cases} \tag{7}$$

$e^{W(\beta)}$.

Therefore, eq. (6) can also be rewritten as

$$x = \Delta e^{W\left(\frac{2(T_{AJ}+y)}{e\Delta}\right)+1}$$

which is, by definition, $b_T(y)$.

In order to derive the closed form of $b_J(x)$ we first solve $\frac{\partial}{\partial y}\mathcal{U}_{\mathcal{J}}(x,y) = 0$. It can be easily proven that $\frac{\partial}{\partial y}\mathcal{U}_{\mathcal{J}}(x,y) = 0$ leads to

$$\log\left(\frac{x}{\Delta}\right) = \eta\left(T_{AJ} + \frac{x}{2} + y\right)^2$$

which can be rewritten as follows:

$$b_J(x) = \sqrt{\frac{\log(\frac{x}{\Delta})}{\eta}} - T_{AJ} - \frac{x}{2}$$

where $\eta = c_T \cdot P \cdot \log 2$.

Therefore, we can write

$$b_T(y) = \Delta e^{\psi(y)+1} \tag{8}$$

$$b_J(x) = \begin{cases} \chi(x), & \text{if } \chi(x) \geqslant 0 \\ 0, & \text{if } \chi(x) < 0 \end{cases} \tag{9}$$

where

$$\psi(y) = W\left(\frac{2[T_{AJ}+y]}{e\Delta}\right) \quad , \quad \chi(x) = \sqrt{\frac{\log(\frac{x}{\Delta})}{\eta}} - T_{AJ} - \frac{x}{2} \tag{10}$$

Note that the best response of the jammer $b_J(x)$ depends on the value of the weight parameter c_T. Also, it can be shown that there exists a critical value of the weight parameter, say $c_T^{(\max)}$, such that $b_J(x) < 0\ \forall x \in \mathcal{S}_{\mathcal{T}}$, $\forall c_T \geq c_T^{(\max)}$. In fact, since the function $\chi(x)$ is strictly decreasing in c_T, $\lim_{c_T\to+\infty}\chi(x) < 0$ and $\lim_{c_T\to 0}\chi(x) = +\infty$, the *intermediate value theorem* ensures the existence of $c_T^{(\max)}$. By looking at the first derivative of the $\chi(x)$ function in eq. (10), it can be shown that $c_T^{(\max)} = \frac{1}{P\log(2)}\frac{1}{2\Delta(\Delta+T)}$. Therefore, if $c_T \geq c_T^{(\max)}$ the only possible strategy of the jammer is $b_J(x) = 0$, and then, as the strategy set of the jammer ($\mathcal{S}_{\mathcal{J}}$) is a singleton, the game has a trivial outcome.

A. Existence of the Nash Equilibrium

It is well known that the intersection points between $b_T(y)$ and $b_J(x)$ are the NEs of the game. Therefore, to demonstrate the existence of at least one NE, it suffices to prove that $b_T(y)$ and $b_J(x)$ have one or more intersection points. In other words, it is sufficient to find one or more pairs $(x^*, y^*) \in \mathcal{S}$ such that

$$(b_T(y^*), b_J(x^*)) = (x^*, y^*) \tag{11}$$

To this aim, in the following we provide some structural properties of the utility functions, $\mathcal{U}_{\mathcal{T}}(x,y)$ and $\mathcal{U}_{\mathcal{J}}(x,y)$, that will be useful in solving eq. (11).

Lemma 1. *For the utility functions $\mathcal{U}_{\mathcal{T}}(x,y)$ and $\mathcal{U}_{\mathcal{J}}(x,y)$, the following properties hold* [5]*:*

- *$\mathcal{U}_{\mathcal{T}}(x,y)$ is strictly concave for $x \in [2\Delta, x']$ and is monotonically decreasing for $x > x'$ where $x' = b_T(y)$*
- *$\mathcal{U}_{\mathcal{J}}(x,y)$ is strictly concave $\forall y \in \mathcal{S}_{\mathcal{J}}$.*

Theorem 1 (NE existence)**.** *The game $\mathcal{G}$ admits at least an NE.*

Proof: If we limit the strategy of the target node to $[2\Delta, x']$, it follows from Lemma 1 that there exists at least an NE since both the utility functions are concave in the restraint strategy set [23]. However, this does not still prove the existence of the NE in the non-restraint strategy set $\mathcal{S}_{\mathcal{T}}$. Let (x^*, y^*) denote the NE with a restraint strategy set $[2\Delta, x']$; we can easily observe that (x^*, y^*) is also the NE of the jamming game with non-restraint strategy set. To show this, recall Lemma 1 that states that $\mathcal{U}_{\mathcal{T}}(x,y)$ is monotonically decreasing for $x > x'$. The transmitter has thus no incentive to deviate from (x^*, y^*) and the jammer has no incentive to deviate from it either. Therefore, (x^*, y^*) is the NE of the jamming game. ∎

B. Uniqueness of the Nash Equilibrium

After proving the NE existence in Theorem 1, let us prove the uniqueness of the NE, that is, there is only one strategy profile such that no player has incentive to deviate unilaterally.

Theorem 2 (NE uniqueness)**.** *The game $\mathcal{G}$ admits a unique NE that can be expressed as given in eq. (7), where $\eta = c_T \cdot P \cdot \log 2$ and*

$$\tilde{c_T} = \frac{4}{\Delta^2 P \log 2} e^{-2[W(\frac{2T}{e\Delta})+1]} / (W(\frac{2T}{e\Delta}) + 1) \tag{12}$$

The proof consists in exploiting formal and structural properties of the best response functions to show that their intersection is unique, that is, eq. (11) admits a unique solution. For a detailed proof see Appendix A

C. Convergence to the Nash Equilibrium

We now analyze the convergence of the game to the NE when players follow *Best Response Dynamics* (BRD). In BRD the game starts from any initial point $(x^{(0)}, y^{(0)}) \in \mathcal{S}$ and, at each successive step, each player plays its strategy by following its best response function. Thereby, at the i-th iteration the strategy profile $(x^{(i)}, y^{(i)})$ can be formally expressed by the following BRD iterative algorithm:

$$\begin{cases} x^{(i)} = b_T(y^{(i-1)}) \\ y^{(i)} = b_J(x^{(i-1)}) \end{cases}$$

[5]The proof of Lemma 1 which is straightforward (although quite long), consists in calculating the first and second derivatives of the utility functions and studying them.

Let $\mathbf{b}(x,y) = (b_T(y), b_J(x))^T$ be the best response vector and $J_\mathbf{b}$ be the Jacobian of $\mathbf{b}(x,y)$ defined as follows

$$J_\mathbf{b} = \begin{bmatrix} \frac{\partial}{\partial x} b_T(y) & \frac{\partial}{\partial y} b_T(y) \\ \frac{\partial}{\partial x} b_J(x) & \frac{\partial}{\partial y} b_J(x) \end{bmatrix} = \begin{bmatrix} 0 & \frac{\partial}{\partial y} b_T(y) \\ \frac{\partial}{\partial x} b_J(x) & 0 \end{bmatrix} \quad (13)$$

It has been demonstrated [24] that, if the Jacobian infinity matrix norm $||J_\mathbf{b}||_\infty < 1$, the BRD always converges to the unique NE. In the following we prove the following theorem:

Theorem 3 (NE convergence - sufficient condition)**.** *The relationship*

$$c_T > \frac{1}{9\Delta^2 \log 2P} \frac{1}{\left(W(\frac{2T_{AJ}}{e\Delta}) + 1\right) e^{2(W(\frac{2T_{AJ}}{e\Delta})+1)}} \quad (14)$$

is a sufficient condition for the game $\mathcal{G}$ to converge to the NE. Furthermore, it converges to the NE in at most $\log_{J_b^{max}} \frac{\epsilon}{||s^1 - s^0||}$ iterations for any ϵ, where $J_b^{max} = \max J_\mathbf{b}$ and $s^i = (x^i, y^i)$.

To demonstrate the theorem,

1) we prove that the relationship
$$\max_{x \in \mathcal{S}_\mathcal{T}} \left(\frac{1}{\eta x^2 \log(\frac{x}{\Delta})} \right) < 9 \quad (15)$$
is a sufficient condition for the BRD to converge to the NE in at most $\log_{J_b^{max}} \frac{\epsilon}{||s^1 - s^0||}$ iterations. This is the focus of Lemma 2;
2) we define a game $\mathcal{G}'$ and demonstrate that $\mathcal{G}$ converges to $\mathcal{G}'$ in two iterations at most. This is the focus of Lemma 3;
3) we demonstrate that the condition in eq. (14) is a sufficient condition for $\mathcal{G}'$ to satisfy eq. (15) and converge to the same NE of $\mathcal{G}$. This is the focus of Lemma 4.

Lemma 2. *The BRD converges to the unique NE from any $(x^{(0)}, y^{(0)}) \in \mathcal{S}$ if $\max_{x \in \mathcal{S}_\mathcal{T}} \left(\frac{1}{\eta x^2 \log(\frac{x}{\Delta})} \right) < 9$ in at most $\log_{J_b^{max}} \frac{\epsilon}{||s^1 - s^0||}$ iterations.*

The proof is based on showing that the above relationship is a sufficient condition for the Jacobian infinity matrix norm $||J_\mathbf{b}||_\infty$ to be always lower than 1, and thus, according to [24], convergence of the BRD follows. We refer the reader to Appendix B for a detailed proof of Lemma 2.

Let us now observe that $b_J(x)$ is lower-bounded as it is non-negative ($b_J(x) \geqslant 0$) and, since it is concave, it has a maximum, say y_M, for $\hat{x} = \Delta e^{\frac{1}{2} W(\frac{2}{\eta \Delta^2})}$, and thus it is upper-bounded ($b_J(x) \leqslant y_M = b_J(\hat{x})$). Also, it is easy to prove that $b_T(y)$ is a non-negative strictly increasing function, hence, it is lower-bounded by $x_m = b_T(0)$. We can thus define a new strategy set $\mathcal{S}' = \mathcal{S}_\mathcal{T}' \times \mathcal{S}_\mathcal{J}' = [x_m, x_M] \times [0, y_M]$, where $\mathcal{S}' \subset \mathcal{S}$ and $x_M = b_T(y_M)$, which is relevant in the following lemma:

Lemma 3. *Given any starting point $(x^{(0)}, y^{(0)}) \in \mathcal{S}$, the BRD is bounded in $\mathcal{S}'$ in at most two iterations. That is, $(x^{(i)}, y^{(i)}) \in \mathcal{S}'$ for $i = 2, 3, ..., +\infty$.*

Proof: Let $\mathcal{S}^{(1)}$ be the strategy set at the first iteration. From eqs. (8) and (9) we have that $b_J(x)$ is lower and upper-bounded by $y = 0$ and $y = y_M$, respectively, thus $y^{(1)} \in [0, y_M]$. Furthermore, as $b_T(x)$ is lower-bounded by $x = x_m$ and $y^{(0)} \in \mathcal{S}_\mathcal{J} = [0, +\infty[$, it follows that $x^{(1)} \in [x_m, +\infty)$. Hence, we have that $\mathcal{S}^{(1)} = \mathcal{S}_\mathcal{T}^{(1)} \times \mathcal{S}_\mathcal{J}^{(1)} = [x_m, +\infty) \times [0, y_M]$, $\mathcal{S}^{(1)} \subset \mathcal{S}$. Due to the boundedness of $y^{(1)}$ which assumes values in $\mathcal{S}_\mathcal{J}^{(1)}$, it can be shown that at the second iteration $x^{(2)} \in [x_m, x_M]$ while $y^{(2)} \in [0, y_M]$, thus, we have that $(x^{(2)}, y^{(2)}) \in \mathcal{S}'$. We can extend the same reasoning to the j-th iteration ($\forall j = 3, 4, ..., \infty$) to obtain that $(x^{(j-1)}, y^{(j-1)}) \in \mathcal{S}'$. Therefore, it follows that $(x^{(j)}, y^{(j)})$ is still in $\mathcal{S}'$, which concludes the proof. ∎

Lemma 4 (NE convergence)**.** *If the parameter c_T satisfies the condition:*

$$c_T > c_T' = \frac{1}{9\Delta^2 \log 2P} \frac{1}{\left(W(\frac{2T_{AJ}}{e\Delta}) + 1\right) e^{2(W(\frac{2T_{AJ}}{e\Delta})+1)}} \quad (16)$$

then $\mathcal{G}'$ converges to the NE of $\mathcal{G}$.

Proof: Since the function on the left-hand side of eq. (15) is non-negative and strictly decreasing, and the minimum value of $\mathcal{S}_\mathcal{T}$ is $x_m = \Delta e^{W\left(\frac{2T_{AJ}}{e\Delta}\right)+1}$, then

$$\max_{x \in \mathcal{S}_\mathcal{T}} \left(\frac{1}{\eta x^2 \log(\frac{x}{\Delta})} \right) = \frac{1}{\eta x_m^2 \log(\frac{x_m}{\Delta})} \quad (17)$$

It is easy to show that if eq. (16) holds, then

$$\frac{1}{\eta x_m^2 \log(\frac{x_m}{\Delta})} < 9$$

and therefore, recalling eq. (17), eq. (15) holds. From Lemma 2 we thus obtain that $\mathcal{G}'$ converges to its NE.

We still need to demonstrate that $\mathcal{G}$ and $\mathcal{G}'$ converge to the same equilibrium point. To this purpose it is sufficient to prove that the equilibrium point of $\mathcal{G}$ is in $\mathcal{S}'$. Theorem 2 guarantees that the game $\mathcal{G}$ admits a unique equilibrium, which has to be in $\mathcal{S}$. Let $(x_{\mathrm{NE}}, y_{\mathrm{NE}})$ be the NE, i.e., the unique intersection point between $b_T(y)$ and $b_J(x)$. As $b_J(x)$ takes values in $[0, y_M]$ it follows that $y_{\mathrm{NE}} \in [0, y_M]$; therefore, $x_{\mathrm{NE}} = b_T(y_{\mathrm{NE}}) \in [x_m, x_M]$. It follows that $(x_{\mathrm{NE}}, y_{\mathrm{NE}}) \in \mathcal{S}'$, which concludes the proof. ∎

V. Stackelberg Game

In a Stackelberg game one of the players acts as the leader by anticipating the best response of the follower. In our scenario, the jammer plays its strategy when a communication from the target node is detected on the monitored channel; thus, it is natural to assume that the target node acts as the leader followed by the jammer. Obviously, given the strategy of the target node x, the jammer will play the strategy that maximizes its utility, that is, its best response $b_J(x)$[6]. This hierarchical structure of the game allows the leader to achieve a utility which is at least equal to the utility achieved in the ordinary game $\mathcal{G}$ at the NE, if we assume *perfect knowledge*, that is, the target node is completely aware of the utility function of the jammer and its parameters, and thus it is able to evaluate $b_J(x)$. Whereas, if some parameters in the utility

[6]In the following, given that the value of c_{T^*} does not impact on the game, for worth of simplicity we assume that $c_{T^*} = 0$.

$$\mathcal{U}_T(x, b_J(x)) = \begin{cases} \sqrt{c_T P \log_2(\frac{x}{\Delta})} - c_{T^*} \cdot T_P \cdot P_T & \text{if } \chi(x) > 0 \quad (18a) \\ \log_2(\frac{x}{\Delta})/(T_{AJ} + \frac{x}{2}) - c_{T^*} \cdot T_P \cdot P_T & \text{if } \chi(x) \leqslant 0 \quad (18b) \end{cases}$$

function of the jammer are unknown at the target node, i.e., the *imperfect knowledge* case, the above result is no more guaranteed as it is impossible to evaluate the exact form of $b_J(x)$. In this section we analyze the Stackelberg game and provide useful results about its equilibrium points, referred to as Stackelberg Equilibria (SEs).

Definition 2. *A strategy profile $(x^*, y^*) \in \mathcal{S}$ is a Stackelberg Equilibrium (SE) if $y^* \in \mathcal{S_J}^{\mathrm{NE}}(x)$ and*

$$x^* = \arg\max_{x'} \mathcal{U}_T(x', y^*)$$

where $\mathcal{S_J}^{\mathrm{NE}}(x)$ is the set of NE for the follower when the leader plays its strategy x.

In the following we will prove that, in the case of perfect knowledge, there is a unique SE for any value of the weight parameter c_T, and we demonstrate that the target node can inhibit the jammer under the perfect knowledge assumption. Next, we will investigate the implications of imperfect knowledge on the game outcome.

A. Perfect Knowledge

Under the perfect knowledge assumption, the target node selects x in such a way that $\mathcal{U}_T(x, b_J(x))$ is maximized, where $\mathcal{U}_T(x, b_J(x))$ is calculated in eqs. (18a) and (18b) by replacing expression (9) in eqs. (4) and (1). By analyzing the first derivative of $\chi(x)$, it can be shown that $\chi(x)$ has a maximum in $\hat{x} = \Delta e^{\frac{1}{2}W(\frac{2}{\eta\Delta^2})}$ and, consequently, $\chi(x)$ is strictly decreasing for $x > \hat{x}$ and strictly increasing for $x < \hat{x}$.

In the following we show that for any value of c_T there exists a unique Stackelberg Equilibrium, and this is when the jammer does not jam the timing channel[7]. Furthermore, we show that the leader can improve its utility at the Stackelberg equilibrium if and only if $c_T < \tilde{c_T}$.

Theorem 4. *For any value of the parameter c_T, the Stackelberg game $\mathcal{G_T}$ has a unique equilibrium.*

Proof: First, we prove that the game admits a unique equilibrium for $c_T \geq c_T^{(\max)}$. Recall that $c_T \geq c_T^{(\max)}$ implies $b_J(x) = 0$; therefore, $\mathcal{S_J}$ is singleton and the unique feasible strategy for the jammer at the SE is $y_{\mathrm{SE}} = 0$. In fact, due to the high cost associated to the emission of the jamming signal, the jammer is inhibited $\forall x \in \mathcal{S_T}$. Hence, it can be easily proved that the strategy profile at the SE is $(x_{\mathrm{SE}}, y_{\mathrm{SE}}) = (\Delta e^{W(\frac{2T_{AJ}}{e\Delta})+1}, 0)$, that is, at the SE the target node selects the strategy that maximizes the capacity of the non-jammed timing channel (where indeed $y_{\mathrm{SE}} = 0$).

[7]In this case the jammer is expected to transmit the interference signal for a short time interval only because this suffices to disrupt communications, as occurs in traditional communication channels.

Instead, if $c_T < c_T^{(\max)}$, from eq. (10) we have that $\chi(\hat{x}) > 0$. Thus, for the intermediate value theorem there exist $x_1 < \hat{x}$ and $x_2 > \hat{x}$ such that $\chi(x_1) = \chi(x_2) = 0$, as shown in Fig. 6.

Let us denote $\mathcal{S_{T}}_1 = \{x \in [2\Delta, x_1]\}$, $\mathcal{S_T}_2 = \{x \in [x_1, x_2]\}$, $\mathcal{S_T}_3 = \{\mathcal{S_T} \setminus (\mathcal{S_T}_1 \cup \mathcal{S_T}_2)\}$, and $x' = \Delta e^{W(\frac{2T_{AJ}}{e\Delta})+1}$. It can be easily proved that x' maximizes eq. (18b) and, since $\chi(x') > 0$, it follows that $x' \in \mathcal{S_T}_2$. Therefore, the utility function of the target node as defined in eq. (18b) increases for $x < x'$ and decreases for $x > x'$. The latter is fundamental to prove the theorem; in fact, as shown in Fig. 6, for $x \in \mathcal{S_T}_1$ the utility of the target node is defined by eq. (18b) and strictly increases as x increases; therefore, we have that in $\mathcal{S_T}_1$ the maximum utility is achieved in x_1. On the contrary, in $\mathcal{S_T}_2$ the utility is defined by eq. (18a), which is a strictly increasing function that achieves its maximum value for $x = x_2$. Finally, for $x \in \mathcal{S_T}_3$ we have that the utility of the transmitter defined by eq. (18b) strictly decreases as $x > x'$; hence, the maximum value is achieved for $x = x_2$.

Since $\mathcal{U}_T(x, b_J(x)) < \mathcal{U}_T(x_2, b_J(x_2))$ with $x \neq x_2$, it follows that, to maximize its own utility, the target node must play the unique strategy $x = x_2$. Note that $\chi(x_2) = 0$ by definition, thus from eq. (9) we have that the strategy of the jammer at the equilibrium is $y_{\mathrm{SE}} = 0$. Therefore, $x_{\mathrm{SE}} = x_2$ is the strategy of the target node at the SE, and we can identify the unique SE as $(x_{\mathrm{SE}}, y_{\mathrm{SE}}) = (x_2, 0)$, which concludes the proof. ■

Let us remark that the above Theorem also highlights an insightful side-effect: at the Stackelberg equilibrium, pursuing the goal of inhibiting the jammer makes the target node prefer to increase transmission delay rather than reduce its achievable capacity.

Let us also note that, although an analytical closed form for x_{SE} cannot be easily derived, its value can be determined by means of numerical search algorithms such as the bisection search algorithm. Obviously, such algorithms will not give the exact value of x_{SE}; in fact, they will return an interval $[x_m, x_M]$ small as desired, containing the solution, i.e., $x_{\mathrm{SE}} \in [x_m, x_M]$, and eventually the target node will select the minimum or the maximum value of the interval which gives the highest utility function. Let $\epsilon(x_m, x_M)$ denote the loss in the utility of the target node due to the fact that it cannot determine the exact value of x_{SE}. Given that the utility function is continuous and that its derivative is upperbounded by $u_{\max} = \sqrt{c_T \cdot P}/(4\Delta \log 2)$ in $[x_1, x_{\mathrm{SE}}]$, it is possible to show that selecting the interval size in such a way that

$$x_M - x_m \leq \epsilon^*/u_{\max} \quad (19)$$

the loss in the utility of the target node, $\epsilon(x_m, x_M)$, is lower than ϵ^*. In other terms, by using numerical search algorithms

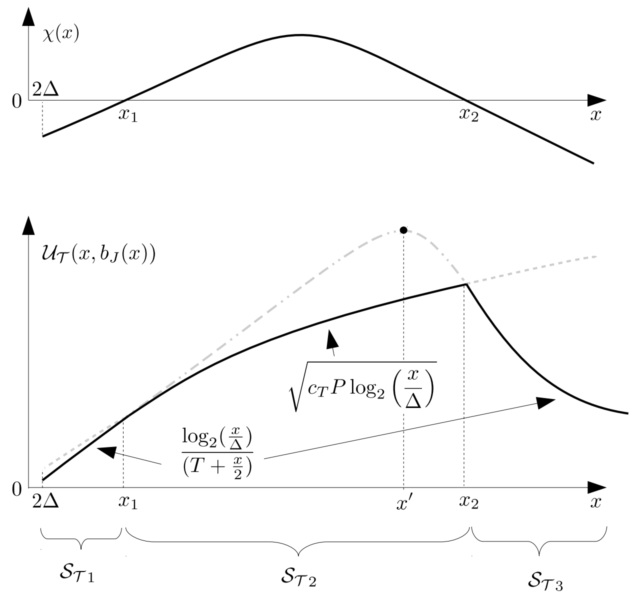


Fig. 6: Graphical representation of $\chi(x)$ and $\mathcal{U}_T(x, b_J(x))$ in the Stackelberg game. The solid line is the actual utility of the target node in each strategy subset.

such as the bisection search algorithm, the target node can make the loss in its utility as small as desired.

In the following we provide an approximation $x'_{\rm SE}$ that can be helpful from a practical point of view. Let us assume that $\left(T_{AJ} + \frac{x}{2}\right) \approx \frac{x}{2}$, therefore, eq. (10) can be rewritten as follows

$$\frac{\log(\frac{x}{\Delta})}{\log(2) c_T P} = \left(\frac{x}{2}\right)^2 \tag{20}$$

By means of simple manipulations it can be easily shown that eq. (20) admits the following solution:

$$x'_{\rm SE} = \Delta e^{-\frac{1}{2} W\left(-\frac{\log(2) c_T P \Delta^2}{2}\right)} \tag{21}$$

In Section VI we will provide numerical results that show how much the approximation in eq. (21) affects the outcome of the Stackelberg game.

Theorem 5. *In the Stackelberg game the target node improves its utility as compared to the NE if and only if* $0 < c_T < \tilde{c_T}$.

Proof: Let us start with the proof of the sufficient condition implied by the Theorem 5. According to eqs. (7) and (18a), proving that $\mathcal{U}_T(x_{\rm SE}, b_J(x_{\rm SE})) > \mathcal{U}_T(x_{\rm NE}, y_{\rm NE})$ is equivalent to showing that

$$\sqrt{c_T P \log_2(\frac{x_{\rm SE}}{\Delta})} > \frac{1}{\log 2} \frac{2}{\Delta} e^{-\frac{1}{2} W(\frac{8}{\eta \Delta^2})}$$

that is

$$\frac{1}{2} W(\frac{8}{\eta \Delta^2}) < \log(\frac{x_{\rm SE}}{\Delta})$$

This only holds if $x_{\rm SE} > \Delta e^{\frac{1}{2} W(\frac{8}{\eta \Delta^2})} = x_{\rm NE}$. Recall that if $0 < c_T < \tilde{c}_T$, the NE is an interior NE, that is, $\chi(x_{\rm NE}) > 0$. Therefore, as $\chi(x_{\rm SE}) = 0$, it must hold that $x_{\rm NE} < x_{\rm SE}$, which proves the sufficiency condition. As for the necessary condition, we have to show that, if $c_T \geqslant \tilde{c_T}$, no improvement can be achieved by the target node. In fact, if $c_T \geqslant \tilde{c_T}$ it is straightforward to prove that the NE and the SE coincide, and thus, the utilities of the target node at the SE and NE are equal. ■

B. Imperfect knowledge

We now investigate the implications of *imperfect knowledge* on the weight parameter c_T in eq. (1). In Theorem 4 we proved that the optimal strategy in the Stackelberg game is $x_{\rm SE}$ such that $\chi(x_{\rm SE}) = 0$. According to eq. (10) the value of c_T is needed to evaluate $x_{\rm SE}$. However, it is reasonable to assume that in realistic scenarios the value of c_T is not available at the target node, while instead, only statistical information on the distribution of c_T is likely known. Let us denote as $f_{c_T}(\xi)$ the probability density function (pdf) of the random variable representing the weight parameter c_T. We also denote as $g(\xi)$ the function returning the strategy of the target node at the SE, $x_{\rm SE}$, when the weight parameter for the jammer is $c_T = \xi$.

The resulting utility function of the target node $\mathbf{U}_T^\xi = \mathcal{U}_T(g(\xi), b_J(g(\xi))$ can be calculated as

$$\mathbf{U}_T^\xi = \begin{cases} \sqrt{c_T P \log_2\left(\frac{g(\xi)}{\Delta}\right)} & \text{if } \xi > c_T \quad \text{(22a)} \\ \log_2\left(\frac{g(\xi)}{\Delta}\right) / \left(T_{AJ} + \frac{g(\xi)}{2}\right) & \text{if } \xi \leq c_T \quad \text{(22b)} \end{cases}$$

Let us refer to $E\{\mathbf{U}_T^\xi\}$ as the expected value of the utility function of the target node. Assuming that $f_{c_T}(\xi)$ is a continuous function, it follows that

$$E\{\mathbf{U}_T^\xi\} = \int_{-\infty}^{+\infty} \mathcal{U}_T(\xi | c_T = \alpha) f_{c_T}(\alpha) d\alpha =$$
$$= \int_{-\infty}^{\xi} \mathcal{U}_T(\xi | c_T = \alpha) f_{c_T}(\alpha) d\alpha + \int_{\xi}^{+\infty} \mathcal{U}_T(\xi | c_T = \alpha) f_{c_T}(\alpha) d\alpha$$

From eqs. (22a) and (22b) we have

$$\begin{aligned} E\{\mathbf{U}_T^\xi\} &= \int_{-\infty}^{\xi} \sqrt{\alpha P \log_2(\frac{g(\xi)}{\Delta})} f_{c_T}(\alpha) d\alpha + \\ &+ \int_{\xi}^{+\infty} \frac{\log_2(\frac{g(\xi)}{\Delta})}{(T_{AJ} + \frac{g(\xi)}{2})} f_{c_T}(\alpha) d\alpha = \\ &= \sqrt{P \log_2(\frac{g(\xi)}{\Delta})} \int_{-\infty}^{\xi} \sqrt{\alpha} f_{c_T}(\alpha) d\alpha + \\ &+ \frac{\log_2(\frac{g(\xi)}{\Delta})}{(T_{AJ} + \frac{g(\xi)}{2})} \int_{\xi}^{+\infty} f_{c_T}(\alpha) d\alpha \end{aligned} \tag{23}$$

By exploiting the relationship in eq. (10), eq. (23) can be rewritten as

$$\begin{aligned} E\{\mathbf{U}_T^\xi\} &= P\left(T_{AJ} + \frac{g(\xi)}{2}\right) \sqrt{\xi} \left[\int_{-\infty}^{\xi} \sqrt{\alpha} f_{c_T}(\alpha) d\alpha + \right. \\ &\left. + \sqrt{\xi} \int_{\xi}^{+\infty} f_{c_T}(\alpha) d\alpha \right] \end{aligned} \tag{24}$$

Note that the target node has first to find $\xi^* = \arg\max_\xi E\{\mathbf{U}_T^\xi\}$, and then, the optimal strategy is evaluated as $x_{\rm SE}(\xi^*)$ such that $\chi(x_{\rm SE}(\xi^*)) = 0$.

In the following we analyze the especially relevant case when the random variable ξ is uniformly distributed in a closed interval[8], that is, the pdf of ξ is defined as

$$f_{c_T}(\xi) = \begin{cases} \frac{1}{\xi_{max}-\xi_{min}} & \text{if } \xi \in [\xi_{min}, \xi_{max}] \\ 0 & \text{otherwise} \end{cases} \tag{25}$$

By substituting eq. (25) in eq. (24), we obtain the following expression

$$E\{\mathbf{U}_T^{\xi}\} = P\frac{\left(T_{AJ} + \frac{g(\xi)}{2}\right)}{\xi_{max} - \xi_{min}}\left[\xi\xi_{max} - \frac{1}{3}\xi^2 - \frac{2}{3}\xi^{\frac{1}{2}}\xi_{min}^{\frac{3}{2}}\right] \tag{26}$$

In order to maximize the expected utility we study the first derivative of eq. (26), which leads to:

$$\frac{W\left(-\frac{P\log(2)\Delta^2}{2}\xi\right)}{1+W\left(-\frac{P\log(2)\Delta^2}{2}\xi\right)}\left(\xi_{max} - \frac{1}{3}\xi - \frac{2}{3}\frac{\xi_{min}^{\frac{3}{2}}}{\sqrt{\xi}}\right) = \\ =2\xi_{max} - \frac{4}{3}\xi - \frac{2}{3}\frac{\xi_{min}^{\frac{3}{2}}}{\sqrt{\xi}} \tag{27}$$

The solution of eq. (27), say ξ_{opt}, is the value of ξ that maximizes the expected utility of the target node. Regrettably, ξ_{opt} can be evaluated only numerically. Thus, in the aim of providing practical methods to choose ξ, in the next section we will discuss some analytical results that show how $\xi = \xi_{max}$ well approximates ξ_{opt}. In fact, if we assume $W\left(-\frac{P\log(2)\Delta^2}{2}\xi\right)/\left[1+W\left(-\frac{P\log(2)\Delta^2}{2}\xi\right)\right] \approx 1$, then, eq. (27) can be reformulated as

$$\xi_{max} - \frac{1}{3}\xi = 2\xi_{max} - \frac{4}{3}\xi$$

whose solution is $\xi = \xi_{max}$. Furthermore, we will show that the above approximation guarantees high efficiency at the SE even if the uncertainty on the actual value of c_T is high, as in the case of a uniform distribution.

VI. Numerical Results

In this section we apply the theoretical framework developed in the previous sections to numerically analyze the equilibrium properties for both the ordinary and Stackelberg games. As introduced in Section III, the settings of the relevant parameters are those in Table I.

A. Ordinary Game

In Fig. 7 we show the best response functions of both the target node and the jammer for different values of the weight parameter c_T. As already said, the NE is the intersection point between the best response functions. As expected, the best response of the target node does not depend on the value of c_T, while this is not true for the best response of the jammer. Note that for high c_T values the jammer reduces its jamming signal duration y, and the strategy of the target node consists in reducing the maximum silence duration x.

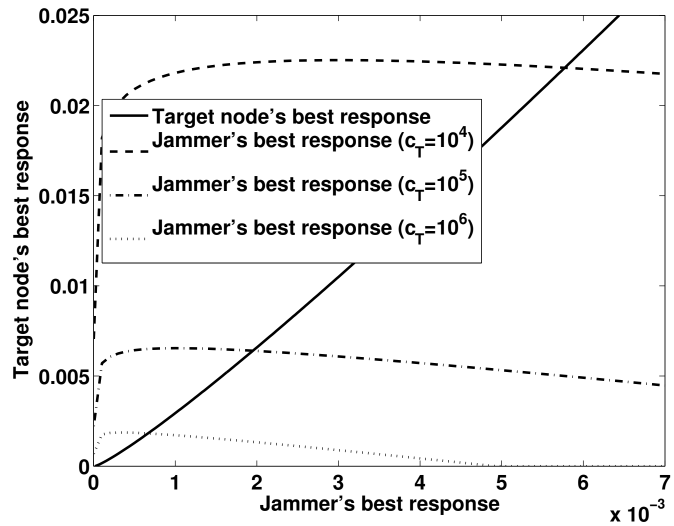


Fig. 7: Best response functions for both the target node and the jammer.

Figs. 8(a) and 8(b) illustrate the strategy of the players at the NE as a function of c_T for different values of the transmitting power P. Note that, as c_T increases, the target node decreases the maximum silence duration and the jammer reduces the jamming signal duration as well. In fact, upon increasing c_T the jammer acts in an energy preserving fashion and this causes a decrease in y. Such a behavior allows the target node to behave more aggressively by reducing the maximum silence duration x. Furthermore, upon increasing P, the strategies x and y decrease as higher P values force the jammer to reduce the jamming signal duration and, thus, the energy consumption. Also, the target node can reduce x, thus increasing its achieved capacity.

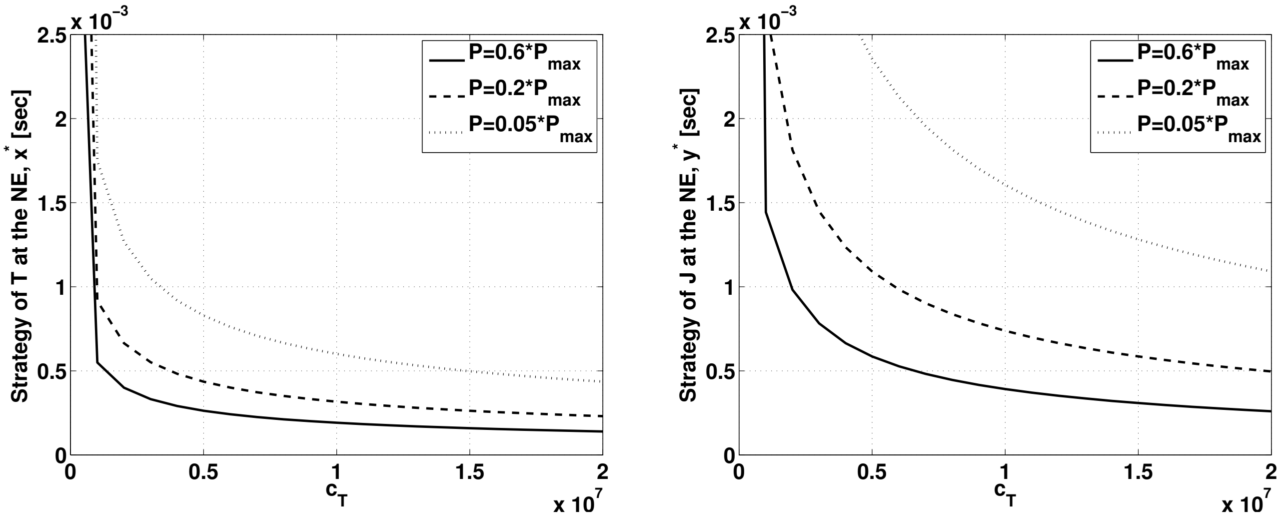
Figs. 9(a) and 9(b) illustrate how the BRD evolves at each iteration for different values of the weight c_T. Since we proved that the game converges to the NE, Figs. 9(a) and 9(b) show how, as expected, the players' strategies converge to the strategy set $\mathcal{S}'$ in 2 iterations (as discussed in Lemma 3) and to the NE in at most 7 iterations[9]. It is also shown that an increase in the value of c_T causes a decrease in the strategies of both players due to the aggressive behavior of the jammer.

B. Stackelberg Game

We now turn to the analysis of the Stackelberg game, where the target node anticipates the jammer's reaction. In this regard, Fig. 10 compares the utilities achieved by each player at the NE and SE. Note that, as proven in Theorem 5, the utility achieved by the target node at the SE is higher than, or at least equal to, the utility achieved at the NE. Moreover, at the SE the utility is higher than at the NE for the jammer as well. In fact, the target node increases the maximum silence duration x, that is, it increases transmission delay, and inhibits the jammer. Accordingly, the jammer stops its disrupting attack, and thus, it saves energy; as a result, its utility increases when

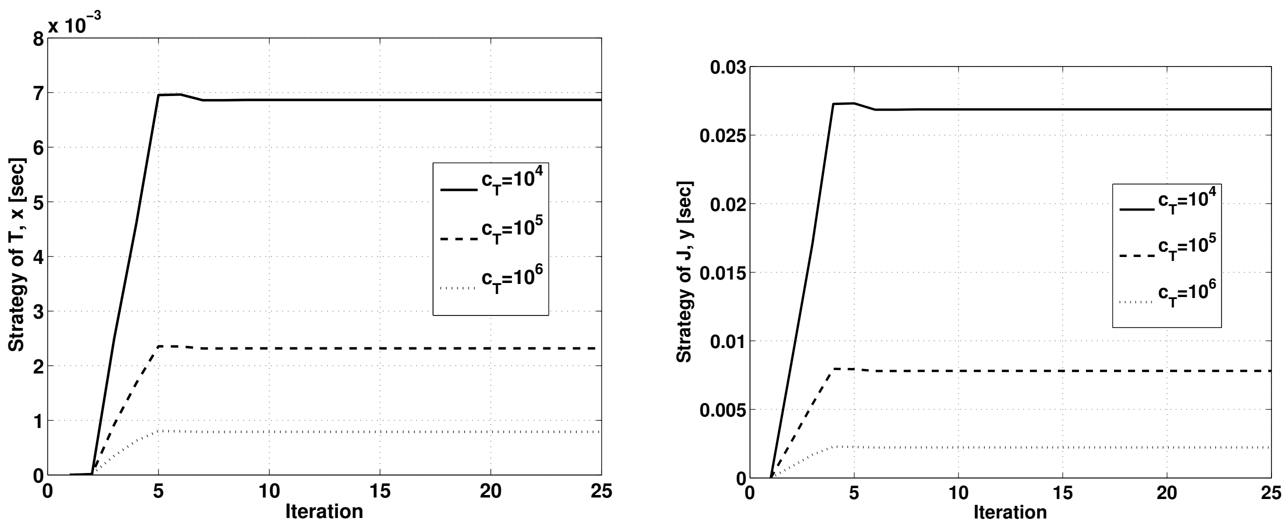
[8] Note that the uniform distribution represents the worst case, as it is the distribution that maximizes the uncertainty on the actual value of c_T, given that a minimum and a maximum values are given.

[9] Note that, although we proved that the convergence to the NE is guaranteed only if $c_T < \tilde{c_T}$, in our simulations the game always converges to the NE in a few iterations, independently of the value of c_T.



(a) (b)

Fig. 8: a) Strategy of the target node at the NE as a function of the weight parameter c_T for different values of the transmitting power P b) Strategy of the jammer at the NE as a function of the weight parameter c_T for different values of the transmitting power P.



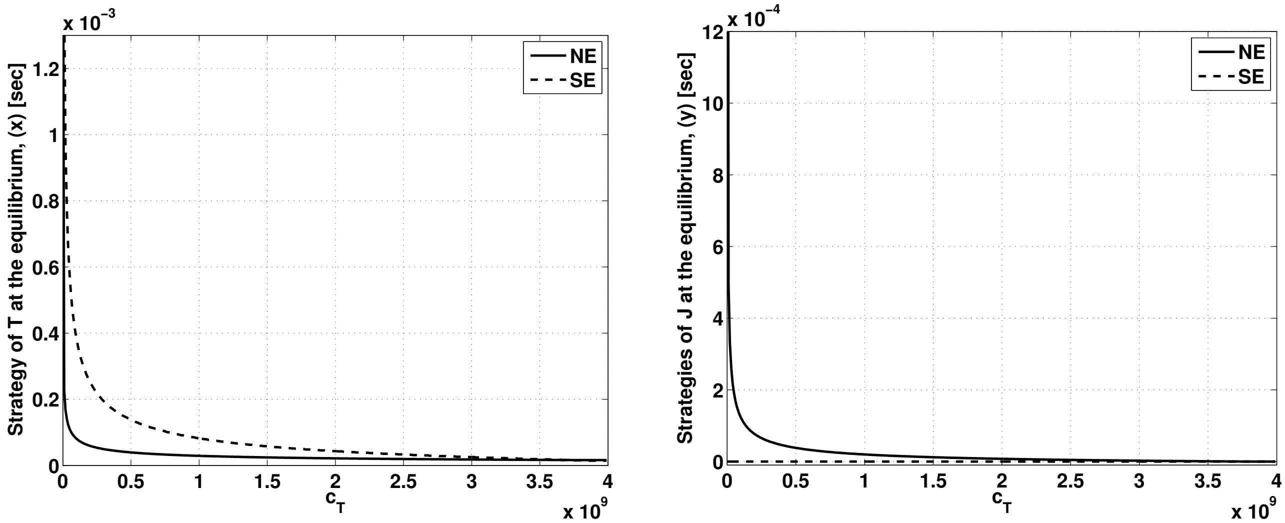
(a) (b)

Fig. 9: a) Strategy of the target node at each iteration b) Strategy of the jammer at each iteration.

compared to that at the Nash Equilibrium. We further observe that, as expected, for high values of c_T, the improvement in the achieved utility becomes negligible, as already proven in Theorem 5.

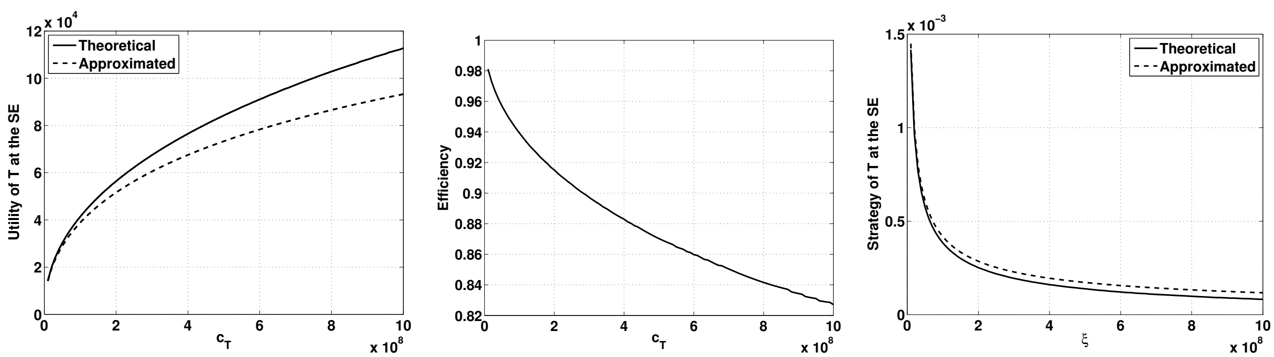
Figs. 11(a) and 11(b) illustrate the strategy at the equilibrium points of the target node and the jammer as a function of the parameter c_T, and show how the strategies of both players decrease as c_T increases. In fact, high values of the weight parameter c_T suggest a conservative behavior of the jammer at the NE (e.g. the jammer is more energy constrained), so that the jammer prefers to decrease the duration of the jamming signal y in order to reduce its energy consumption. Instead, as proven in Theorem 4, at the SE the target node forces the jammer in stopping its jamming attack, thus, $y_{\mathrm{SE}} = 0$. Furthermore, for high values of the parameter c_T, the strategy x of the target node consists in choosing low silence duration at both the NE and SE. This is because by increasing c_T the strategy of the jammer consists in reducing the duration of the jamming signal. Hence, the target node decreases the maximum duration of the silence intervals x, that is, T reduces the transmission delays while achieving a higher transmission capacity. Note that when the value of c_T approaches $\tilde{c_T}$, the NE and SE become equal.

Under the perfect knowledge assumption, at the SE the strategy of the target node, x_{SE}, coincides with the solution of $\chi(x) = 0$, which can also be approximated to x'_{SE} as given in eq. (21). Accordingly, in Fig. 12(a) we compare the utilities of the target node at the SE, in its exact and approximated strategies x_{SE} and x'_{SE}, respectively. Fig. 12(b) shows that the approximation accuracy of x'_{SE}, defined as the ratio between



(a) (b)

Fig. 11: a) Strategy of the target node T at NE and SE as a function of the weight parameter c_T b) Strategy of the jammer J at NE and SE as a function of the weight parameter c_T.



(a) (b) (c)

Fig. 12: Impact of the approximation $x'_{\rm SE}$ in eq. (21) on the Stackelberg game outcome as a function of the weight parameter c_T ($c_T \cdot P = 2 \cdot 10^6$).

$\mathcal{U}_T(x'_{\rm SE}, b_J(x'_{\rm SE}))$ and $\mathcal{U}_T(x_{\rm SE}, b_J(x_{\rm SE}))$, strongly depends on the value of c_T. As shown in Fig. 12(c), the error introduced by the approximation $\left(T_{AJ} + \frac{x_{\rm SE}}{2}\right) \approx \frac{x'_{\rm SE}}{2}$ is low when low values of c_T are considered, because, in this case, the strategy of T at the SE, $x_{\rm SE}$, consists in choosing larger silence durations, and thus $\frac{x_{\rm SE}}{2} \gg T_{AJ}$. On the contrary, when c_T is high, there is no need for the target node to choose high $x_{\rm SE}$ values, thus the above approximation introduces a non-negligible error on the estimate of $x'_{\rm SE}$. Note that, although the approximation is affected by errors, Fig. 12(b) shows that the approximation accuracy is still high (i.e. larger than 82%).

To evaluate the impact of imperfect knowledge on the utility of the target node, let us now define the *equilibrium efficiency* $e(\xi)$ as follows:

$$e(\xi) = \frac{\mathbf{U}_T^{\xi}}{\mathbf{U}_T^{c_T}} \tag{28}$$

Fig. 13 illustrates the equilibrium efficiency of the target node as a function of c_T for different choices of ξ. More in detail, we considered $\xi \in \{\xi_{opt}, \xi_{mean}, \xi_{max}, \xi_{min}\}$, where $\xi_{mean} = (\xi_{max} + \xi_{min})/2$, $\xi_{min} = 10^5$ and $\xi_{max} = 10^9$. Note that in our simulations $\xi_{min} = 10^5$ and $\xi_{max} = 10^9$ are realistic setting assumptions. In fact, lower values of ξ_{min} or higher values of ξ_{max} lead to unbalanced settings as one of the terms in eq. (1) will always dominate the other. The most important result is that the equilibrium efficiency when $\xi \in \{\xi_{opt}, \xi_{mean}, \xi_{max}\}$ is always higher than 75%, while the case $\xi = \xi_{min}$ achieves a very low equilibrium efficiency (and thus, it is not reported in Fig. 13). As demonstrated in Section V-B, Fig. 13 shows that ξ_{max} well approximates ξ_{opt}, i.e., $e(\xi_{opt}) \simeq e(\xi_{max})$. Therefore, from a practical point of view, if the computation of ξ_{opt} is not feasible (e.g., high computational cost and low hardware capabilities) it is still possible to achieve a high equilibrium efficiency by choosing $\xi = \xi_{max}$.

Finally, in Fig. 14 we compare the utility functions of the target node and the jammer obtained at the NE and SE with what is obtained in the cases the two players select their strategies without considering the strategies of each other.

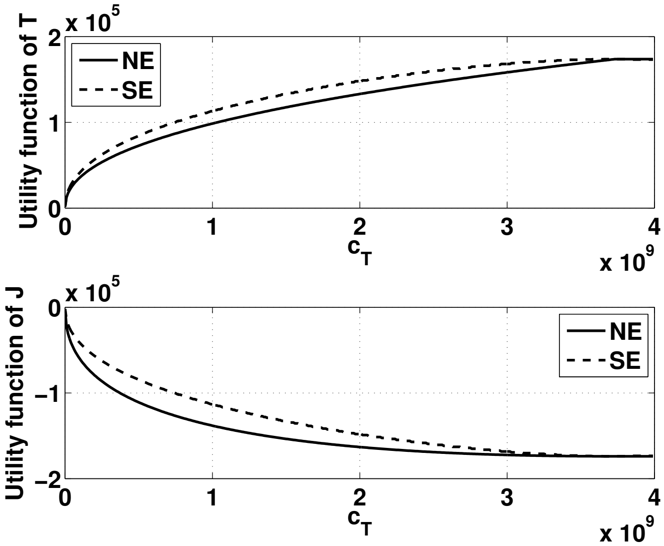


Fig. 10: Comparison between the utilities achieved by each player at the NE and SE as a function of the weight parameter c_T ($c_{T^*} \cdot P = 2 \cdot 10^6$).

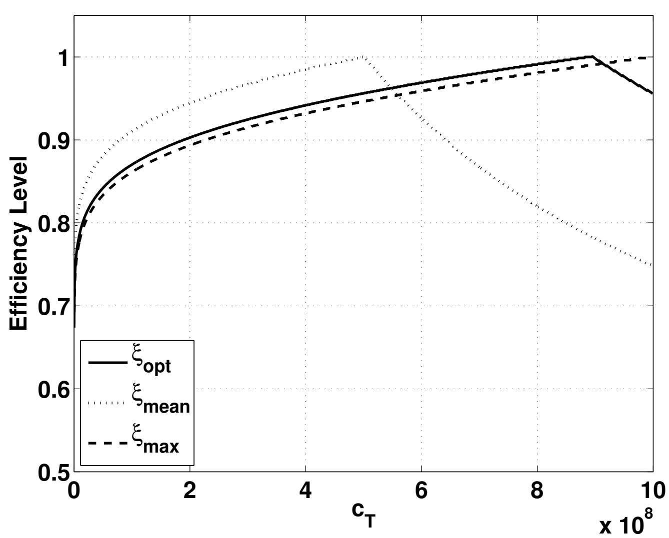


Fig. 13: Equilibrium efficiency $e(\xi)$ as a function of the weight parameter c_T ($c_T \cdot P = 2 \cdot 10^6$).

More specifically we will consider the two following cases:

- *Case A*: The target node selects its strategy x in such a way that its capacity is maximized without considering that the jammer will try to disrupt the communication in the timing channel as well. In other terms, the target node will assume that $y \approx 0$.
- *Case B*: The jammer selects its strategy y assuming that the target node is not aware that it (the jammer itself) is trying to disrupt the communication in the timing channel. In other terms, the jammer will assume that $x \approx b_T(0)$.

When compared to the NE and SE cases the utility function of the target node will decrease in Case A and increase in Case B. The viceversa holds for the utility function of the jammer. We observe that the gap between the utility functions

Name	Value	Unit
T_{AJ}	15	μs
Δ	1	μs
P	2	W
T_P	20	μs
c_T	$8 \cdot 10^9$	$bit/(sec \cdot J)$
c_{T^*}	10^6	$bit/(sec \cdot J)$

TABLE II: Parameter settings used in our simulations.

obtained in Cases A and B compared to the NE and SE decrease when the cost c_T increases. This is because when the cost c_T increases the jammer becomes more concerned about the energy consumption and therefore the value y_{NE} becomes smaller. Accordingly, the assumptions considered in Cases A and B become accurate and consequently the behavior approaches what is obtained when each player takes the strategy of the opponent into account.

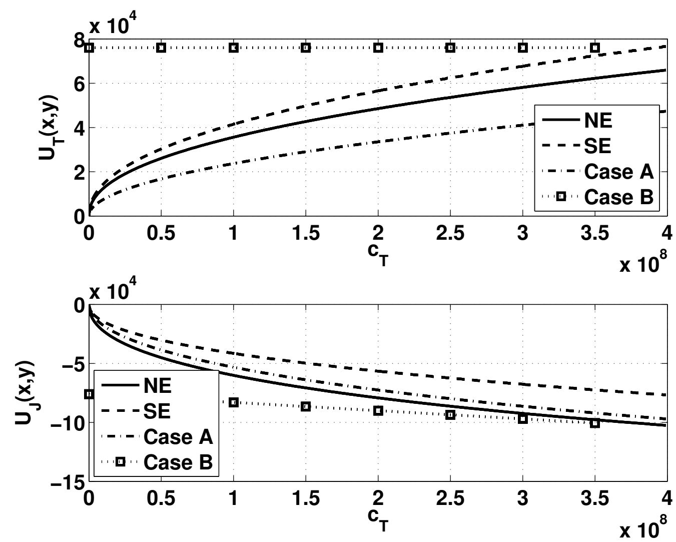


Fig. 14: Comparison between the utility of the target node and the jammer when they work at the NE, at the SE and what is obtained in Case A and B.

C. Simulation results

To assess the accuracy of the theoretical results derived in the previous sections, we implemented a simulator that shows how players' behavior dynamically evolves and how players choose their strategies. In the simulations we assume that each player chooses its own initial strategy randomly. Then, players update their strategies each 10 cycles during which each player estimates the opponent's strategy. Players update their strategies according to the BRD discussed in Section IV-C. The simulation parameter setup is summarized in Table II. Note that we chose $c_T > c_T^{max}$ so that NE is on the border, i.e., the strategy of the jammer at the NE is $y^* = 0$. In Fig. 15 we show an example of the simulation results that illustrates how players dynamically change their strategies depending on the opponent's one. The figure shows that after three iterations, players reach the NE, that is, due

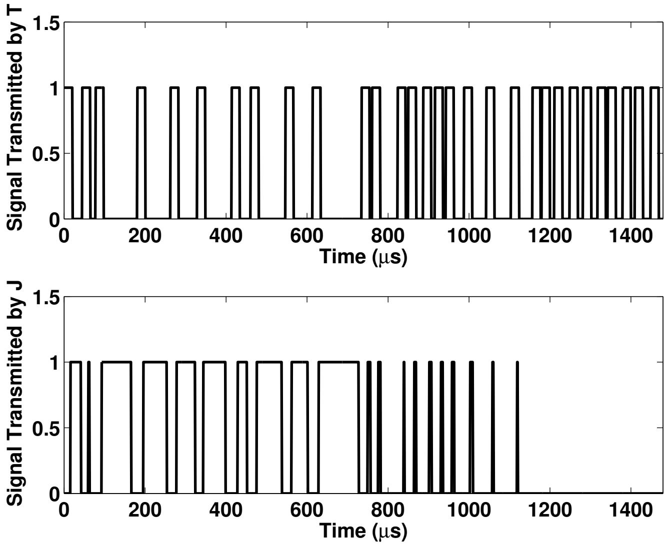


Fig. 15: Strategies chosen by the players vs. time.

to the high energy cost, the jammer stops its attack while the target node chooses its strategy according to its best response function,i.e., $x^* = b_T(0)$.

VII. Conclusions

In this paper we have proposed a game-theoretic model of the interactions between a jammer and a communication node that exploits a timing channel to improve resilience to jamming attacks. Structural properties of the utility functions of the two players have been analyzed and exploited to prove the existence and uniqueness of the Nash Equilibrium. The convergence of the game to the Nash Equilibrium has been studied and proved by analyzing the best response dynamics. Furthermore, as the reactive jammer is assumed to start transmitting its interference signal only after detecting activity of the node under attack, a Stackelberg game has been properly investigated, and proofs on the existence and uniqueness of the Stackelberg Equilibrium has been provided. Finally, the case of imperfect knowledge about the parameter c_T has been also discussed. Numerical results, derived in several real network settings, show that our proposed models well capture the main factors behind the utilisation of timing channels, thus representing a promising framework for the design and understanding of such systems.

References

[1] V. Anantharam and S. Verdu, "Bits through queues," *Information Theory, IEEE Transactions on*, vol. 42, no. 1, pp. 4–18, 1996.

[2] G. Morabito, "Exploiting the timing channel to increase energy efficiency in wireless networks," *Selected Areas in Communications, IEEE Journal on*, vol. 29, no. 8, pp. 1711–1720, 2011.

[3] L. Galluccio, G. Morabito, and S. Palazzo, "TC-Aloha: A novel access scheme for wireless networks with transmit-only nodes," *Wireless Communications, IEEE Transactions on*, vol. 12, no. 8, pp. 3696–3709, August 2013.

[4] W. Xu, W. Trappe, and Y. Zhang, "Anti-jamming timing channels for wireless networks," in *Proceedings of the first ACM conference on Wireless network security*. ACM, 2008, pp. 203–213.

[5] S. D'Oro, L. Galluccio, G. Morabito, and S. Palazzo, "Efficiency analysis of jamming-based countermeasures against malicious timing channel in tactical communications," in *Communications (ICC), 2013 IEEE International Conference on*. IEEE, 2013, pp. 4020–4024.

[6] W. Xu, K. Ma, W. Trappe, and Y. Zhang, "Jamming sensor networks: attack and defense strategies," *Network, IEEE*, vol. 20, no. 3, pp. 41–47, 2006.

[7] R. Saranyadevi, M. Shobana, and D. Prabakar, "A survey on preventing jamming attacks in wireless communication," *International Journal of Computer Applications*, vol. 57, no. 23, pp. 1–3, November 2012, published by Foundation of Computer Science, New York, USA.

[8] R. Poisel, *Modern Communications Jamming Principles and Techniques*, ser. Artech House information warfare library. Artech House, 2004. [Online]. Available: http://books.google.it/books?id=CZDXton6vaQC

[9] R.-T. Chinta, T. F. Wong, and J. M. Shea, "Energy-efficient jamming attack in IEEE 802.11 MAC," in *Military Communications Conference, 2009. MILCOM 2009. IEEE*. IEEE, 2009, pp. 1–7.

[10] Y. W. Law, L. Van Hoesel, J. Doumen, P. Hartel, and P. Havinga, "Energy-efficient link-layer jamming attacks against wireless sensor network MAC protocols," in *Proceedings of the 3rd ACM workshop on Security of ad hoc and sensor networks*. ACM, 2005, pp. 76–88.

[11] M. Wilhelm, I. Martinovic, J. B. Schmitt, and V. Lenders, "Short paper: reactive jamming in wireless networks: how realistic is the threat?" in *Proceedings of the fourth ACM conference on Wireless network security*. ACM, 2011, pp. 47–52.

[12] W. Xu, W. Trappe, Y. Zhang, and T. Wood, "The feasibility of launching and detecting jamming attacks in wireless networks," in *Proceedings of the 6th ACM international symposium on Mobile ad hoc networking and computing*. ACM, 2005, pp. 46–57.

[13] M. Strasser, B. Danev, and S. Čapkun, "Detection of reactive jamming in sensor networks," *ACM Transactions on Sensor Networks (TOSN)*, vol. 7, no. 2, p. 16, 2010.

[14] M. Strasser, S. Capkun, and M. Cagalj, "Jamming-resistant key establishment using uncoordinated frequency hopping," in *Security and Privacy, 2008. SP 2008. IEEE Symposium on*. IEEE, 2008, pp. 64–78.

[15] B. Wang, Y. Wu, K. R. Liu, and T. C. Clancy, "An anti-jamming stochastic game for cognitive radio networks," *Selected Areas in Communications, IEEE Journal on*, vol. 29, no. 4, pp. 877–889, 2011.

[16] D. Yang, G. Xue, J. Zhang, A. Richa, and X. Fang, "Coping with a smart jammer in wireless networks: A stackelberg game approach," *Wireless Communications, IEEE Transactions on*, vol. 12, no. 8, pp. 4038–4047, 2013.

[17] Y. Liu and P. Ning, "Bittrickle: Defending against broadband and high-power reactive jamming attacks," in *INFOCOM, 2012 Proceedings IEEE*. IEEE, 2012, pp. 909–917.

[18] Y. Liu, D. Ghosal, F. Armknecht, A.-R. Sadeghi, S. Schulz, and S. Katzenbeisser, "Hide and seek in time—robust covert timing channels," in *Computer Security–ESORICS 2009*. Springer, 2009, pp. 120–135.

[19] E. Altman, K. Avrachenkov, and A. Garnaev, "A jamming game in wireless networks with transmission cost," in *Network Control and Optimization*. Springer, 2007, pp. 1–12.

[20] S. Anand, S. Sengupta, K. Hong, and R. Chandramouli, "Power control game in multi-terminal covert timing channels," *Selected Areas in Communications, IEEE Journal on*, vol. 30, no. 1, pp. 44–53, 2012.

[21] S. Anand, S. Sengupta, and R. Chandramouli, "An attack-defense game theoretic analysis of multi-band wireless covert timing networks," in *INFOCOM, 2010 Proceedings IEEE*. IEEE, 2010, pp. 1–9.

[22] G. Morabito, "Increasing capacity through the use of the timing channel in power-constrained satellite networks," in *INFOCOM 2007. 26th IEEE International Conference on Computer Communications. IEEE*. IEEE, 2007, pp. 580–588.

[23] J. B. Rosen, "Existence and uniqueness of equilibrium points for concave n-person games," *Econometrica: Journal of the Econometric Society*, pp. 520–534, 1965.

[24] D. G. Luenberger, "Complete stability of noncooperative games," *Journal of Optimization Theory and Applications*, vol. 25, no. 4, pp. 485–505, 1978.

VIII. Appendices

Appendix A. Proof of Theorem 2

Proof: In order to prove the theorem we have to solve eq. (11), that is, find a pair (x, y) which solves the following system of equations:

$$\begin{cases} y = \chi(\Delta e^{\psi(y)+1}) \\ x = \Delta e^{\psi(y)+1} \end{cases} \tag{29}$$

By exploiting the Lambert W-function definition and the relationship $z/W(z) = e^{W(z)}$, where $z = \left[\frac{2(T_{AJ}+y)}{e\Delta}\right]$, it can be proven that the above system leads to

$$(y + T_{AJ})^2 = \frac{1}{\eta} \cdot \frac{\psi^2(y)}{\psi(y)+1} \tag{30}$$

Given that the first derivative of the Lambert W-function is defined as

$$W'(z) = \frac{W(z)}{z(W(z)+1)} \tag{31}$$

eq. (30) can also be rewritten as

$$e^{W\left(\frac{2(T_{AJ}+y)}{e\Delta}\right)} = \frac{1}{\eta} \cdot \frac{2}{\Delta e} \cdot W'\left(\frac{2(T_{AJ}+y)}{e\Delta}\right) \tag{32}$$

Note that the function on the left-hand side is strictly increasing, while the one on the right-hand side is strictly decreasing. These structural properties imply that the two functions have no more than one intersection point. Therefore, the game admits a unique NE.

Now we focus on finding a closed form for the unique NE.

To this purpose, eq. (32) can be reformulated as

$$e^{2W\left(\frac{2(T_{AJ}+y)}{e\Delta}\right)}\left(W\left(\frac{2(T_{AJ}+y)}{e\Delta}\right)+1\right) = \frac{1}{\eta}\left(\frac{2}{e\Delta}\right)^2$$

which, by exploiting the relation $z = W(z)e^{W(z)}$, can be rewritten as follows:

$$W\left(\frac{2(T_{AJ}+y)}{e\Delta}\right) = \frac{1}{2}W\left(\frac{8}{\eta\Delta^2}\right) - 1 \tag{33}$$

It is easy to prove that eq. (33) has the following solution

$$y^* = \frac{\Delta}{2}\left(\frac{1}{2}W\left(\frac{8}{\eta\Delta^2}\right) - 1\right)e^{\frac{1}{2}W\left(\frac{8}{\eta\Delta^2}\right)} - T_{AJ} \tag{34}$$

By substituting eq. (34) in eq. (8) we obtain $x^* = \Delta e^{\frac{1}{2}W(\frac{8}{\eta\Delta^2})}$. As the point (x^*, y^*) has been obtained as the intersection between the best response functions in eqs. (8) and (9), it follows that $(x_{\text{NE}}, y_{\text{NE}}) = (x^*, y^*)$ is the unique NE.

Finally, we prove that the NE $(x_{\text{NE}}, y_{\text{NE}})$ is an *interior* NE. An interior NE happens when it is not on the border of the strategy set; therefore, we aim at proving that $x_{\text{NE}} > 2\Delta$ and $y_{\text{NE}} > 0$. As $x_{\text{NE}} = \Delta e^{\frac{1}{2}W(\frac{8}{\eta\Delta^2})}$, proving that x_{NE} is not on the border is trivial; from eq. (34) it can also be easily proven that the condition $y_{\text{NE}} > 0$ implies $0 < c_T < \tilde{c_T}$, where $\tilde{c_T}$ is given in eq. (12); therefore, an interior NE exists only if $0 < c_T < \tilde{c_T}$. Theorem 1 states that an NE must exist for any given weight parameter c_T. Since we already proved that an interior NE exists only if $0 < c_T < \tilde{c_T}$, we can deduce that the NE is on the border if $c_T \geqslant \tilde{c_T}$.

From eq. (9) we know that for $c_T \geqslant \tilde{c_T}$ the best response function of the jammer, $b_J(x)$, is continuous, and it is upper-bounded by $b_J(\hat{x})$ where $\hat{x} = \Delta e^{\frac{1}{2}W(\frac{2}{\eta\Delta^2})}$, and lower-bounded by 0; thus, as the NE has to be at the border, it follows that the only feasible solution is $y_{\text{NE}} = 0$. Hence, from eqs. (8) and (9), it is easy to derive closed form solutions on the border NE, $(x_{\text{NE}}, y_{\text{NE}}) = \left(\Delta e^{W(\frac{2T_{AJ}}{e\Delta})+1}, 0\right)$, which concludes the proof. ∎

Appendix B. Proof of Lemma 2

Proof: To prove the Lemma, it will be shown that the condition in eq. (15) implies that the Jacobian matrix norm $||J_{\mathbf{b}}||_\infty$ in eq. (13) is lower than 1. In fact, the condition $||J_{\mathbf{b}}||_\infty < 1$ leads to:

$$\max\left(\left|\frac{\partial}{\partial y}b_T(y)\right|, \left|\frac{\partial}{\partial x}b_J(x)\right|\right) < 1$$

Note that $\left|\frac{\partial}{\partial y}b_T(y)\right|$ can be calculated as

$$\left|\frac{\partial}{\partial y}b_T(y)\right| \quad = \quad \frac{2}{W(\frac{2(T_{AJ}+y)}{e\Delta})+1}$$

The above function is non-negative and strictly decreasing, thus it achieves its maximum value when $y = 0$. Accordingly, it is sufficient to show that

$$\max_{y \in \mathcal{S}_{\mathcal{J}}}\left(\frac{2}{W(\frac{2(T_{AJ}+y)}{e\Delta})+1}\right) < 1 \qquad , \qquad \forall y \geq 0$$

or, equivalently, that

$$\max_{y \in \mathcal{S}_{\mathcal{J}}}\left(\frac{2}{W(\frac{2(T_{AJ}+y)}{e\Delta})+1}\right) = \frac{2}{W(\frac{2T_{AJ}}{e\Delta})+1} < 1 \qquad , \qquad \forall y \geq 0$$

which is indeed satisfied for all values of y in the strategy set; therefore, $\left|\frac{\partial}{\partial y}b_T(y)\right| < 1, \forall y \in \mathcal{S}_{\mathcal{J}}$.

Concerning the condition $\left|\frac{\partial}{\partial x}b_J(x)\right| < 1$, by deriving $b_J(x)$, it follows that

$$\left|\frac{1}{2}\left[\frac{1}{x\sqrt{\eta \log \frac{x}{\Delta}}} - 1\right]\right| < 1 \tag{35}$$

The expression on the right-hand side of eq. (35) is a non-negative strictly decreasing function, so again eq. (35) results in

$$\max_{x \in \mathcal{S}_{\mathcal{T}}}\left(\left|\frac{1}{2}\left[\frac{1}{x\sqrt{\eta \log\left(\frac{x}{\Delta}\right)}} - 1\right]\right|\right) < 1 \tag{36}$$

Note that eq. (36) can be rewritten in the form given in eq. (15) and $||J_{\mathbf{b}}||_\infty = ||J_{\mathbf{b}}||$ as $J_{\mathbf{b}}$ is diagonal. Let $s^i = (x^i, y^i)$, it then follows that

$$||s^{i+1} - s^i|| \leq ||J_{\mathbf{b}}^{max}|| \cdot ||s^i - s^{i-1}|| \leq \cdots \leq ||J_{\mathbf{b}}^{max}||^i ||s^1 - s^0||$$

where $||J_{\mathbf{b}}^{max}|| = \max J_{\mathbf{b}}$. The above equation indicates that given any $\epsilon > 0$, after at most $\log_{J_{\mathbf{b}}^{max}} \frac{\epsilon}{||s^1 - s^0||}$ iterations, the game converges to the NE as $||s^{i+1} - s^i|| \leq \epsilon$ which thus concludes the proof. ∎