A Game Theoretic Approach for Distributed Resource Allocation and Orchestration of Softwarized Networks

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***Abstract*—*Softwarization* of networks allows simplifying deployment, configuration and management of network functions. The driving force towards this evolution is represented by Software Defined Networking (SDN) that allows more flexible and dynamic network resource allocation and management. Efficient resource allocation and orchestration are two primary targets of this softwarization process; however, centralized methodologies result complex, and exhibit scalability issues. So, distributed solutions are to be preferred but, in order to be effective, should quickly converge towards equilibrium solutions. In this paper, we focus on making distributed resource allocation and orchestration a viable approach, and prove convergence of the relevant mechanisms. Specifically, we exploit game theory to model interactions between users requesting network functions and servers providing these functions. Accordingly, a two-stage Stackelberg game is presented where servers act as leaders of the game and users as followers. Servers have conflicting interests and try to maximize their utility; users, on the other hand, use a replicator behavior and try to imitate other users decisions to improve their benefit. The framework proves the existence and uniqueness of an equilibrium, and a learning mechanism to converge to such equilibrium is proposed. Numerical results show the effectiveness of the approach.**

***Index Terms*—Game Theory, Softwarized Networks, Orchestration, Resource Management.**

I. Introduction

The proliferation of new services and applications in the Internet with different requirements in terms of availability, service quality and resilience, is making management of network infrastructures a key challenge. In this evolving scenario, Telco Operators (TOs) show increasing interest in *softwarizing* their networks, so making deployment, configuration, management and updating of network functions faster and easier, and thus achieving numerous advantages in terms of both Capital Expenditure (CAPEX) and Operational Expenditure (OPEX).

Two relevant key enablers of this evolution are Software-Defined Networking (SDN) [1–3] and Network Functions Virtualization (NFV) [4]. SDN allows a flexible management of the network resources thanks to its peculiarity of separating the network control from the forwarding plane. NFV, on the other hand, brings virtualization concepts from cloud computing to the network in order to let software-based network functions, also called virtualized network functions (VNFs), run on commodity hardware infrastructures.

The introduction of the joint SDN/NFV paradigm is seen by Telco Operators (TOs) as the way to move to more flexible networks where services can be instantly monitored, controlled, billed, and managed on the fly, rather than requiring a set of complex, manual changes [5]. However, as compared to purpose-built networking hardware or middle boxes devices, deterrents to this approach are the achievable performance and the scalability. Key elements for the design of these systems are resource allocation, and network function orchestration. Although similar design problems have been studied in cloud computing scenarios [6–9], there are important differences stemming from the fact that servers in data centers are connected to each other through high-capacity and high-speed networks, so making the specifics of the underlying network less important. On the contrary, in network function deployment, network constraints (e.g. bandwidth and latency) are of crucial importance. The choice of where running network functions has to be made by accounting not only for the increased load in the nodes hosting the functions, but also for the latency experienced to reach these nodes, which can be different for each flow [10, 11].

The first step towards management and function allocation in an NFV scenario was made in [12], where the VNF-P algorithm was introduced to handle traffic load variations by dynamically instantiating VNFs. In the same context [13] and [14] discuss placement policies for specific VNFs. Instead, the work [15] considers a heterogeneous scenario with VNFs characterized by different scalability, reliability, and availability requirements, and proposes an extension of the Openstack orchestrator to translate the individual deployment requirements into a placement of the VNFs in the cloud infrastructure. Two works that are very close to this paper in some aspects are [16] and [17]. In [16] VNF placement is made in a way that minimizes the overall network cost, expressed in terms of the distance between users and the locations where services are provided, and the cost of service setup. Instead, the work in [17] considers the multi-commodity facility location by considering the existence of more than one VNF instance in the same network.

The problem of placement of network functions implemented as middle boxes is considered in [18], with the target of

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optimizing network operational costs and utilization, without violating service level agreements. This VNF Orchestration Problem (VNF-OP) is addressed and formulated as an Integer Linear Programming (ILP) problem that is then solved through heuristics.

Another aspect that has to be considered is system scalability w.r.t. the number of functions and the customers. In fact, the complex tasks of management, orchestration and resource allocation are in charge of only one entity, the *Orchestrator*, which therefore requires sophisticated algorithms which generally results in NP-hard problems [16, 19, 20]. This makes the deployment of the softwarized paradigm unfeasible if TOs aim at designing and managing their networks while optimizing costs and performance. So centralized solutions (see Case A in Fig. 1(a)) or virtually centralized solutions (see Case B in Fig. 1(b)) could result unfeasible, and distributed approaches to resource allocation and orchestration have to be pursued.

With all this in mind, the reference scenario addressed in this paper addresses a single TO network domain where some customers, in the following referred to as *Servers*, give their availability to support the TO in providing network services to the *Users*, not only sharing their hardware facilities where running network functions, but also strongly alleviating management and orchestration burden in the decision tasks of both placing functions and assigning them to each active user flow. We consider that the Server role is played by customers of the TO network and network functions are located in the CN premises (see Case C in Fig. 1(c)). So, both resource provisioning and management are distributed within the considered TO network domain. With respect to this, let us note that, at the best of our knowledge, there is no work so far where the main tasks performed by the orchestrator, and thus related to management issues, are distributed among customers of the network, i.e., the VNF Servers.

A relevant issue with this distributed approach is related to system convergence towards equilibrium. In fact, the possibility to converge towards an equilibrium as well as the rapidness of this convergence should be investigated to prove the feasibility of the approach. Accordingly, game theory is the natural way to model and characterize the system. In particular, a new market is associated to the proposed system, where the main actors are: 1) the Servers, that are the sellers of the network functions; 2) the Users, that play the role of buyers; 3) the TO, that coordinates the whole system. In such a context, Servers autonomously decide the price and the bandwidth to be requested to the TO network in order to provide the network services. Users, on the other hand, according to the price specified by each Server, and the corresponding expected performance in terms of both experienced latency and provided bandwidth, choose one Server for each VNF. In this way the task of associating each flow to a Server is not decided by the Orchestrator, but in an autonomous and distributed way, as a consequence of the interaction between Users and Servers.

To model and support interactions between Servers and Users, we exploit hierarchical and evolutionary game theoretic tools. Specifically, since Servers naturally act and make decisions by anticipating the Users, we define a two-stage Stackelberg game where Servers act as the leaders of the game, and Users as the followers. Servers have conflicting interests among themselves, as their objective is to individually and selfishly maximize a utility function. Also, as commonly assumed in multi-player markets, Servers are expected not to cooperate with each other, and do not exchange any information with other competitors. Therefore, their interactions are modeled by non-cooperative game theory. Instead, Users are influenced by social and imitation behavior, i.e., they observe other Users' decisions and imitate those decisions if this is expected to improve their benefit. Thus, their interactions are modeled by using the replicator dynamics from Evolutionary Game Theory (EGT) [21].

In more detail, we derive a closed-form solution for the equilibrium condition of the replicator dynamics which is then used to solve the Stackelberg game. Accordingly, we show that the considered game admits a Stackelberg Equilibrium (SE), and we prove that the SE is unique. We also propose a reinforcing learning procedure that provably converges to the unique SE, and illustrate an algorithmic implementation. We show that the learning procedure can be implemented in a privacy-preserving and distributed fashion. Finally, we present an extensive numerical result analysis to highlight the dependency of the dynamic interactions among players on the main system parameters, and evaluate the proposed market model, in the view of providing some insights in setting system parameters to maximize revenues. A part of the numerical analysis is also aimed to show that the proposed learning procedure is scalable w.r.t. the number of Servers, and quickly converges to the SE.

At our best knowledge, this is the first work where a game-theoretic approach is used in a softwarized network scenario to support interactions between Servers and Users.

Moreover, exploiting the game-theoretic approach allows us to demonstrate that adopting a distributed framework, instead of a traditional centralized approach, is beneficial for all the involved stakeholders. On the one hand, the simplified orchestration and the possibility to offload VNFs on third-parties Servers is beneficial to the TO. Servers have economic benefit participating to the VNF Market as sellers. Finally, Users can choose the Servers that most fit their needs.

The remaining of this paper is organized as follows. In Section II, the considered network scenario is described. In Section III, the game-theoretic framework which models the considered resource allocation and orchestration problem is proposed and studied. A numerical analysis of the proposed game-theoretic framework is presented in Section IV. Finally, in Section V conclusions are drawn.

II. System Model

The considered reference scenario is sketched in Fig. 2. It consists of a network domain of a Telco Operator (TO) that provides customers of this domain with network services according to the NFV paradigm. The main roles in the system are played by the *Orchestrator*, the *VNF Servers*, and the *Users*.

Users are the customers that generate flows and request VNFs for each of their flows. They are located in Customer Networks (CNs) where a *Customer Premise Equipment* (CPE)

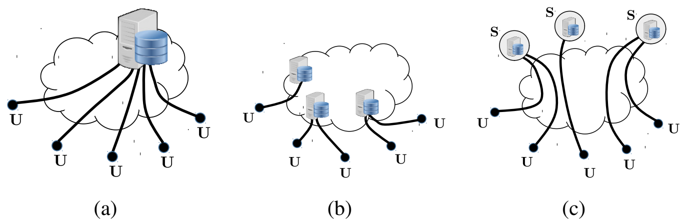


Fig. 1. a) Case A: Centralized market scenario where all network functions are executed on a single server owned by the TO; b) Case B: Centralized market scenario where network functions are distributed among several servers owned by the TO; c) Case C: Proposed distributed market scenario where network functions are executed on several third-parties servers.

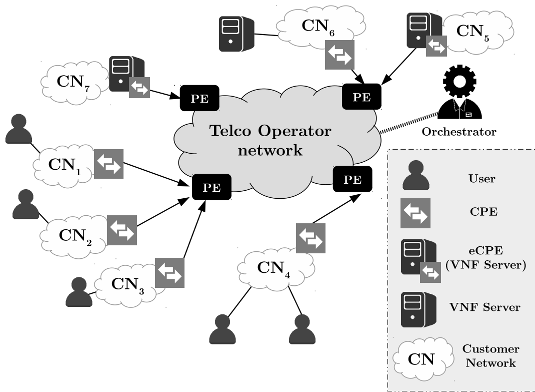


Fig. 2. The reference network scenario.

device allows them to be connected to an access node of the TO core network, in the figure indicated as *Provider Edge* (PE) node.

VNF Servers are NFV-compliant nodes [22, 23] owned by network customers that have decided to run VNFs in order to serve the TO network domain they belong to, and obtain economic benefits. A VNF Server can be either a stand-alone computer, such as the one connected to the CN_6 in Fig. 2, a set of servers organized as a data center, whose resources are partially or totally dedicated to run VNFs, or an enhanced CPE (eCPE) node. As described in [24], the latter is a CPE device that is able to run VNFs in a virtualized environment (e.g. the eCPE nodes connecting CN_5 and CN_7 to the TO network). Besides the hardware facilities, VNF Servers need an amount of bandwidth that is provided them by the TO network. A VNF Server can provide more than one VNF and decide the selling prices autonomously. In general, a VNF Server can also provide, manage and sell an entire service chain realized by connecting local component VNFs. However, for the sake of simplicity and without loss in generality, in the sequel we will refer to VNF Servers as servers that provide VNFs only[1].

A very important role in the system is played by the *Orchestrator*, which is in charge of management and orchestration of the whole system. It runs on a dedicated server and communicates with all nodes through the TO network. The main tasks performed by the Orchestrator are:

[1]The problem of distributed service chain composition is out of the scope of this paper and is addressed in [25].

TABLE I
NOTATION

Variable	Description
$\mathcal{S}, \mathcal{U}$	Sets of VNF Servers and User Groups
n_{ip}	Fraction of users in $\mathcal{U}_p$ connected to VNF Server i
p	User Groups $p \in \mathcal{U}$
b_{ip}	Bandwidth requested by $i \in \mathcal{S}$ to serve users from p
M	Number of VNF Servers
$p_{ip}^{(\mathcal{F})}$	Price imposed by $i \in \mathcal{S}$ to $p \in \mathcal{U}$ to access VNF $f_p \in \mathcal{F}$
N_p	Population size of User Group p
$p_i^{(\mathcal{B})}$	Bandwidth-unit price imposed by the TO to VNF Server i
$\mathcal{F}$	Set of VNFs
$U_{ip}^{(\mathcal{U})}$	Utility function of a user in p connected to VNF Server i
$f_p \in \mathcal{F}$	VNF function requested by User Group p
γ_m	Step-size of the learning procedure
c_{ip}	Cost of $i \in \mathcal{S}$ to serve a single user in $p \in \mathcal{U}$
$\alpha_1,\ \alpha_2,\ \alpha_3$ $\beta_1,\ \beta_2$	Weighing parameters

- Exposing a list of VNFs that the TO wants to provide to its Users;
- Providing the VNF Servers with the VNF templates, containing the deployment and operational behavior requirements necessary to realize each VNF and manage its lifecycle;
- Assigning a slice of bandwidth to the VNF Servers according to their bandwidth request;
- Providing each User with the current list of VNF Servers that are running the requested VNFs, including information regarding the price applied by each VNF Server and the relevant performance parameters, in terms of experienced latency and received bandwidth.
- Allowing Users to choose a VNF Server for each requested VNF function by setting the flow table of the SDN switches in the TO network in such a way that User flows traverse the chosen VNF Servers.

Policies for management and orchestration of the resources are a key element of the system as they strongly influence its performance. The main performance parameter that depends on the applied policies is the latency. In fact, placing a VNF on a specific node in the network determines that all the flows using it have to pass through that node; thus, if that node is very far from the sources of some flows, latency may result unacceptable for them. It is evident that each VNF Server is characterized by almost the same performance latency parameter for all the Users that enter the network through the same PE node, or through different PE nodes which are close to each other in the core network, i.e., are connected to each other by high-speed links of a few miles. Another important parameter is the bandwidth that each VNF Server provides to Users, which depends on both the amount of bandwidth the VNF Server requests to the TO and the number of User flows using its VNFs.

In the following, we use the term *User Group* to indicate the set of Users requesting the same VNF, which are characterized by the same latency from the VNF Servers providing that VNF, and exhibit the same requirements in terms of delay and bandwidth.

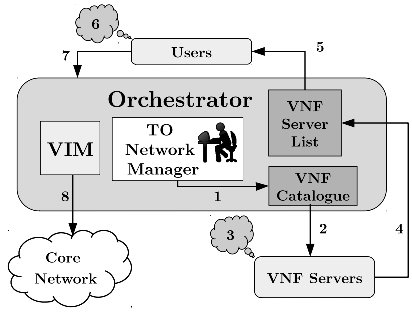


Fig. 3. Management process flow diagram.

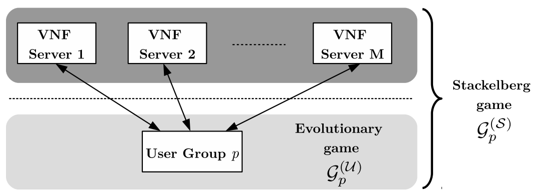


Fig. 4. The proposed game-theoretic framework for User Group $p \in \mathcal{U}$.

A. Distributed System Management

In this section, we describe the management framework of the distributed reference system. Let us define $\mathcal{F}$ as the set of VNFs provided by the TO. The main entities involved in the management operations from its onboarding to its usage by User flows are sketched in Fig. 3. Accordingly, for each VNF $f \in \mathcal{F}$, the relative steps can be synthesized as follows:

1) The Network Manager (i.e. a human operator) creates a VNF template for f. All templates are onboarded and stored on the Orchestrator, and listed in a VNF Catalogue;
2) Servers that are interested in providing the VNF f, download the template from the Orchestrator, and create an instance of f, assigning to it a set of local resources.
3) Each VNF Server individually decides the amount of bandwidth required to serve its Users, participating to the game $\mathcal{G}^{(\mathcal{S})}$ described in Section III-B;
4) Each VNF Server that has launched the VNF f, requests to the Orchestrator to be registered on the VNF Server list as providing the VNF f, also specifying the decided price to be applied to the Users and the requested bandwidth;
5) Each User that is interested in the VNF f, in the following indicated as User u, contacts the Orchestrator to receive information concerning the VNF market of f. More specifically, the User receives the list of all the VNF Servers that are running f and, for each of them, the bandwidth that this VNF Server would assign to u, the price that it has decided to be applied to the Users of the same User Group, and the IP address of the considered VNF server, so that the User u can autonomously derive the latency from it.
6) Thanks to the information received in the previous item, the User u chooses to which VNF Server to connect to. This is done by participating to the game $\mathcal{G}^{(\mathcal{U})}$ that will be introduced in Section III-A.
7) The User u communicates to the Orchestrator the VNF Server chosen for f during the previous step.
8) The Virtual Infrastructure Manager (VIM) block of the Orchestrator configures the SDN switches in the TO network in such a way that the specific flow for which the User u has chosen a given Server, traverses this Server.

B. The Market Model

Let us now discuss the market model that supports the above management framework[2]. Let $\mathcal{F}$ be the set of VNFs provided by the network, and $\mathcal{U}$ the set of User Groups. For each VNF $f_p \in \mathcal{F}$, let $p \in \mathcal{U}$ a User Group composed by N_p Users that are interested in f_p, and $\mathcal{S}$ be the set of VNF Servers that provide it.

Let d_{ip} be the latency encountered by the flows of the Users belonging to p to reach the VNF Server $i \in \mathcal{S}$.

It is realistic to assume that User Groups act as distinct *tenants* which are allowed in principle to connect to the same VNF Server, but, for security reasons, cannot mix their traffic with those generated by other User Groups. Accordingly, requests from each User Group p have to be individually accommodated.

The entities that will participate to this market are the VNF Servers that have installed and run f_p, the Users that need f_p for some of their flows, and the Orchestrator that has decided to provide its customers with f_p and that intend to gain some economic benefit from it.

For each User flow traversing a given VNF Server, this Server has to allocate a given amount of computing and storage resources, and this represents a cost for the VNF Server. Considering the generic VNF Server $i \in \mathcal{S}$, we will refer to the incremental cost incurred by the VNF Server i to guarantee the required resources to a new flow requesting function f_p as c_{ip}. For example, the cost c_{ip} may be an energy cost as in [24], that depends on the price applied by the energy provider.

Therefore, the cost for a VNF Server i to manage all the User flows in p, $C_{ip}^{(\mathcal{F})}$, is proportional to the number of flows n_{ip}, that is

$$C_{ip}^{(\mathcal{F})} = c_{ip} \cdot n_{ip} \tag{1}$$

Another cost for the VNF Servers is due to the bandwidth that they receive from the TO network according to the requests issued to the Orchestrator. Let b_{ip} be the bandwidth received by the VNF Server i to manage the User Group p, and $p_i^{(\mathcal{B})}$ the bandwidth-unit price applied by the TO network to the VNF Server i. Note that the value of $p_i^{(\mathcal{B})}$ does not depend on the User Group. Accordingly, the cost of the overall bandwidth used by that VNF Server is:

$$C_{ip}^{(\mathcal{B})} = p_i^{(\mathcal{B})} \cdot b_{ip} \tag{2}$$

On the other hand, the revenue for the VNF Server i associated to the provision of VNF f_p is proportional to both the number n_{ip} of Users that are using it, and the price $\hat{p}_{ip}^{(\mathcal{F})}$ applied by this VNF Server. Now, if we assume that the VNF Servers have to pay a commission (or fee) to the TO, represented by the *commission parameter* $\psi \in [0, 1]$, the actual revenue of the

[2] In Table I we provide a list of the symbols used throughout the paper with their meaning.

VNF Server i related to the provision of VNF f_p to the User Group p is

$$R_{ip} = p_i^{(\mathcal{F})} \cdot n_{ip} \quad (3)$$

where $p_i^{(\mathcal{F})} = \hat{p}_{ip}^{(\mathcal{F})}(1-\psi)$.

The mechanism to decide the amount of bandwidth that each VNF Server requests to the TO network will be discussed in Section III. It is aimed at maximizing a utility function defined as follows:

$$U_{ip}^{(\mathcal{S})}(\mathbf{b}_p) = \beta_1 R_{ip} - \beta_2 \left[C_{ip}^{(\mathcal{F})} + C_{ip}^{(\mathcal{B})}\right] \quad (4)$$

where $\mathbf{b}_p = (b_{1p}, b_{2p}, \ldots, b_{Mp})$ is the *bandwidth vector* that contains the bandwidth b_{ip} requested to the TO network by each VNF Server, and β_1 and β_2 are appropriate constants weighing the relative relevance of revenues and costs.

On the other hand, Users in User Group p, choose the VNF Server by also taking into account the latency experienced to reach it, d_{ip}, and the current price it is applying to the VNF f_p. However, the higher the number of User flows using the same VNF Server, the lower the bandwidth allocated to each of them. Specifically, the benefit function of each user in p is expected to be increasing in the amount of resource allocated to that user, i.e., b_{ip}/n_{ip}; and to be decreasing in both the price $\hat{p}_{ip}^{(\mathcal{F})}$, and the latency d_{ip}. Specifically, and in line with standard economic assumptions [26], we assume that Users experience *diminishing returns* as the value of b_{ip}/n_{ip} increases. Such an assumption can be modeled through concave functions, e.g., logarithmic functions, and has been widely used in economics theory to reflect the concept of risk-aversion or satisfaction behavior of rational decision-makers.

With all this in mind, each User selects the VNF Server that maximizes the following utility function [26, 27]:

$$U_{ip}^{(\mathcal{U})}(n_{ip}) = \ln\left(\alpha_1 \frac{b_{ip}}{n_{ip}}\right) - \alpha_2 \hat{p}_{ip}^{(\mathcal{F})} - \alpha_3 d_{ip} \quad (5)$$

where $\mathbf{n}_p = (n_{1p}, n_{2p}, \ldots, n_{Mp})$ is the *state vector* that contains the number n_{ip} of flows from the User Group p served by each VNF Server in $\mathcal{S}$; α_1, α_2 and α_3 are appropriate constants that weigh the contributions to the utility function of the bandwidth received, the price applied by the VNF Server, and the latency encountered to reach that Server, respectively[3]. In the following of the paper, we will refer to α_1, α_2, α_3, β_1 and β_2 as the *weighing parameters*.

C. Telco Operator revenues

An important topic that deserves particular attention is related to the revenues generated by the TO. In centralized markets such as in Cases A and B, depicted in Figs. 1(a) and 1(b), respectively, the whole amount paid by Users is received by the TO itself. Therefore, the TO is able to monopolize revenues generated by the VNF provisioning. Instead, by distributing the VNF provisioning process such as in Case C (see Fig. 1(c)), a part of the User payments go to VNF Servers because they give a commission on the sale of VNFs, and also pay the TO to get the necessary amount of bandwidth to serve its users. Accordingly, let $U_A^{(\mathrm{TO})}$, $U_B^{(\mathrm{TO})}$ and $U_C^{(\mathrm{TO})}$ be the revenues in Cases A, B and C, respectively.

In Case A, we have that all VNFs required by all User Groups are executed on a single server. Therefore, the revenue of the TO can be expressed as follows:

$$U_A^{(\mathrm{TO})} = \sum_{p \in \mathcal{U}} N_p \left(\hat{p}_p^{(\mathcal{F})} - c_p\right) \quad (6)$$

where N_p is the number of Users in User Group $p \in \mathcal{U}$, $\hat{p}_p^{(\mathcal{F})}$ is the price charged to users in User Group p to access function f_p on the centralized server, and c_p is the analogous of c_{ip} for the centralized server.

In Case B, VNFs are all provided by servers managed by the TO. In this latter case, the revenue of the TO is:

$$U_B^{(\mathrm{TO})} = \sum_{p \in \mathcal{U}} \left[N_p \hat{p}_p^{(\mathcal{F})} - \sum_{i \in \mathcal{S}^{(P)}} \left(n_{ip}^{(\mathrm{OPT})} c_{ip}\right)\right] \quad (7)$$

where $\mathcal{S}^{(P)}$ is the set of proprietary servers and $n_{ip}^{(\mathrm{OPT})}$ is the optimal number of users in the User Group p on the TO-proprietary server i, for all $p \in \mathcal{U}$ and $i \in \mathcal{S}^{(P)}$.

Finally, in Case C, VNF provisioning is distributed among different third-party VNF Servers, and the overall revenue of the TO is

$$U_C^{(\mathrm{TO})} = \sum_{p \in \mathcal{U}} \left(\sum_{i \in \mathcal{S}} C_{ip}^{(\mathcal{B})} + \psi \sum_{i \in \mathcal{S}} \hat{p}_{ip}^{(\mathcal{F})} n_{ip}\right) \quad (8)$$

The first term in (8) depends on the bandwidth requested by the VNF Servers, while the second term depends on the commissions paid by those VNF Servers to the TO.

The analysis on the efficiency of TO's revenues in all the above cases will be investigated in Section IV-E where we will show that, in many cases, distributing the VNF market is much more profitable than centralizing it.

III. Game Model

In this section, we illustrate the proposed game-theoretic model of the interactions between VNF Servers and Users in the distributed management framework.

Decisions taken by VNF Servers and Users depend on both individualistic interests, e.g., maximize their own utility, and decisions taken by counterparts, e.g., opponents' strategies. For example, Users connect to one of the available VNF Servers depending on the offered bandwidth and other relevant parameters such as proposed price and expected communication delay. On the contrary, VNF Servers aim to maximize their revenues and are not likely to cooperate with each other. Also, their actions depend on the number of Users that are connected to them to use their VNFs.

In real scenarios, VNF Servers naturally act and make decisions by anticipating the Users. Accordingly, interactions among VNF Servers and Users can be modeled as a two-stage Stackelberg game where VNF Servers act as the *leaders* of the game and Users as the *followers*. In the addressed problem we should also consider Users that replicate other Users' decisions. Such replicative behavior naturally arises in those scenarios

[3]Note that the use of a logarithmic function in (5) is justified by the fact that such class of functions have been shown to be proportionally fair.

where multiple entities make decisions by replicating other Users' behavior [28–31].

In Section III-A, we first define a game $\mathcal{G}_p^{(\mathcal{U})}$ among users of the same User Group $p \in \mathcal{U}$ where we exploit Evolutionary Game Theory (EGT) and replicator dynamics to model the decision-making process of Users. Then, in Section III-B, we use non-cooperative game theory to define the game $\mathcal{G}_p^{(\mathcal{S})}$ which models competitive interactions among the VNF Servers to serve users in the User Group p. Finally, in Section III-C we propose a distributed and privacy preserving reinforment learning procedure to compute the equilibrium of the game $\mathcal{G}_p^{(\mathcal{S})}$.

The illustrated games will be played each time some conditions of the system change. More specifically, a variation of either the latency of the VNF Servers from the Users of a User Group, or the price decided by one of the VNF Servers for a given function f_p, or the number of Users interested in the function f_p, determines a variation in the utility function of some User, and this stimulates the Users to start playing the $\mathcal{G}_p^{(\mathcal{U})}$ game to change the Server. The consequent distribution variation of the Users on the Servers, in its turn, stimulates the Servers to play the $\mathcal{G}_p^{(\mathcal{S})}$ game in order to decide a modification of the bandwidth to be requested to the TO network. The games are iterated until all the entites in the system reach a new steady state. In Section IV we will numerically analyze this transient period, and show that it lasts some 10 iterations.

The considered game-theoretic model and its hierarchical structure are shown in Fig. 4.

A. Evolutionary game $\mathcal{G}_p^{(\mathcal{U})}$ among Users

Each User is intrinsically selfish as it makes decisions with the aim of maximizing its own utility $U_{ip}^{(\mathcal{U})}$, as defined in (5). However, the higher the number n_{ip} of Users in the User Group p connected to the i-th VNF Server, the lower the utility $U_{ip}^{(\mathcal{U})}$ of that User. Therefore, the decision-making process of each User is also influenced by decisions taken by the other Users belonging to the same User Group p. Also, if a User is aware that another User is achieving a better utility, he can decide to imitate that User and migrate to the same VNF Server to which that User is connected [32]. In the rest of this paper, we refer to this phenomenon as *imitation behavior*.

Imitation behavior often arises when considering interactions among entities that rationally try to maximize their benefit by imitating other entities' decisions that provide better benefit. For example, imitation is at the basis of a variety of decision making problems in both wired and wireless networks [29–31] that are often modeled by exploiting theoretical tools from evolutionary game theory.

In line with a vast body of literature, we consider the well-known and widely used *replicator dynamics* [33] as the imitation dynamics which describe the interactions among Users. Accordingly, for each User Group $p \in \mathcal{U}$, we define the evolutionary game $\mathcal{G}_p^{(\mathcal{U})}$ as follows:

- *Population*: it consists of the set of the N_p Users in User Group $p \in \mathcal{U}$.
- *Strategy*: it is defined as the choice of the VNF Server $i \in \mathcal{S}$ to whom each User in the population p decides to connect; the *strategy set* of each User is $\mathcal{S}$.
- *Utility*: the utility, or benefit, achieved by each User connected to the VNF Server $i \in \mathcal{S}$ is equal to $U_{ip}^{(\mathcal{U})}$ as defined in (5).

We can now define the *replicator equation* that describes how the number of Users in the population p that connect to available VNF Servers varies

$$\dot{n}_{ip} = n_{ip}\left[U_{ip}^{(\mathcal{U})}(n_{ip}) - \frac{1}{N_p}\sum_{j\in\mathcal{S}} n_{jp}U_{jp}^{(\mathcal{U})}(n_{ip})\right] \quad (9)$$

where $n_{ip} \in \mathbf{n}_p$ denotes the number of Users in the User Group p which have chosen as a strategy to connect to the i-th VNF Server.

The first term in the right-hand side of (9) represents the utility of a User that connects to the i-th VNF Server, while the second term represents the average utility of the population which depends on the current distribution $\mathbf{n}_p$ of the population. Therefore, the growth rate $\dot{n}_{ip}/n_{ip}$ of the number of Users in the User Group p connected to the i-th VNF Server is equal to the difference between the benefit when choosing the strategy i, and the average benefit of the whole population.

A general result from EGT shows that an equilibrium point for the replicator dynamics is a fixed point of the replicator dynamics such that all Users experience the same benefit, i.e., $U_{ip}^{(\mathcal{U})} = U_{jp}^{(\mathcal{U})}$ for all $i, j \in \mathcal{S}$.

In Proposition 1, we will show that the replicator equation (9) for each User Group p admits a unique solution for any bandwidth vector $\mathbf{b}_p$. Furthermore, we characterize the equilibrium point by deriving the resulting state vector $\mathbf{n}_p^*$ at the equilibrium. To this purpose, and for notation purposes, let us define an auxiliary variable $\phi_{i,j}^{(p)}$ as follows:

$$\phi_{i,j}^{(p)} = e^{\left[\alpha_2\left(\hat{p}_{ip}^{(\mathcal{F})} - \hat{p}_{jp}^{(\mathcal{F})}\right) + \alpha_3(d_{ip} - d_{jp})\right]} \quad (10)$$

From (10), it can be easily shown that the following relationships hold for all $i, j, k \in \mathcal{S}$ and $p \in \mathcal{U}$

$$\phi_{i,i}^{(p)} = 1, \quad \phi_{i,j}^{(p)} = 1/\phi_{j,i}^{(p)}, \quad \text{and} \quad \phi_{k,j}^{(p)} = \frac{\phi_{i,j}^{(p)}}{\phi_{i,k}^{(p)}} \quad (11)$$

Proposition 1. *For all $p \in \mathcal{U}$ and any given bandwidth vector $\mathbf{b}_p$, the replicator equation* (9) *admits a unique evolutionary equilibrium $\mathbf{n}_p^*$. Also, the number of Users n_{ip}^* in User Group p connected to the generic VNF Server $i \in \mathcal{S}$ at the equilibrium point can be derived as follows:*

$$n_{ip}^* = N_p \frac{b_{ip}}{\sum_{j\in\mathcal{S}} b_{jp}\phi_{i,j}^{(p)}} \quad (12)$$

where $b_{ip} \in \mathbf{b}_p$.

Proof: The replicator equation can be reduced to an equivalent system of ordinary differential equations (ODEs). Thus, to show that the replicator dynamics admits a unique equilibrium point, it suffices to note that the right-hand side of the mean dynamic in (9) is continuously differentiable. Therefore, Lipschitz continuity, and thus uniqueness of the equilibrium, follow [21].

Now, in order to determine the unique equilibrium, it is well known that it is reached when $\dot{n}_{ip} = 0$. Such condition implies that $U_{ip}^{(\mathcal{U})} = U_{jp}^{(\mathcal{U})}$ for all $i, j \in \mathcal{S}$, i.e., all Users receive the same benefit. Accordingly, we can build a system of equations with $N_p(N_p - 1)/2$ equations that can be solved by exploiting the relationship $N_p = \sum_{i\in\mathcal{S}} n_{ip}$. Thus, after some easy analytical derivations, we obtain the result in (12).

For the sake of illustration, in the following we show how to derive (12) when $N_p = 2$. However, the more general case can be treated in a similar way. From (5) and (10), and by imposing $U_{1p}^{(\mathcal{U})} = U_{2p}^{(\mathcal{U})}$ we get

$$\ln\left(\frac{b_{1p}}{b_{2p}}\frac{n_{2p}}{n_{1p}}\right) = \ln(\phi_{1,2}^{(p)}) \tag{13}$$

Recall that $N_p = n_{1p} + n_{2p}$. Thus, we get

$$n_{1p}^* = N_p \frac{b_{1p}}{b_{1p} + b_{2p}\phi_{1,2}^{(p)}} \quad \text{and} \quad n_{2p}^* = N_p \frac{b_{2p}}{b_{2p} + b_{1p}\phi_{2,1}^{(p)}}$$

which is a specific case of (12). ■

An important result that stems from the uniqueness of the equilibrium point is that the replicator dynamics converges towards a unique stable point. Therefore, uniqueness avoids possible oscillations among two or more equilibrium points. Also, since the equilibrium is unique, complex equilibrium selection mechanisms to determine the most efficient NE in the set of the feasible multiple NEs are not required.

To estimate the replicator equation in (9), each User in the User Group p has to evaluate its utility $U_{ip}^{(\mathcal{U})}(n_{ip})$ and get the average utility $\frac{1}{N_p}\sum_{j\in\mathcal{S}} n_{jp}U_{jp}^{(\mathcal{U})}(n_{ip})$ of the group. The utility $U_{ip}^{(\mathcal{U})}(n_{ip})$ is defined in (5) and only requires the value of the ratio b_{ip}/n_{ip}, i.e., the amount of bandwidth that will be assigned to the User. As already stated at point 5) in Section II.A, it is worth noting that the value of b_{ip}/n_{ip} is already available in the VNF Server List shown in Fig. 3. Instead, since no User is aware of the strategies of the other Users in p, the average utility of Users in p is broadcast by the Orchestrator to all Users. It is important to note that the above values do not carry any private information about decisions taken by other Users. In fact, users do not know the number N_p of Users in the group, therefore it is not possible to derive any private information from both b_{ip}/n_{ip} and $\frac{1}{N_p}\sum_{j\in\mathcal{S}} n_{jp}U_{jp}^{(\mathcal{U})}(n_{ip})$.

B. Stackelberg game $\mathcal{G}_p^{(\mathcal{S})}$ between VNF Servers and Users

As already discussed before, VNF Servers act as leaders of the game between VNF Servers and Users. Also, in Proposition 1 we have derived the distribution $\mathbf{n}_p^*$ of the population $p \in \mathcal{U}$ at the equilibrium of the replicator dynamics.

For the sake of notation, let us first define the two following auxiliary variables

$$\tilde{p}_{ip} = N_p\left(\beta_1 p_{ip}^{(\mathcal{F})} - \beta_2 c_{ip}\right) \tag{14}$$

and

$$\pi_i = \beta_2 p_i^{(\mathcal{B})} \tag{15}$$

Accordingly, we can incorporate (3), (12), (14) and (15) in (4) to rewrite the utility function $U_{ip}^{(\mathcal{S})}$ of the generic VNF Server $i \in \mathcal{S}$ as follows:

$$U_{ip}^{(\mathcal{S})}(\mathbf{b}_p) = \tilde{p}_{ip}\frac{b_{ip}}{\sum_{k\in\mathcal{S}} b_{kp}\phi_{i,k}^{(p)}} - \pi_i b_{ip} \tag{16}$$

For each User Group $p \in \mathcal{U}$, we define the non-cooperative game $\mathcal{G}_p^{(\mathcal{S})}$ as follows:

- *Player set*: it consists of the set $\mathcal{S}$ of VNF Servers.
- *Strategy*: it is defined as the amount of bandwidth b_{ip} to be requested to the TO network to serve its connected Users from User Group p. For each User Group in $\mathcal{U}$, we assume that such amount of bandwidth is bounded by B_i. Thus, the strategy set is $\mathcal{B} = \prod_{i\in\mathcal{S}} \mathcal{B}_i$, where $\mathcal{B}_i = [0, B_i]$ and $\prod$ identifies the Cartesian product[4].
- *Utility*: the utility of each VNF Server $i \in \mathcal{S}$ is equal to $U_{ip}^{(\mathcal{S})}$ as defined in (16).

By calculating the first-order derivative of (16), it can be easily shown that $\tilde{p}_{ip} \leq 0$ leads to a non-positive first-order derivative of the utility function $U_{ip}^{(\mathcal{S})}$. In other words, the best strategy for the i-th VNF Server is not to participate in the game $\mathcal{G}_p^{(\mathcal{S})}$ for User Group p, i.e., $b_{ip} = 0$. Therefore, those VNF Servers with $\tilde{p}_{ip} \leq 0$ exit the game and they can be removed from the player set $\mathcal{S}$. Accordingly, without loss of generality, in our model we assume that the player set $\mathcal{S}$ is composed by only those VNF Servers such that $\tilde{p}_{ip} > 0$.

In the following, we analyze the Stackelberg game $\mathcal{G}_p^{(\mathcal{S})}$ and provide useful results about its equilibrium points, referred to as SEs.

Definition 1. *Let $\mathbf{b}_p^* \in \mathcal{B}$. The strategy profile $(\mathbf{b}_p^*, \mathbf{n}_p^*)$ is a SE for the game $\mathcal{G}_p^{(\mathcal{S})}$ if for all $\mathbf{b}_p \in \mathcal{B}$ and $i \in \mathcal{S}$, we have*

$$U_{ip}^{(\mathcal{S})}(\mathbf{b}_p^*, \mathbf{n}_p^*) \geq U_{ip}^{(\mathcal{S})}(\mathbf{b}_p, \mathbf{n}_p^*)$$

where $\mathbf{n}_p^*$ is defined as in (12)*.*

Definition 2. *Let $\mathbf{b}_p^* = (b_{ip}^*, \mathbf{b}_{p_{-i}}^*)$, where $\mathbf{b}_{p_{-i}}^*$ is the bandwidth vector of all players except i, i.e., $\mathbf{b}_{p_{-i}}^* = (b_{jp}^*)_{j\in\mathcal{S},\, j\neq i}$ with $b_{jp}^* \in \mathbf{b}_p^*$. The strategy $\mathbf{b}_p^* = (b_{1p}^*, b_{2p}^*, \ldots, b_{Mp}^*)$ is said to be the "Stackelberg strategy" for the game $\mathcal{G}_p^{(\mathcal{S})}$ if for all $i \in \mathcal{S}$ we have that*

$$b_{ip}^* = \arg\max_{b_{ip}\in\mathcal{B}_i} U_{ip}^{(\mathcal{S})}(b_{ip}, \mathbf{b}_{p_{-i}}^*, \mathbf{n}_p^*)$$

Also, the value $U_{ip}^{(\mathcal{S})}(\mathbf{b}_p^*, \mathbf{n}_p^*)$ is denoted as the "Stackelberg utility" of leader i in game $\mathcal{G}_p^{(\mathcal{S})}$.

In Proposition 2, we prove that the game $\mathcal{G}_p^{(\mathcal{S})}$ admits a unique SE.

Proposition 2. *The game $\mathcal{G}_p^{(\mathcal{S})}$ admits a unique SE.*

Proof: The main steps of the proof are as follows. First, we prove the existence of the equilibrium by exploiting concavity properties of VNF Server utility functions in (16). Then,

[4]We do not consider the variable $\hat{p}^{(\mathcal{F})}$ as a strategy for the VNF Server as we limit our study to the case where, while the bandwidth varies in time, the pricing policy remains constant during each game execution.

we show that the Diagonal Strict Concavity (DSC) property holds. The DSC property implies that VNF Servers experience diminishing returns along any direction, i.e., along all $b_{ip} \in \mathbf{b}_p$. Finally, we exploit results contained in [34, 35] to prove that a unique equilibrium exists.

Let the *marginal utility* $v_{ip}(\mathbf{b}_p)$ of each player $i \in \mathcal{S}$ be defined as $v_{ip}(\mathbf{b}_p) = \frac{\partial U_{ip}^{(\mathcal{S})}(\mathbf{b}_p)}{\partial b_{ip}}$. Therefore, from (16) it follows that the marginal utility of the generic VNF Server i is

$$v_{ip}(\mathbf{b}_p) = \tilde{p}_{ip} \frac{\sum_{k \in \mathcal{S}, k \neq i} b_{kp} \phi_{i,k}^{(p)}}{\left(\sum_{k \in \mathcal{S}} b_{kp} \phi_{i,k}^{(p)}\right)^2} - \pi_i \tag{17}$$

where $\tilde{p}_{ip}$ is defined in (14).

To show that the DSC property holds, it must be shown that: i) $U_{ip}^{(\mathcal{S})}(\mathbf{b}_p)$ is strictly concave in b_{ip}; ii) $U_{ip}^{(\mathcal{S})}(\mathbf{b}_p)$ is convex in $\mathbf{b}_{p_{-i}}$; and iii) the function $\rho(\mathbf{b}_p, \mathbf{r}_p)$ defined as

$$\rho(\mathbf{b}_p, \mathbf{r}_p) = \sum_{i \in \mathcal{S}} r_{ip} U_{ip}^{(\mathcal{S})}(\mathbf{b}) \tag{18}$$

is concave in $\mathbf{b}_p$ for some $\mathbf{r}_p = (r_{1p}, r_{2p}, \ldots, r_{Mp})$ such that $r_{ip} > 0$ for all $i \in \mathcal{S}$.

From (16), it can be shown that property i) holds as $U_{ip}^{(\mathcal{S})}(\mathbf{b}_p)$ is defined as the difference between a strictly concave function and a concave function. To prove ii), it suffices to note that the Hessian matrix of $U_{ip}^{(\mathcal{S})}(\mathbf{b}_p)$ has all non-negative eigenvalues, i.e., the Hessian matrix is positive semidefinite.

Let $r_{ip} = 1/\tilde{p}_{ip}$ for all $i \in \mathcal{S}$. Accordingly, (18) can be rewritten as follows:

$$\begin{aligned} \rho(\mathbf{b}_p, \mathbf{r}_p) &= \sum_{i \in \mathcal{S}} \frac{b_{ip}}{b_{ip} + \sum_{j \neq i} b_{jp} \phi_{i,j}^{(p)}} - \sum_{i \in \mathcal{S}} r_{ip} \pi_i b_{ip} \\ &= \frac{b_{1p}}{b_{1p} + \sum_{j \neq 1} b_{jp} \phi_{1,j}^{(p)}} + \sum_{k \neq 1} \frac{b_{kp}}{b_{kp} + \sum_{j \neq k} b_{jp} \phi_{k,j}^{(p)}} \\ &\quad - \sum_{i \in \mathcal{S}} r_{ip} \pi_i b_{ip} \end{aligned} \tag{19}$$

From (11), we have that

$$\begin{aligned} \rho(\mathbf{b}_p, \mathbf{r}_p) &= \frac{b_{1p}}{b_{1p} + \sum_{k \neq 1} b_{kp} \phi_{1,k}^{(p)}} + \sum_{k \neq 1} \frac{b_{kp} \phi_{1,k}^{(p)}}{b_{1p} + \sum_{j \neq 1} b_{jp} \phi_{1,j}^{(p)}} \\ &- \sum_{i \in \mathcal{S}} r_{ip} \pi_i b_{ip} = 1 - \sum_{i \in \mathcal{S}} r_{ip} \pi_i b_{ip} \end{aligned} \tag{20}$$

Observe that $\rho(\mathbf{b}_p, \mathbf{r}_p)$ is a concave function in $\mathbf{b}_p$ as required in iii). Therefore, we have that DSC property holds and the general theory in [34, 35] ensures the uniqueness of the equilibrium. ■

In (16), we use the equilibrium condition in (12). Therefore, interactions between Users (i.e., the followers) and VNF Servers (i.e., the leaders) modeled through the game $\mathcal{G}_p^{(\mathcal{S})}$ produce a unique SE $(\mathbf{b}_p^*, \mathbf{n}_p^*)$. However, recall that VNF Servers compete with each other in the Stackelberg game. Accordingly, the strategy profile $\mathbf{b}_p^*$ discussed above also represents a Nash Equilibrium (NE) [35] for the competitive game among VNF Servers.

Algorithm 1 Exponential Reinforcement Learning (XL)

Parameter: step-size sequence γ_m (default: $\gamma_m = 1/m$).
Initialize: $m \leftarrow 0$; $z_{ip} \leftarrow 0$ for all $i \in \mathcal{S}$.
Repeat
    $m \leftarrow m + 1$;
    for each VNF Server $i \in \mathcal{S}$ **do simultaneously**
        requested bandwidth $b_{ip} \leftarrow B_i \left[1 + \exp(-z_{ip})\right]^{-1}$;
        measure marginal utility v_{ip} from (17);
        update scores: $z_{ip} \leftarrow z_{ip} + \gamma_m v_{ip}$;
    until termination criterion is reached.

C. *Reinforcement Learning Procedure for game $\mathcal{G}_p^{(\mathcal{S})}$*

In Proposition 2, we have shown that the game $\mathcal{G}_p^{(\mathcal{S})}$ admits a unique equilibrium. Unfortunately, we are not able to find a proper characterization of the equilibrium and provide closed-form expressions. Thus, we need to provide a robust mechanism to allow VNF Servers to individually reach the equilibrium of the game. Accordingly, in the following we propose an *exponential reinforcing learning* [36, 37] procedure, which provably converges to the unique equilibrium of the game.

For each VNF Server $i \in \mathcal{S}$ that serves Users in the User Group $p \in \mathcal{U}$, we define the following learning procedure

$$\begin{cases} z_{ip}(m+1) = z_{ip}(m) + \gamma_m v_{ip}\left(\mathbf{b}_p(m)\right) \\ b_{ip}(m+1) = B_i \frac{e^{z_{ip}(m+1)}}{1 + e^{z_{ip}(m+1)}} \end{cases} \tag{21}$$

where m represents the iteration index, $\mathbf{b}_p(m)$ is the bandwidth vector at iteration m, and γ_m is the step-size of the learning procedure whose importance will be explained later. For each User Group $p \in \mathcal{U}$, the algorithmic implementation of (21) is shown in Algorithm 1.

In the following Proposition 3, we show that the proposed exponential reinforcing learning procedure converges to the equilibrium of the game.

Proposition 3. *Let γ_m be the step-size of the learning procedure and $\sum_m \gamma_m^2 < \sum_m \gamma_m = +\infty$. For any feasible initial condition in $\mathcal{B}$ and User Group $p \in \mathcal{U}$, Algorithm 1 always converges to the unique SE of $\mathcal{G}_p^{(\mathcal{S})}$.*

The proof consists in showing that i) the *mean dynamic* of (21), i.e., its continuous-time version, converges to the equilibrium of the game as time goes to infinity, and ii) (21) is an *asymptotic pseudo-trajectory* (APT) [38] for the continuous-time version of (21). For a detailed and rigorous proof, we refer the reader to Appendix A.

From Proposition 3, we have that any variable step-size rule in the form $\gamma_m = 1/m^\beta$ with $\beta \in (0.5, 1]$ will converge to the unique SE of the game $\mathcal{G}_p^{(\mathcal{S})}$.

Let us note that, in order to compute $b_{ip}(m+1)$ in (21), each VNF Server $i \in \mathcal{S}$ is required to know $v_{ip}\left(\mathbf{b}_p(m)\right)$ in (17), which only depends on the term $\sum_{k \in \mathcal{S}} b_{kp}(m) \phi_{i,k}^{(p)}$. To this purpose, the Orchestrator has full access to the VNF Server parameters (e.g., d_{ip}, $p_{ip}^{(\mathcal{F})}$, etc...). Then, at each iteration and for each VNF Server $i \in \mathcal{S}$, the Orchestrator is able to compute the overall sum $\sum_{k \in \mathcal{S}} b_{kp}(m) \phi_{i,k}^{(p)}$ and send it to the corresponding i-th VNF Server. Note that, by so doing, the i-th VNF Server cannot extract any private information on other VNF Servers from the sum $\sum_{k \in \mathcal{S}} b_{kp}(m) \phi_{i,k}^{(p)}$. Thus, it

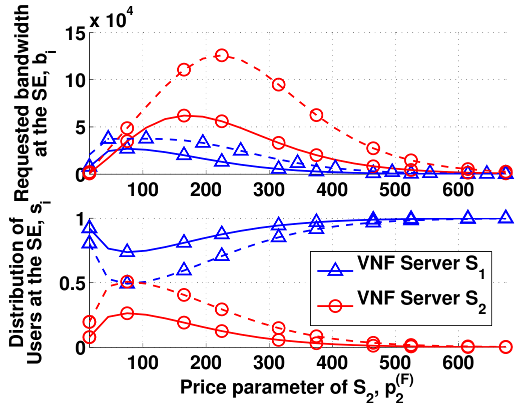


Fig. 5. Requested bandwidth and population distribution at the equilibrium as a function of the price $p_2^{(\mathcal{F})}$ charged by S_2 (Solid lines: $d_1 = 5$ DUs and $d_2 = 40$ DUs; Dashed lines: $d_1 = d_2 = 40$ DUs)

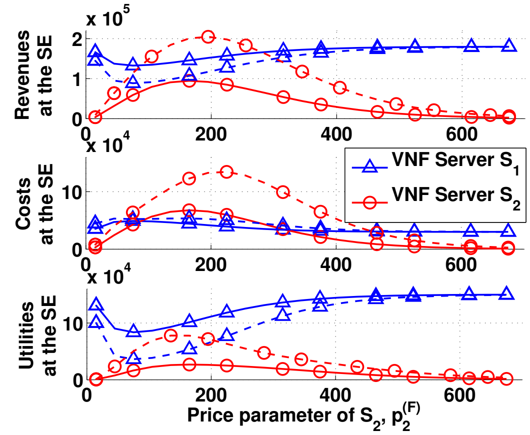


Fig. 6. Revenues, costs and utilities of VNF Servers S_1 and S_2 as a function of the price $p_2^{(\mathcal{F})}$ charged by S_2 (Solid lines: $d_1 = 5$ DUs and $d_2 = 40$ DUs; Dashed lines: $d_1 = d_2 = 40$ DUs).

means that the learning procedure (21) can be implemented in a privacy-preserving and distributed fashion.

IV. Numerical analysis

In this section, we present a numerical analysis of the proposed distributed orchestration and resource allocation scheme.

In our simulations, we assume a population size of $N = 3000$ Users and, unless otherwise stated, we consider the following weighing parameters: $\alpha_1 = 1, \alpha_2 = 0.015, \alpha_3 = 0.035$, $\beta_1 = 1$ and $\beta_2 = 1$. Finally, and unless explicitly mentioned otherwise, we assume that the commission parameter is $\psi = 0$, and the bandwidth-unit price $p_i^{(\mathcal{B})}$ is equal for all VNF Servers in $\mathcal{S}$ and is set to $p_i^{(\mathcal{B})} = 1$ Price Units (PUs). For illustrative purposes, we primarily focus on the two VNF Servers case (i.e., $M = 2$) as it allows us to highlight the dynamics of the interactions among Users and VNF Servers together with the impact of the various system parameters on the outcome of the game $\mathcal{G}^{(\mathcal{S})}$. Moreover, we also provide extensive results also for the case $M > 2$, which makes possible to show the feasibility of the proposed learning procedure and to analyze the impact of latency when multiple VNF Servers are willing to provide the considered VNF.

A. Impact of pricing on the SE

In this section, we preliminarily study the impact of the pricing applied by the VNF Servers. To this purpose, in Fig. 5 we show the outcome of the game as a function of the price $p_2^{(\mathcal{F})}$ charged by VNF Server S_2 to its Users, when the price applied by VNF Server S_1 is assumed constant and equal to $p_1^{(\mathcal{F})} = 60$ PUs. Specifically, we show the amount of bandwidth b_i that each VNF Server requests to the TO network and the number n_i of Users that connect to each VNF Server at the SE. Also, we consider two different configurations of the latencies d_i experienced by Users connected to the i-th VNF Server. For the sake of generality, we will express latency in terms of Delay Units (DUs). In more detail, solid lines illustrate the outcome of the game when $d_1 = 5$ DUs and $d_2 = 40$ DUs, respectively. Instead, dashed lines refer to the case when $d_1 = d_2 = 40$ DUs. As expected, when $p_2^{(\mathcal{F})}$ is high, Users are likely to connect to VNF Server S_1 because it applies a lower price (i.e., $p_1^{(\mathcal{F})} = 60$ PUs). Accordingly, Users get higher payoffs when they connect to VNF Server S_1 independently of the experienced connection latencies d_i. On the contrary, when $p_2^{(\mathcal{F})}$ is low, in order to attract more Users, the strategy of VNF Server S_2 consists in requesting a high amount of bandwidth to the TO network. In this way, as evident from (5), the utility of the Users increases as a consequence of the increase in the shared bandwidth. Such behavior holds for values of $p_2^{(\mathcal{F})}$ that are below a given threshold, above which requesting more bandwidth is no more the optimal choice. For values of $p_2^{(\mathcal{F})}$ higher than this threshold[5], the optimal strategy of the VNF Servers consists in reducing the amount of requested bandwidth. Such behavior is motivated by the fact that an increase in the requested bandwidth causes an increase in the costs, which also leads to a reduction in the utilities achieved by the VNF Servers. Accordingly, when the cost to provide more resources to the Users is higher than the expected revenues, VNF Servers prefer to reduce the amount of shared resources to reduce costs and keep high revenues. Finally, it is worth noting that when $d_1 = 5$ DUs and $d_2 = 40$ DUs (solid lines), both VNF Servers request to the TO network a lower amount of bandwidth than in the case when $d_1 = d_2 = 40$ DUs. This is due to the fact that, when latencies are equal (or similar), there is no monopolistic behavior and VNF Servers have to compete to attract more Users, which results in higher requested bandwidth.

In Fig. 6 we show revenues, costs and utilities achieved by VNF Servers S_1 and S_2 at the SE as a function of the pricing parameter $p_2^{(\mathcal{F})}$. More in detail, revenues and costs are defined as the first and second terms in (4), respectively. Instead, utilities are equal to $U_i^{(\mathcal{S})}$, and are defined as in (4). From (3), we have that revenues achieved by each VNF Server depend on R_i and are determined by the number n_i of Users that connect to that VNF Server as shown. Accordingly, Fig. 6 shows that revenues vary according to the distribution of Users at the SE being considered in Fig. 5. On the contrary, from (1)

[5] In general, the threshold values are different for the two VNF Servers.

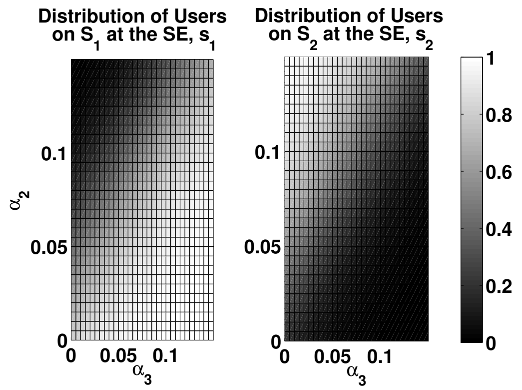


Fig. 7. Distribution of Users at the SE as function of the two weights α_2 and α_3 in (5).

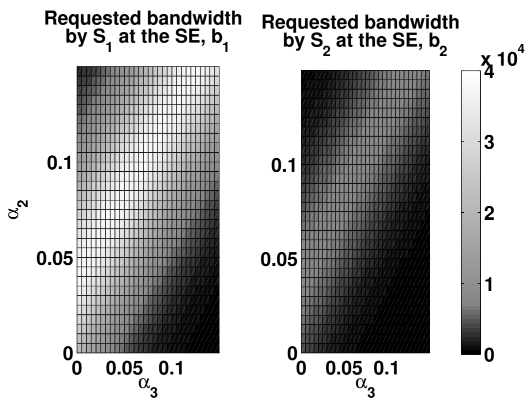


Fig. 8. Requested bandwidth of the two VNF Servers at the SE as a function of the two weights α_2 and α_3 in (5).

and (2) we have that costs depend on both the number n_i of Users connected to VNF Server S_i and the requested bandwidth b_i at the SE. Therefore, as shown in Fig. 6 the resulting trend of experienced costs is a combination of both n_i and b_i, which are shown in Fig. 5. Note that, when the value of the price parameter $p_2^{(\mathcal{F})}$ is high, the number of Users connected to S_1 asymptotically tends to N, i.e., the whole Users' population is likely inclined to connect to the VNF Server S_1 which provides better performance for a lower price $p_1^{(\mathcal{F})}$. Therefore, for high values of $p_2^{(\mathcal{F})}$, S_2 requests a small amount of bandwidth such that its revenues and costs asymptotically tend to zero.

B. Impact of the weighing parameters on the SE

In this section, we estimate the impact of the weights α_2 and α_3 that appear in (5) on the outcome of the game $\mathcal{G}^{(\mathcal{S})}$ when $M = 2$. To this purpose, in Figs. 7 and 8 we show the distribution of Users at the SE as a function of α_2 and α_3. In our simulation we have assumed $p_1^{(\mathcal{F})} = 60$ PUs, $p_2^{(\mathcal{F})} = 20$ PUs, $d_1 = 5$ DUs and $d_2 = 40$ DUs. Fig. 7 illustrates the distribution of Users at the equilibrium. When α_2 is high and α_3 is low, i.e., Users are much more concerned about the price charged by VNF Servers than the experienced latency, Users are much more attracted by the VNF Server S_2 since $p_2^{(\mathcal{F})} << p_1^{(\mathcal{F})}$. On the contrary, when α_3 is high but α_2 is low, i.e., Users weigh the experienced latency more than the cost to obtain a share of the resources, Users are more attracted by the VNF Server S_1. In Fig. 8, we show the bandwidth requested by both VNF Servers at the equilibrium as a function of α_2 and α_3. Fig. 8 shows that the behavior of both VNF Servers is similar. For example, the requested bandwidth increases for low values of both α_2 and α_3, and then decreases when either α_2 or α_3 decrease. Even though the behavior is similar, the requested bandwidth considerably differs for the two VNF Servers. In fact, Fig. 8 shows that the highest value of b_1 is $\approx 3.5 \cdot 10^4$, while the maximum value of b_2 is $\approx 1 \cdot 10^4$.

C. Time-varying and PE positioning analysis

In this section, we discuss the impact of the population size N, the cost c_i to process each flow, and the PE position on the outcome of the game $\mathcal{G}^{(\mathcal{S})}$. To this purpose, we simulated a scenario where the number of Users requesting a given VNF and the cost c_i vary in time according to realistic night/day usage patterns. More specifically, let the number of Users N and the cost c_i vary in a 48-hours long temporal window as shown in Fig. 9(a).

Fig. 9(b) illustrates both the strategy of each VNF Server, i.e., the bandwidth requested to the TO network, and the distribution of the population at the equilibrium when $p_1^{(\mathcal{F})} = p_2^{(\mathcal{F})} = 80$ PUs, $d_1 = 5$ DUs and $d_2 = 40$ DUs. Observe that when the cost to process flows is low, VNF Servers can support more User connections. Accordingly, VNF Servers request more bandwidth to the network to attract a higher number of Users. However, when $d_1 = 5$ DUs and $d_2 = 40$ DUs, even though the VNF Server S_2 provides Users with a higher amount of bandwidth, it also has a high latency. Therefore, to reduce the experienced latency, Users connect to the VNF Server S_1 and thus $n_1 > n_2$. Instead, when both VNF Servers have equal latency, i.e., $d_1 = d_2 = 40$ DUs, Fig. 9(c) shows that the majority of the population chooses the VNF Server which provides the highest amount of bandwidth.

To study the impact of the PE position, i.e., the User entrance points to the network, w.r.t. the position of VNF Servers on the outcome of the game $\mathcal{G}^{(\mathcal{S})}$, we consider five VNF Servers, i.e., $\mathcal{S} = \{S_1, S_2, S_3, S_4, S_5\}$, and five possible positions of the access PE, here denoted as PE_k with $k = 1, 2, \ldots, 5$. We assume $\hat{p}_i^{(\mathcal{F})} = 60$ PUs for all $i \in \mathcal{S}$. Each access PE position corresponds to a different latency configuration. For example, in Fig. 10(a) it is shown that VNF Servers S_1 and S_2 provide low latencies when Users access the network through provider edges PE_1 and PE_2, and high latencies when Users access through PE_4 and PE_5. For VNF Servers S_4 and S_5, what happens is exactly the opposite, whereas S_3 provides low latencies to Users independently of the position of the access PE. When Users access through PE_1 and PE_2, Fig. 10(b) shows that the majority of them decides to connect to S_1 and S_2. Thus, as shown in Fig. 10(c), to attract such an expectedly increasing number of Users, S_1 and S_2 request a high amount of bandwidth to the TO network. As expected, the contrary holds in the case Users access through PE_4 and PE_5.

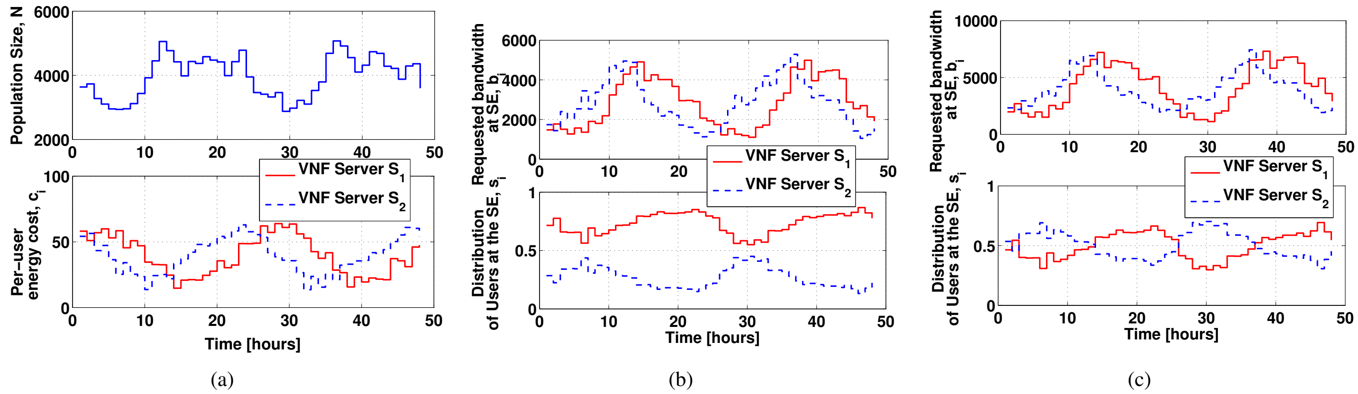


Fig. 9. a) Population size N and price parameter c_i as a function of time; b) Requested bandwidth and distribution of Users at the equilibrium as a function of time when $d_1 = 5$ DUs and $d_2 = 40$ DUs; c) Requested bandwidth and distribution of Users at the equilibrium as a function of time when $d_1 = d_2 = 40$ DUs.

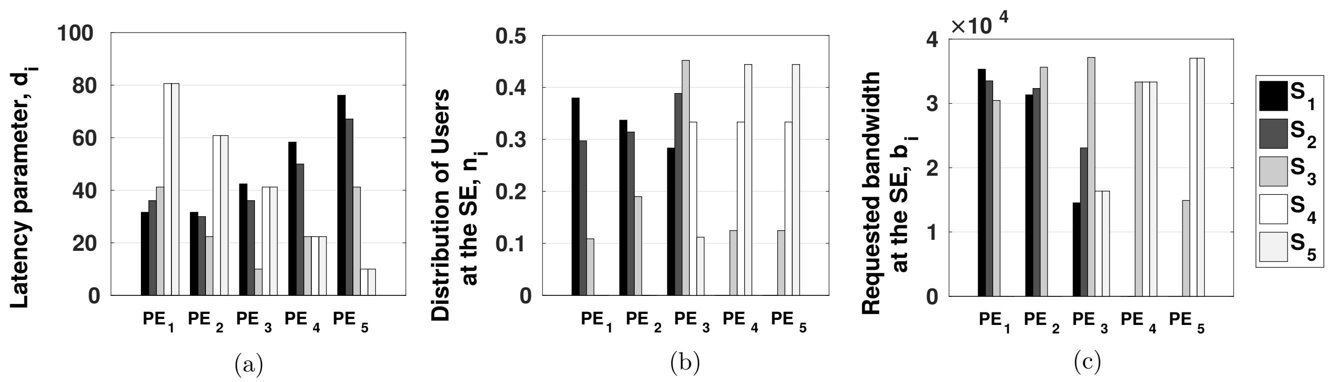


Fig. 10. Latencies (a), distribution of Users (b), and requested bandwidth (c) at the SE for different access PE positions.

D. Convergence Analysis

In this section, we investigate the convergence of the proposed learning procedure in (21). Specifically, we are interested in analyzing the convergence speed of (21) and its scalability w.r.t. the number M of VNF Servers. Results shown in this section are averaged over 100 simulation runs where we have assumed $\hat{p}_i^{(\mathcal{F})} = 900$ PUs for all $i \in \mathcal{S}$, while the latency d_i and the cost c_i parameters have been randomly generated as illustrated below. Also, for illustrative purposes we first focus on single User Group case. Instead, in the sequel we will also provide results for the case of multiple User Groups, which makes possible to show the adaptability to network configuration changes of the proposed learning procedure.

At each simulation run, the latency parameters d_i are randomly generated from two Gaussian distributions. Specifically, the first half of $M/2$ VNF Servers are associated with a Gaussian distribution with mean values $\mu_1^{(d)} = 300$ and standard deviations equal to $\sigma_1 = 1/8\mu^{(d)}$. Instead, the second half of $M/2$ VNF Servers are associated to a Gaussian distribution with $\mu_2^{(d)} = 450$ and $\sigma_2 = 3/16\mu^{(d)}$. The cost parameters c_i are generated from a Gaussian distribution with mean value $\mu^{(c)} = 850$ and standard deviation $\sigma = 30$.

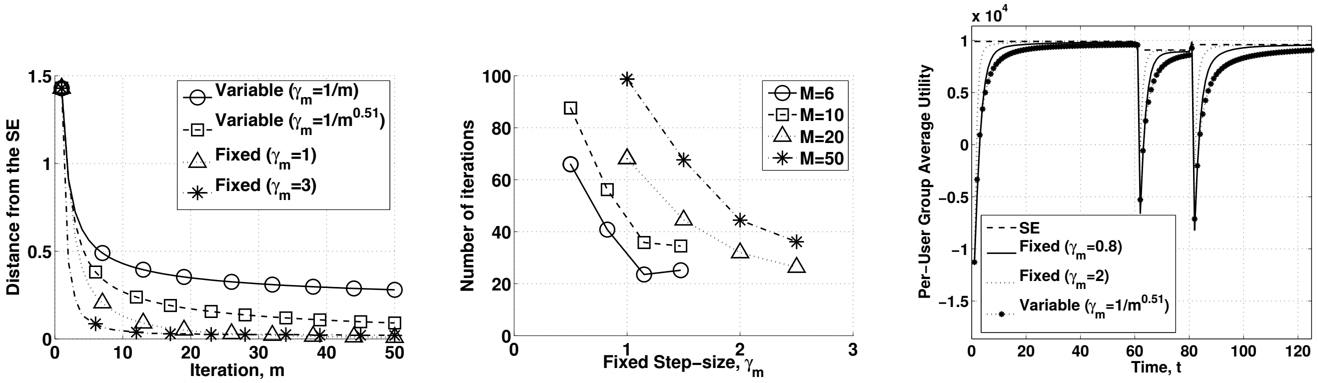
To measure the convergence speed of the proposed learning procedure, at each iteration we consider the normalized Euclidean distance between the bandwidth vector $\mathbf{b}(m)$ computed in (21) and the SE vector $\mathbf{b}^*$ as follows:

$$d(\mathbf{b}(m), \mathbf{b}^*) = \sqrt{\sum_{i \in \mathcal{S}} \left(\frac{|b_i(m) - b_i^*|}{B_i} \right)^2} \qquad (22)$$

In Fig. 11(a), we show how fast the proposed learning procedure converges to the unique SE of the game $\mathcal{G}_p^{(\mathcal{S})}$ when $M = 10$, for different step-size rules. Specifically, we consider both variable step-size (i.e., $\gamma_m = 1/m^{\beta}$) with $\beta \in \{0.51, 1\}$, and fixed step-size rules (i.e., $\gamma_m \in \{1, 3\}$). It is shown that fixed step-size rules converge faster than variable step-size rules. In addition, the convergence speed is faster when high values of the fixed step-size are considered, i.e., $\gamma_m = 3$. Recall that convergence of the learning procedure under variable step-size rules is ensured by Proposition 3. Unfortunately, the same is not true for fixed step-size rules, as in this case convergence to the SE cannot be proven analytically. It is worth noting that very large fixed step-size are prone to generate oscillations around the SE [6]. Therefore, to guarantee convergence to the SE while achieving a fast convergence speed, a variable step-size $\gamma_m = 1/m^{\beta}$ with $\beta = 0.51$ should be considered.

Finally, in Fig. 11(b), we show how many iterations the proposed learning procedure needs to reach the SE as a function of the step-size γ_m for different values of the number M of

[6] To avoid oscillations, if generated, approaches similar to "Search-then-converge" (STC) [36] can be effectively applied.



(a) (b) (c)

Fig. 11. a) Distance from the SE for different step-size rules; b) Number of iterations needed to reach the SE for different number M of VNF Servers and values of the step-size γ_m; c) Adaptability of the proposed learning procedure as a function of different step-size rules.

VNF Servers when we consider a fixed step-size rule. More in detail, we let the learning procedure run until the stopping condition is reached, i.e., $d(b_i(m), b_i^*) \leq 0.01$ for all $i \in \mathcal{S}$. As expected, an increase in the value of the step-size improves the convergence speed of the learning procedure. Furthermore, in Fig. 11(b) we show the scalability of the proposed learning procedure w.r.t. the number M of VNF Servers. It is important to note that an increase in the value of the step-size γ_m allows to improve the convergence speed of the learning procedure even when high number of VNF Servers are considered, e.g., $M = 50$. Thus, by properly increasing the value of the step-size, it is also possible to improve the scalability of the learning procedure.

Now, we investigate the adaptability of the proposed learning procedure when network parameters change over time. Specifically, we consider $M = 10$ VNF Servers which serve five different User Groups each of which is requesting a different function. Fig. 11(c) shows the per-User Group average VNF Server utility defined as $\frac{1}{|\mathcal{U}|M}\sum_{i\in\mathcal{S}}\sum_{p\in\mathcal{U}} U_{ip}^{(\mathcal{S})}$ at the SE and that achieved by using the learning procedure for different step-size rules as a function of time. For each User Group $p \in \mathcal{U}$, at time instant $t = 0$, both the games $\mathcal{G}_p^{(\mathcal{U})}$ and $\mathcal{G}_p^{(\mathcal{S})}$ are played and, as shown in Fig. 11(c), the SE is reached in few iterations. Then, at time instants $t = \{60, 80\}$ we simulate a network configuration change. More in detail, at $t = \{60, 80\}$ we randomly generate a new latency parameter configuration. Note that any change in the latency configurations pushes users in each User Group to re-distribute themselves according to the new network configuration through the evolutionary game $\mathcal{G}_p^{(\mathcal{U})}$. It follows that, to adapt to new network and user distribution conditions, and to evaluate the amount of bandwidth to be requested to the TO for each User Group, VNF Servers need to re-execute the game $\mathcal{G}_p^{(\mathcal{S})}$. As shown in Fig. 11(c), at time instants $t = \{60, 80\}$, users will re-arrange themselves differently, thus causing a deviation from the previous SE. Accordingly, the learning procedure is re-executed by VNF Servers and Fig. 11(c) shows that the system is able to quickly adapt to system fluctuations, i.e., the proposed learning procedure is able to reach the SE in few iterations. As expected, the convergence rate is faster in the case of fixed step-size rules. Specifically, the higher the step-size, the faster the convergence speed and adaptability of the learning procedure. However, even though the considered variable step-size rules show bad performance in terms of number of iteration needed to reach the equilibrium, recall that they assures convergence. Instead, the same does not hold for fixed step-size rules which are fast but their convergence to the equilibrium cannot be analytically proved.

E. TO revenue efficiency analysis

As introduced in Section II, an important aspect that deserves particular attention is related to the revenues generated by the TO. In the following, to measure the efficiency of the proposed distributed mechanism w.r.t. the revenues of the TO we consider the *efficiency ratio*, defined as the ratio between the revenues generated by the TO under our distributed market model and that achieved under monopolistic and centralized models, i.e., Cases A and B in Figs. 1(a), 1(b).

Specifically, the efficiency ratio in Case A and Case B is denoted as ξ_A and ξ_B, respectively, and defined as follows:

$$\xi_A = \frac{U_C^{(\mathrm{TO})}}{U_A^{(\mathrm{TO})}}, \quad \xi_B = \frac{U_C^{(\mathrm{TO})}}{U_B^{(\mathrm{TO})}} \tag{23}$$

where $U_A^{(\mathrm{TO})}$, $U_B^{(\mathrm{TO})}$ and $U_C^{(\mathrm{TO})}$ are defined in (6), (7) and (8), respectively. An efficiency ratio higher than or equal to 1 means that our proposed mechanism provides revenues to the TO which are either higher or equal to that achieved in centralized markets.

In Case A, the position of the unique centralized server is of extreme importance as it will determine the latency that network users will experience when connecting to that server and the cost parameter c. Therefore, to investigate the efficiency of the proposed mechanisms under different network configurations, we consider 50 possible latency and cost configurations. Specifically, the VNF Server position configuration and the cost parameters used by each VNF Server, that is c_{ip}, have been generated by using the same Gaussian distributions described in Section IV-D. In order the Case A to be comparable, also in this case we considered 50 simulation runs, where the latencies

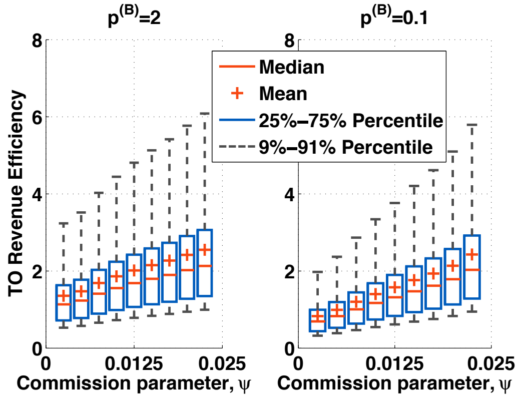


Fig. 12. Efficiency ratio ξ_A as a function of the commission parameter ψ for different values of the bandwidth-unit price p_b.

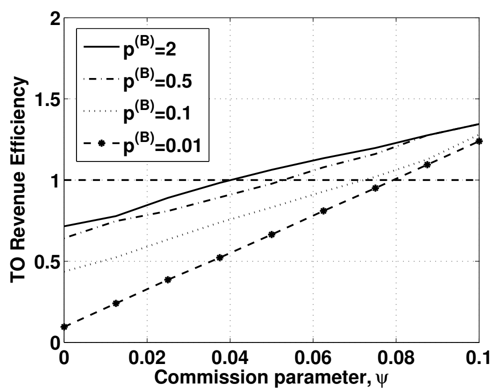


Fig. 13. Efficiency ratio ξ_B as a function of the commission parameter ψ for different values of the bandwidth-unit price p_b.

from the unique VNF Server and the costs, have been calculated as the average value of all the latencies and the costs in the same run for the Case C. For illustrative purposes, in the following we assume that the parameters $p_i^{(\mathcal{B})}$, $\hat{p}_i^{(\mathcal{F})}$ and ψ are fixed and equal for all VNF Servers $i \in \mathcal{S}$ and we consider a single User Group.

We consider all the above possible network configurations and, for each network configuration, we evaluate the efficiency ratio ξ_A. Obtained results are shown in Fig. 12 where we show ξ_A as a function of the commission parameter ψ for different values of the bandwidth-unit price $p^{(\mathcal{B})}$. Fig. 12 shows that, in most realizations, distributing VNF functions is more efficient than having a single server that manages and controls the VNFs. Also, it is shown that only under few network configurations the proposed mechanism is not efficient. Instead, in the majority of the cases high efficiency is obtained, $\xi_A > 1$, if compared to the centralized market scheme in Case A. Furthermore, an interesting result is related to the bandwidth-unit price p_b. Specifically, an increase in the value of p_b also increases the efficiency of the proposed distributed market model.

Instead, in Fig. 13, we consider Case B, where VNFs are executed on M servers which are owned by the TO, and we compare it with our proposed mechanism where the same servers are, instead, VNF Servers owned by customers. Accordingly, for each simulation run we have considered the same position and price configurations of the relative run of Case C. Fig. 13 shows the efficiency ratio ξ_B as a function of the commission parameter ψ for different values of the bandwidth-unit price p_b. The proposed mechanism is not efficient for the TO for small values of the commission parameter ψ. Specifically, when the commissions on VNF provisioning sent by VNF Servers is low, i.e., $U_C^{(\text{TO})} \approx \sum_{i \in \mathcal{S}} C_i^{(\mathcal{B})}$, the proposed mechanism fails in improving the revenues of the TO. Instead, it is worth noting that when small values of the commission parameter are considered, e.g., $4\% - 8\%$ of the overall revenues generated by the sale of VNFs, our proposed solution is more convenient for the TO than the centralized ones. This is an important aspect which show that the proposed model for the distribution of the VNF provisioning is not only profitable for VNF Servers which are now able to enter a new market, but it is also profitable for the TO. In fact, our distributed market allows the TO to receive payments from VNF Server w.r.t. both bandwidth requirements and commission fees on payments submitted by network users. Instead, Case B only allows payments from network users. Accordingly, by allowing third-party VNF Servers to access the VNF market, the TO can improve its revenues and also reduce the computational cost which is outsourced to those VNF Servers. Also, note that an increase in the value of the bandwidth-unit price p_b improves the efficiency of the proposed mechanism against Case B.

The numerical analysis of the proposed system shows that the three stakeholders can take advantage of the framework; indeed the TO can simplify the Orchestration mechanism by using the distributed scheme and employing external servers for VNFs provisioning. The VNF Servers on their side are sellers and can increase their economic benefit. Finally, Users can also increase their benefit in terms of price reduction and performance improvement.

V. Conclusions

In this paper we have discussed how game-theoretic tools can be effectively used to address the problem of distributed management, resource allocation and orchestration of softwarized networks.

Specifically, we exploited hierarchical game theory to design a distributed SDN/NFV system where VNF Servers participate in the VNF market as sellers of VNFs. The interactions among VNF Servers and Users requesting VNFs have been modeled as a two-stage Stackelberg game where the former act as the leaders and the latter as the followers of the game. Uniqueness of the SE has been proved, and a reinforcing learning procedure which provably converges to the unique SE has been proposed. We accounted for imitative and social behaviors of Users and we used the replicator dynamics equation from evolutionary game theory to model their interactions. Furthermore, a closed-form equilibrium condition has been derived.

Through the exploitation of game theory, we have shown that a distributed framework results beneficial for all the involved stakeholders. On the one hand, the simplified orchestration and the possibility to offload VNFs on third-parties VNF Servers is beneficial to the TO. On the other hand, VNF Servers have economic benefit by participating to the VNF Market as sellers. Finally, Users can choose the VNF Servers that most fit their needs.

The numerical analysis carried out has proved the feasibility and efficiency of the proposed game-theoretic framework. In particular, the results obtained show that the framework is scalable and rapidly adapts to network changes.

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VI. APPENDICES

APPENDIX A. PROOF OF PROPOSITION 3

Proof: For the sake of clarity, in the following we will omit the subscript p which identifies the User Group $p \in \mathcal{U}$. The mean dynamics of (21) is

$$\begin{cases} \dot{z}_i = v_i(\mathbf{b}) \\ b_i = B_i \frac{e^{z_i}}{1+e^{z_i}} \end{cases} \tag{24}$$

At any given time t, let $\mathbf{b}(t)$ be a solution for (24). In system theory, such solution is often referred to as *solution orbit* or *trajectory* of the system. In the following, we show that i) $\mathbf{b}(t)$ converges to $\mathbf{b}^*$ as $t \to +\infty$, and ii) (21) is an *asymptotic pseudo-trajectory* (APT) [38] for the mean dynamic (24), and converges to $\mathbf{b}^*$ if some mild conditions on the step-size are satisfied.

From Proposition 2, we have that $U_i^{(\mathcal{S})}(\mathbf{b})$ is a strictly concave function in b_i. Therefore, $v_i(\mathbf{b})(b_i - b_i^*) < 0$ for all $b_i \in [0, B_i]$ by definition. By exploiting this latter result, it can be shown that the function $V(\mathbf{b})$ defined as

$$V(\mathbf{b}) = \sum_{i \in \mathcal{S}} B_i \ln\left(\frac{B_i - b_i^*}{B_i - b_i}\right) + b_i^* \ln\left(\frac{b_i^*}{b_i} \cdot \frac{B_i - b_i}{B_i - b_i^*}\right) \tag{25}$$

is a *strict Lyapunov function* for (24). In fact, we have that $\dot{V} = dV(\mathbf{b})/dt = \sum_{i \in \mathcal{S}} v_i(\mathbf{b})(b_i - b_i^*) < 0$, $V(\mathbf{b}^*) = 0$ and $V(\mathbf{b}) > 0$ for all $\mathbf{b} \neq \mathbf{b}^*$. It can be shown that $V(\mathbf{b})$ is *radially unbounded*, i.e., $V(\mathbf{b}) \to \infty$ when $\|\mathbf{b}\| \to \infty$. Therefore, the equilibrium point $\mathbf{b}^*$ is also *globally asymptotically stable* (GAS), which implies that $\mathbf{b}(t)$ converges to $\mathbf{b}^*$ as $t \to +\infty$.

Now, we prove the second part of the proposition which consists in showing that also the discrete-time algorithm asymptotically converges to the equilibrium. By decoupling (24), we get

$$\dot{b}_i = \frac{db_i}{dt} = b_i \left(1 - \frac{b_i}{B_i}\right) v_i(b) \tag{26}$$

The latter result will be useful to show that the discrete-time algorithm tracks the continuous-time system up to a bounded error that asymptotically tends to 0 as i increases.

A second-order Taylor expansion of (21) leads to

$$b_i(m+1) = b_i(m) + \gamma_m b(m)\left(1 - \frac{b_i(m)}{B_i}\right) v_i(\mathbf{b}(m)) + \frac{1}{2}\mu\gamma_m^2 \quad (27)$$

for some bounded μ . Note that μ is bounded because $\frac{\partial}{\partial b_i} v_i(\mathbf{b})$ is bounded by definition. Intuitively, (27) is the discrete version of (26) up to a bounded error. Since, by assumption, $\sum_m \gamma_m^2 < \sum_m \gamma_m = +\infty$, results in [38] show that $b_i(m)$ is an APT for (24).

It still remains to prove that $b_i(m) \rightarrow b_i^*$. By decoupling z_i and b_i, we obtain $z_i = \ln\left(\frac{b_i}{B_i - b_i}\right)$. By rewriting $V(\mathbf{b})$ in terms of $\mathbf{z}$, we obtain $V(\mathbf{z})$. By considering a Taylor expansion of $V(\mathbf{z})$, we obtain:

$$V(\mathbf{z}(m+1)) = V(\mathbf{z}(m)) + \gamma_m \sum_{i\in\mathcal{S}} (b_i(m) - b_i^*)\, v_i\,(b_i(m)) + \frac{1}{2}\mu'\gamma_m^2$$

for some bounded $\mu' > 0$.

Since $\mathbf{b}^*$ is GAS, it follows that $\mathcal{B}$ is a basin of attraction for $\mathbf{b}^*$. Therefore, there must exist a compact set $\mathcal{L} \subset \mathcal{B}$ containing $\mathbf{b}^*$, where $\mathcal{B}$ is the strategy set of the game $\mathcal{G}^{(\mathcal{S})}$. So, if we prove that there also exists a large enough m' such that $\mathbf{b}(m') \in \mathcal{L}$, then, the proof is concluded. Assume ad absurdum that such m' does not exists. Recall that $v_i(\mathbf{b})(b_i(m) - b_i^*) < 0$ by definition. Therefore, it must exist some $\beta > 0$ such that $\sum_{i\in\mathcal{S}} v_i(\mathbf{b})(b_i(m) - b_i^*) \leq -\beta$ for a large enough m. It follows that

$$V(\mathbf{z}(m+1)) \leq V(\mathbf{z}(m)) - \gamma_m\beta + \frac{1}{2}\mu'\gamma_m^2 \quad (28)$$

which yields to

$$V(\mathbf{z}(m+1)) \leq V(\mathbf{z}(0)) - \beta\sum_m \gamma_m + \frac{1}{2}\mu'\sum_m \gamma_m^2 \quad (29)$$

By assumption $\sum_m \gamma_m^2 < \sum_m \gamma_m = +\infty$. Thus, (29) leads to $V(\mathbf{z}(m+1)) \leq -\infty$, which is a contradiction as $V(\mathbf{z})$ is lower bounded by construction. Therefore, [38] ensures that there must exist m' such that $\mathbf{b}(m') \in \mathcal{L}$ and $\lim_{m\rightarrow+\infty} \mathbf{b}(m) = \mathbf{b}^*$, which concludes the proof.

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